Bayes factors

Evaluating evidence for models

Kass & Raftery 1995

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Imagine that.....

You have data and

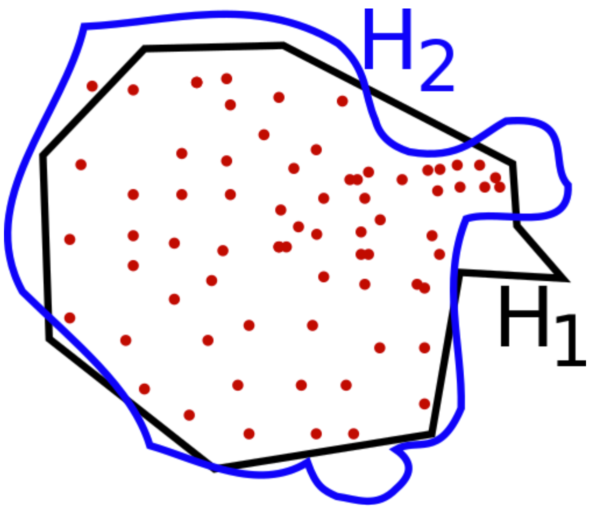
- Competing theories
- Prior opinion on theories

You want to

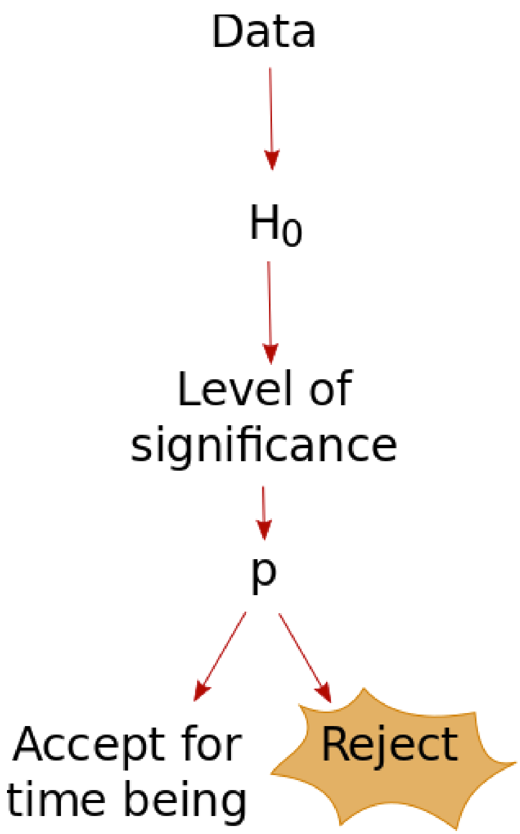
- Evaluate evidence *for* each theory

or...

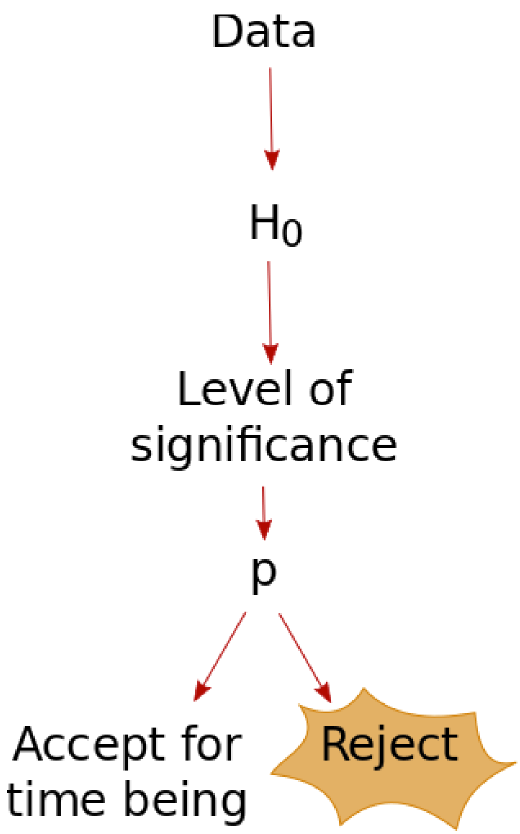
- You just want an estimate of a variable *r* no matter which theory is correct

