Optimal design, Robustness and Risk Aversion

M.E.J. Newman, Michelle Girvan and J. Doyne Farmer,
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From HOT to COLD

- From «Not Normal»: Why do systematic errors follow a power law distribution?
- HOT: «Highly optimized tolerance»
  - High performing systems robust to anticipated perturbations, but fragile to rare ones.
  - Example: Mice (HOT) vs. humans (COLD), rare perturbation: volcano
- The probability of catastrophic failure
  - Gambler's ruin
- Objective of paper:
  - Derive power law for HOT models
  - Suggest an alternative model (COLD)
- Hypothesis: Optimize on utility instead of yield.

Yield Optimization in a Forest

- Forester, random spark and firebreaks
- Trees in patches of s(**r**)
- Spark distribution of p(**r**)
- Cost of firebreaks in terms of yield, F

$$Y = 1 - s(\mathbf{r}) - F$$

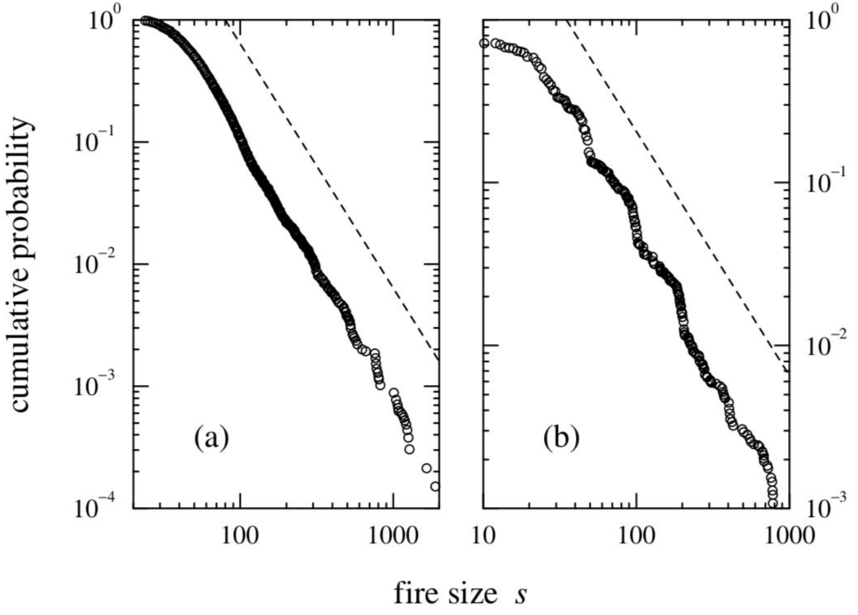
$$Y = 1 - \int p(\mathbf{r}) s(\mathbf{r}) d^d r - agd \int s(\mathbf{r})^{-1/d} d^d r$$

Power laws: Where do they come from?

- Probability distribution of fire sizes:

$$\rho(s) = p(\mathbf{r})\frac{d^d r}{ds} = -ag\frac{d+1}{d}p(\mathbf{r})\frac{d^d r}{dp}s^{-(2+1/d)}$$

$$= C \cdot p(\mathbf{r})\frac{d^d r}{dp} \cdot \text{"}powerlaw\text{"}$$



- For 2D Gaussian distribution of the sparks - A power law!

$$\rho(s) = 3\pi\sigma_x\sigma_y ag s^{-5/2}.$$

Risk aversion and truncation

- How do we incorporate risk aversion?
- From economics: utility functions!

$$u(s) = \frac{(1-s)^\alpha}{\alpha}$$

- Maximizing average utility, with fixed F (lambda is Lagrange multiplier):

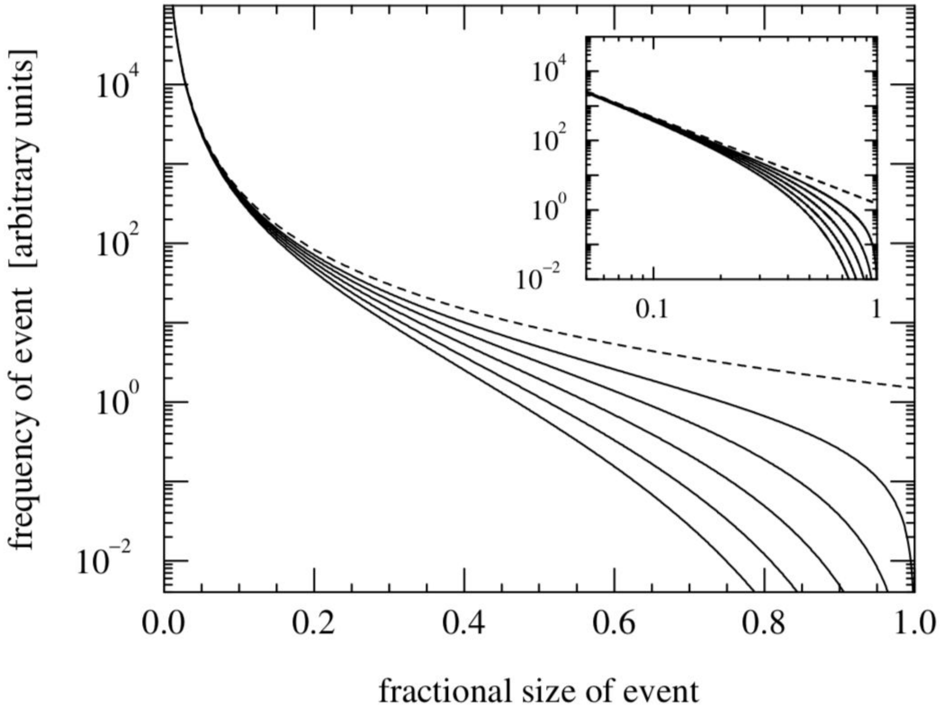
$$U = \int p(\mathbf{r}) u(s(\mathbf{r})) d^d r,$$

$$\frac{dp}{ds} = \lambda \frac{(\alpha + 1/d)s - (1 + 1/d)}{(1-s)^\alpha s^{2+1/d}}$$

- Alpha = 1, then reduced to HOT
- Alpha < 0 is desirable

COLD fixes tails

- Truncated heavy tails
  - Dotted = HOT: probability is small but eventually...
  - Alpha < 0: zero probability of complete ruin



Conclusion

- Very small decrease in yield, but substantial increase in robustness
  - Smaller patches in COLD than in HOT
  - Also robust to rare perturbations
- COLD truncates heavy tails
- COLD designs can give more consistent measurements

Thank you!

References, figures from [3]:

[1] David C. Bailey. Not Normal: the uncertainties of scientific measurements. 2016.

[2] J. M. Carlson and John Doyle. Highly optimized tolerance: Robustness and design in complex systems. *Phys. Rev. Lett.*, 84:2529–2532, Mar 2000.

[3] M. E. J. Newman, Michelle Girvan, and J. Doyne Farmer. Optimal design, robustness, and risk aversion. *Phys. Rev. Lett.*, 89:028301, Jun 2002.