The Kalman Filter

The problem to be solved

- Real time tracking of a dynamical system.
  - position estimation
  - bacterial population
- Accessible information of system state:
  - Noisy observations
  - Model predictions

Explanation

- The observations:
  - observations of state variables with noise
  - direct or indirect measurements
- The model
  - time evolution of system (differential equations)
  - predicts system state next time step
- Combining the two
  - A corrected prediction
  - Minimization of error of state estimate

The details: prediction step

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_k$$

The details: correction step

$$
\begin{aligned}
\tilde{\mathbf{y}}_k &= \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \\
\mathbf{S}_k &= \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k \\
\mathbf{K}_k &= \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1} \\
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\tilde{\mathbf{y}}_k \\
\mathbf{P}_{k|k} &= (I - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}
\end{aligned}
$$

A simple example...

References

- Wikipedia: https://en.wikipedia.org/wiki/Kalman_filter
- R.E. Kalman : A New Approach to Linear Filtering and Prediction Problems (1960)