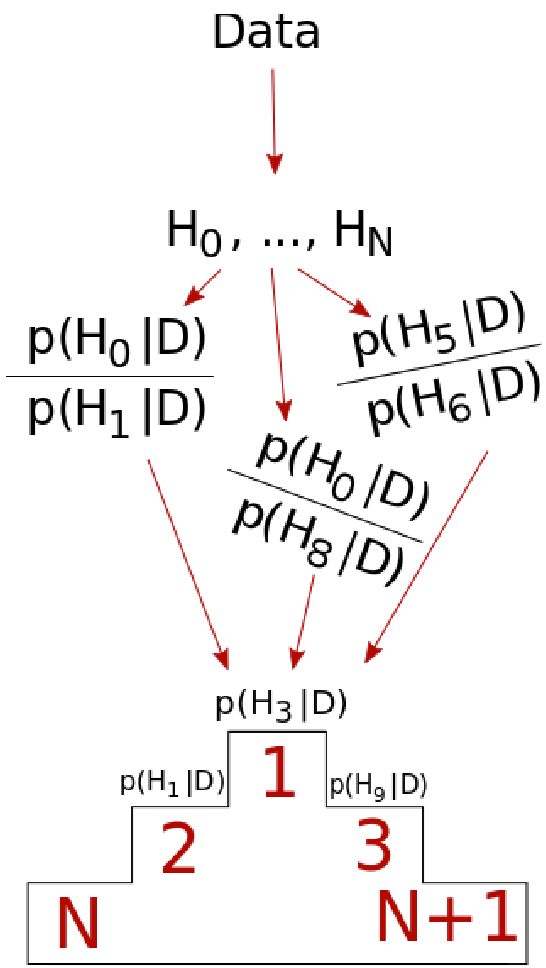


Mindset: Bayes factors



Mindset: Bayes factors





Evaluate evidence *for* hypothesis!

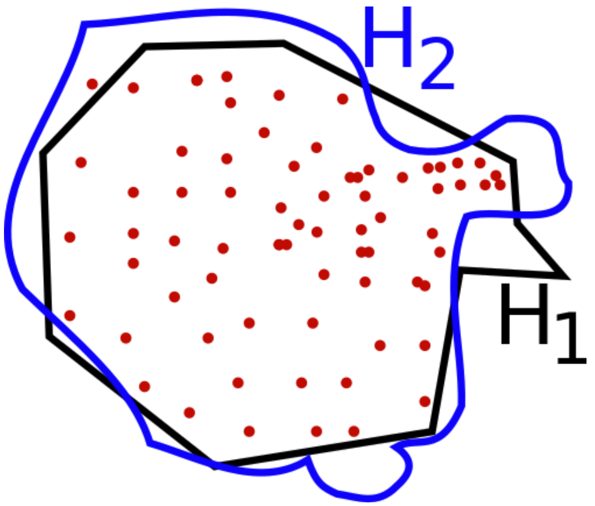
The math!

Bayes Theorem:

$$p(H_i|D) = \frac{p(D|H_i)p(H_i)}{p(D|H_1)p(H_1) + p(D|H_2)p(H_2)}$$

Taking the ratio $p(H_1|D)/p(H_2|D)$ gives

$$\frac{p(H_1|D)}{p(H_2|D)} = \frac{p(D|H_1)}{p(D|H_2)}\frac{p(H_1)}{p(H_2)} = B_{12}\frac{p(H_1)}{p(H_2)}.$$



Bayes factor “transforms”
prior odds to posterior odds

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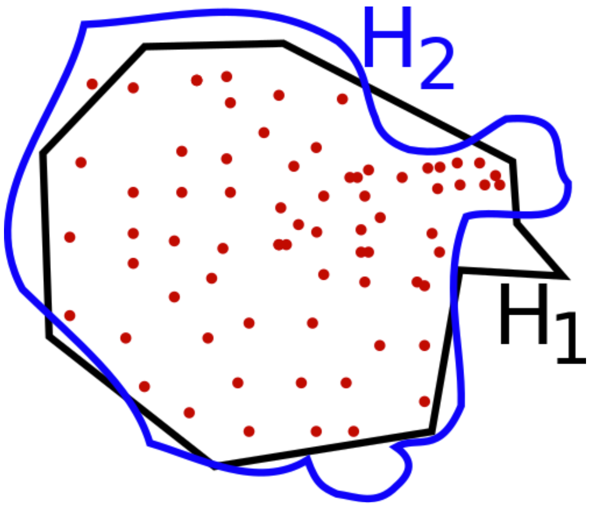


Bayes factor "transforms" prior odds to posterior odds

B_{10}	Evidence against H_0
1 to 3	Not worth more than a bare mention
3 to 20	Positive
20 to 150	Strong
> 150	Very strong

