Bayesian Inference of a Finite Population Mean Under Length-Biased Sampling

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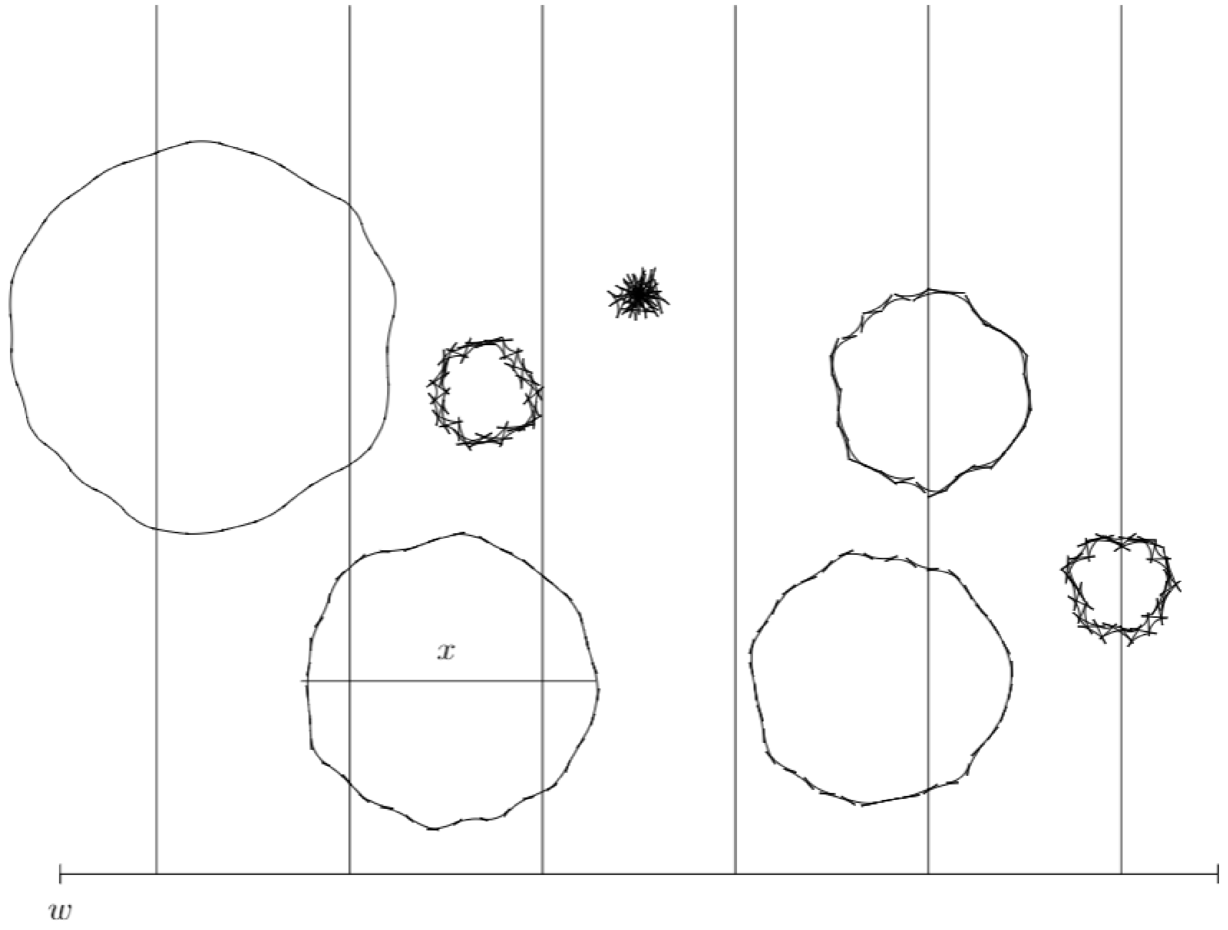
Estimating regrowth in a quarry

- 2 sets of three transects
- We count n from a finite population N
- Sampling biased towards large x