TABLE I. Interpretation of Bayes factor $B_{10} = 1/B_{01}$

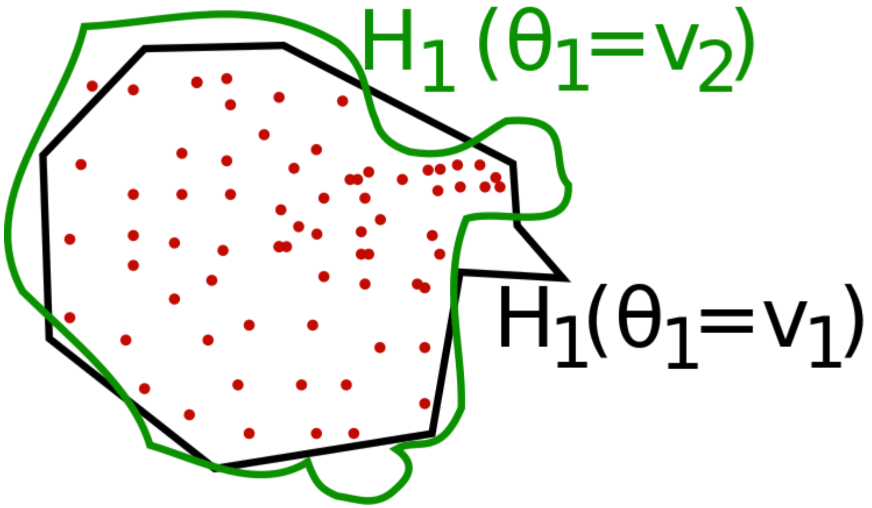
Two important features...

1. If model contains parameters:

$$p(D|H_i) = \int p(D|\theta_i, H_i)p(\theta_i|H_i)d\theta_i$$

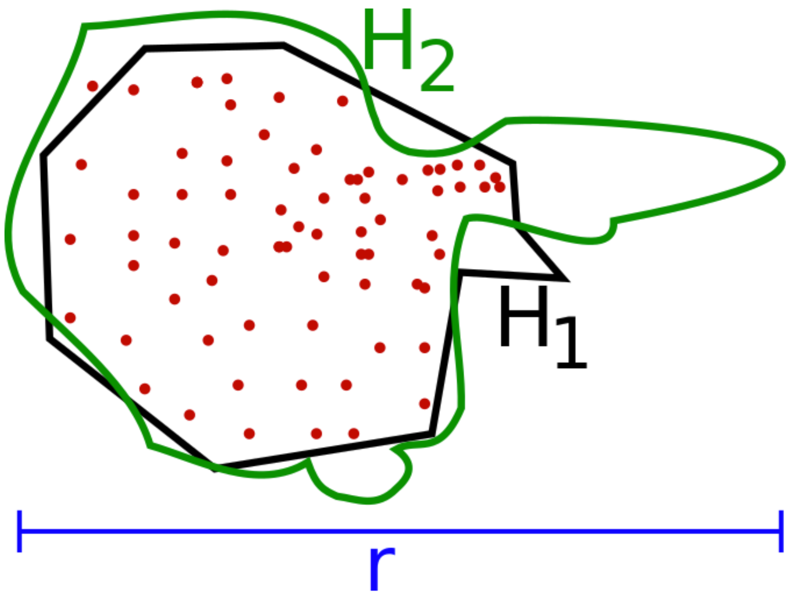
Effectively: Punishment for extra parameters

2. $p(\theta_i|H_i)$ must be chosen



No need for choosing model

Maybe the exact theory is not important.. But you want to estimate *r*



$$p(H_i|D) = B_{i0}\frac{P(H_i)}{P(H_0)} / \sum_{m=0}^{n} B_{m0}\frac{P(H_m)}{P(H_0)}$$

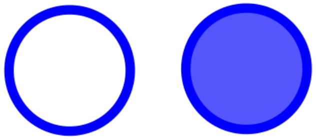
Use these as weights..

$$p(r|D) = \sum_{i=0}^{n} p(r|D, H_i)p(H_i|D)$$

This estimate of r takes model uncertainty into account!

Example 1: Bacteria

Bacteria: 2 strains develop "AUD" with different probabilities



$p_1 < p_2$

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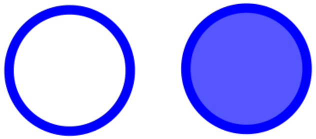

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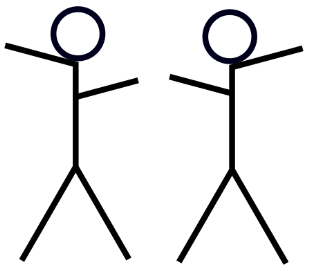
Researchers: Predict that probabilities are equal if trait A is selected for in "1"

HUGE NEWS!!



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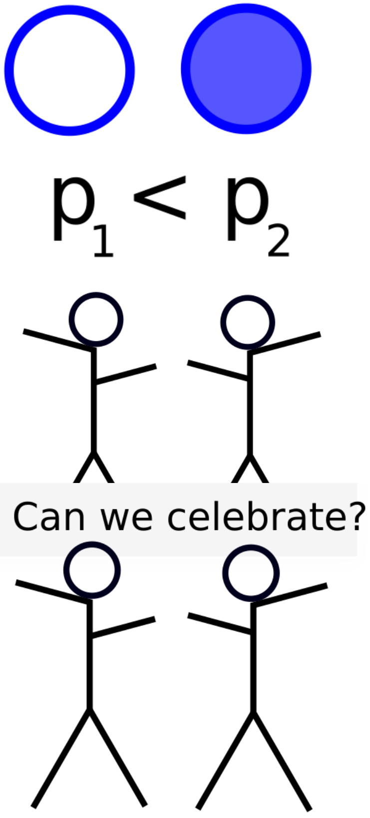

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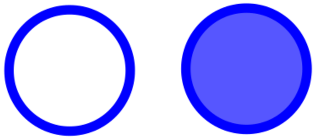

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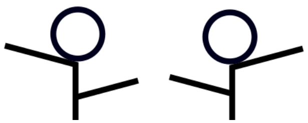
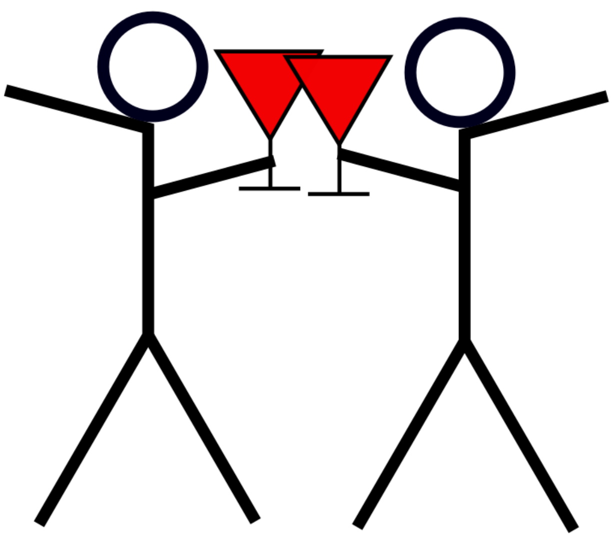


Finally!



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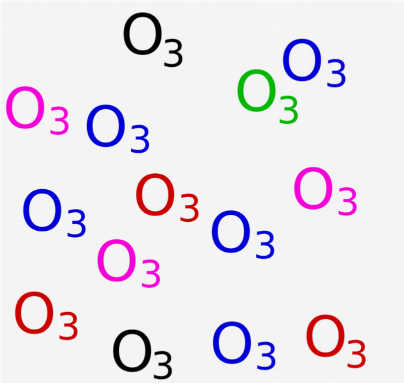
Bayes factor: $B_{10} = 0.065$

("Positive" for H_0)

Example 2: Ozone in TX



Houston, TX: Often occuring high O_3 levels.

Now: Evaluate whether measures taken decreased levels.

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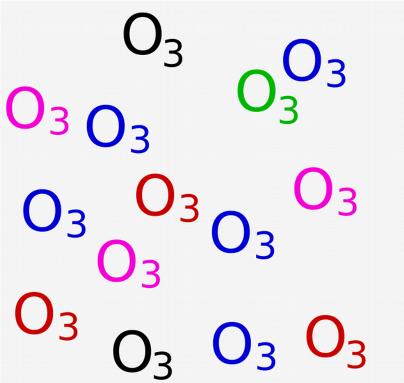

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<u>Now:</u> Evaluate whether measures taken decreased levels.

H_0	No decrease	$O(t) = O_{\text{past}}$
H_1	Gradual decrease	$O(t) = O_{\text{past}} e^{-kt}$
H_2	Abrupt decrease	$O(t) = O_{\text{past}}\Theta(t_0 - t) + O_{\text{new}}\Theta(t - t_0)$

Such functions are usually difficult to handle
Not with Bayes Factors!

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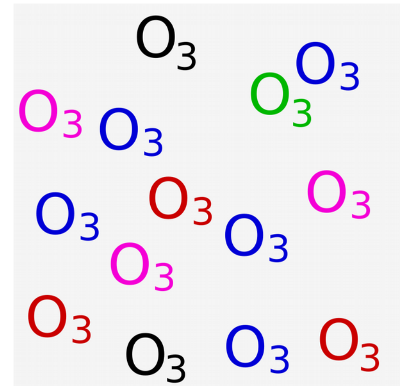

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$$B_{10} = 0.02$$

$$B_{20} = 2.75$$

$$B_{21} = 135$$

Conclusion: *If* decrease, probably abrupt. This could be due to sudden advances in measurement technology, giving less extreme events.
Researchers found such recent advances to have occured.

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Bayes factors might be the BAYEST WAY TO GO!

(sorry)