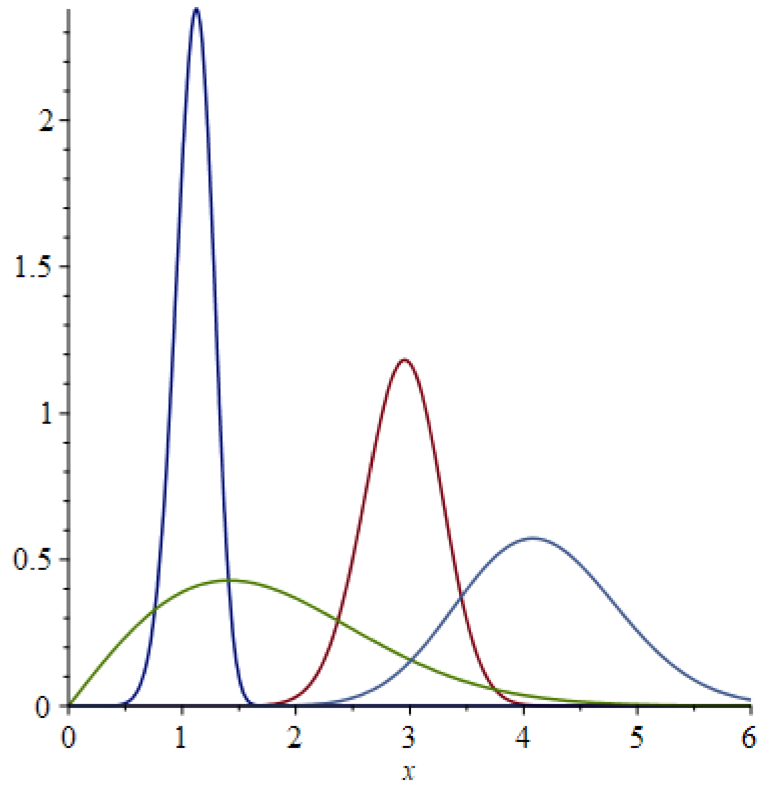
Bayesian methodology

- Distribution of shrubs as a function of width (GG)

$$f(x|\alpha, \beta, \gamma)$$

- Probability of counting, given width x

$$P(x|I=1) = \frac{\frac{x}{w} f(x)}{\int \frac{x}{w} f(x) \mathrm{d}x}$$



- Bayes→Generalised gamma with new parameters.

$$\alpha_{bias} = \alpha + \frac{1}{\gamma}, \qquad \beta_{bias} = \beta, \qquad \gamma_{bias} = \gamma$$

Estimating the finite size distribution

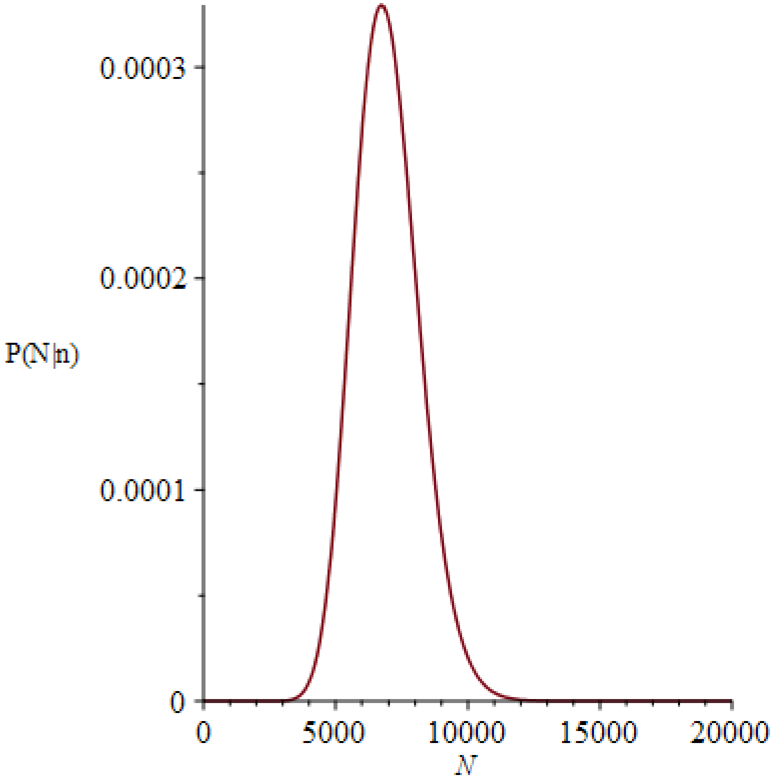
- Best estimate of N (Horvitz-Thompson)

$$\hat{N} = w \sum_{i=1}^{n} \frac{1}{x_i}$$

- Assuming

$$P(n_i|N_i) \sim Binomial(N_i, \mu_0)$$

- Using Bayes yields negative binomial distribution



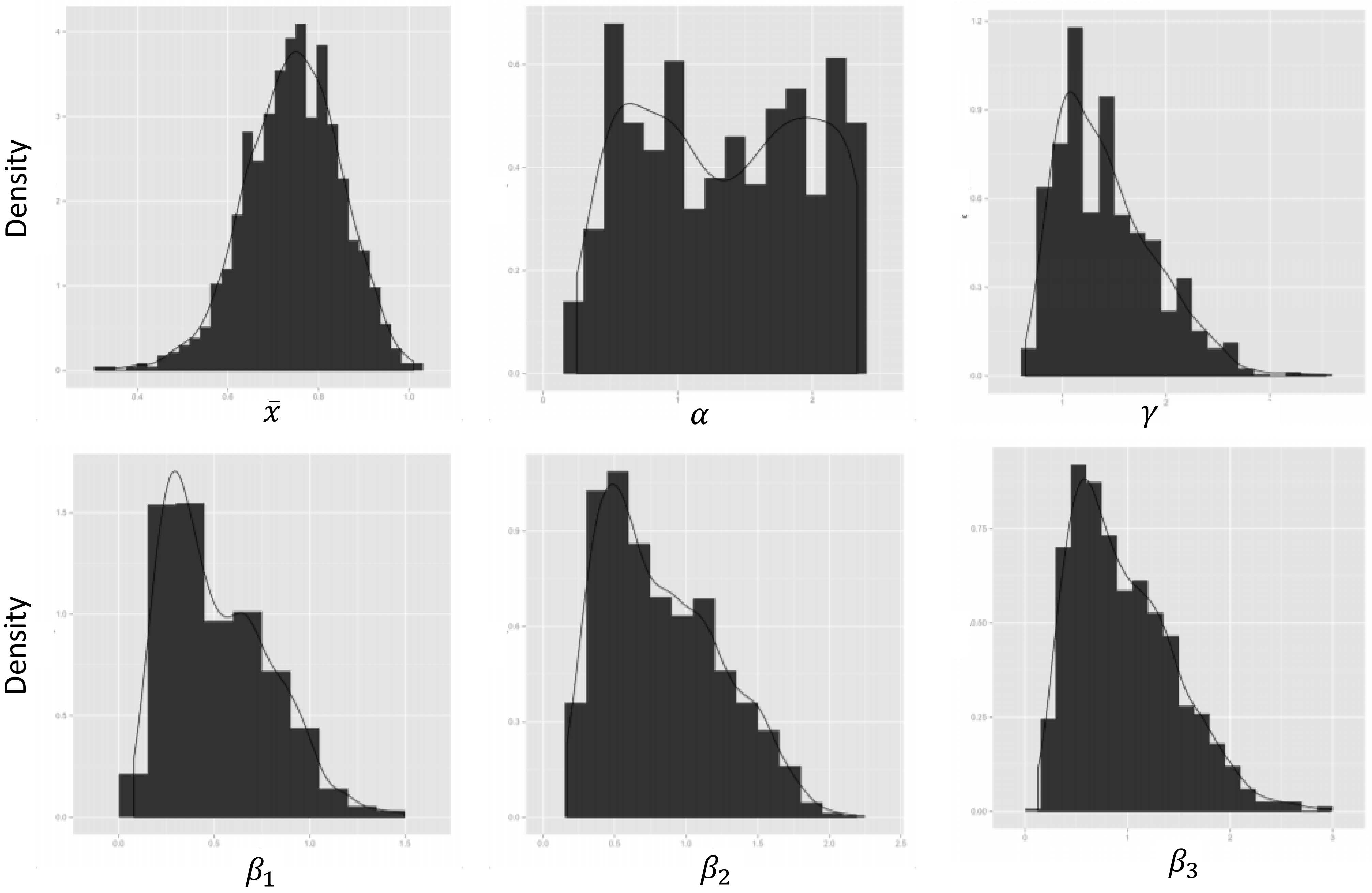
Parameter estimation

- Bayes applied to find posterior for parameters

$$\pi(\alpha, \beta, \gamma | x) = \frac{GG(x|\alpha, \beta, \gamma)\pi(\alpha, \beta, \gamma)}{\int GG(x|\alpha, \beta, \gamma)\pi(\alpha, \beta, \gamma)\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma}$$

- Noninformative priors used
  - Reduces effects from over fitting.

- Can be sampled using Markov-Chain MC or similar sampling schemes


Density
Density
$\bar{x}$
α
γ
β_1
β_2
β_3

Summary

- Biased model fit yields better LH than unbiased
- Better sampling algorithms makes the sampling more effective
- Plans to include covariates in the analysis