Optimization by Simulated Annealing

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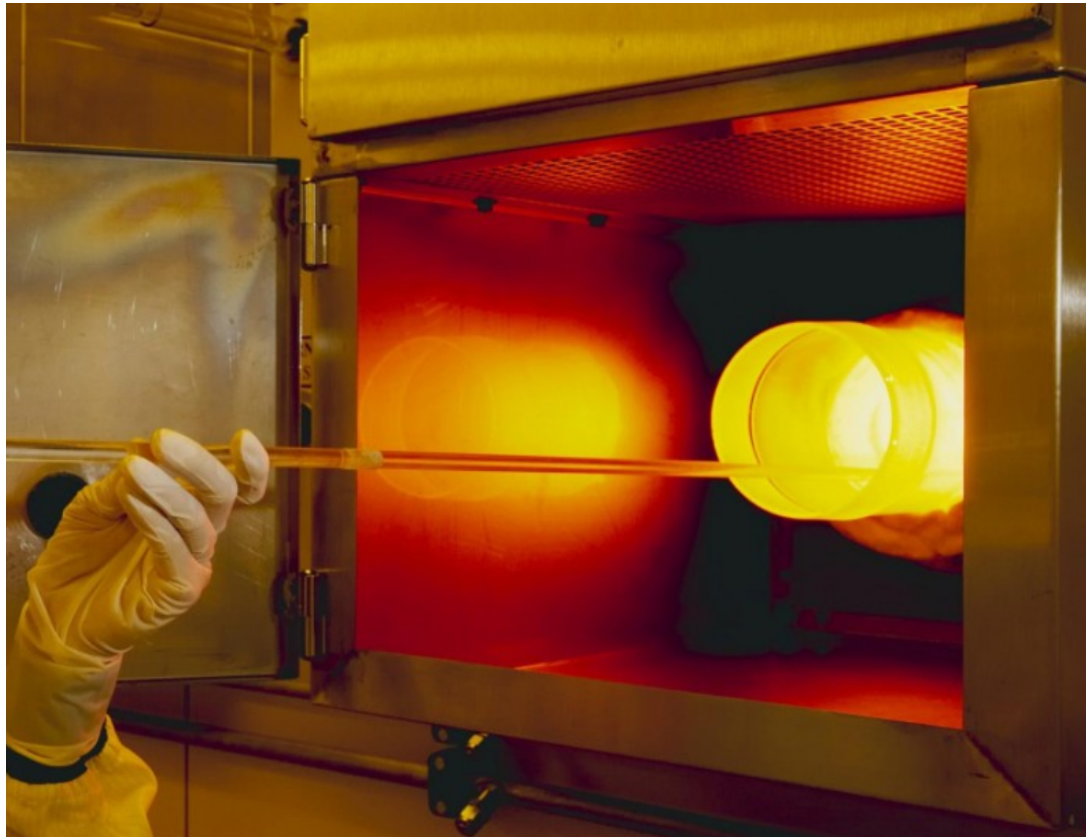
Inspiration – Annealing and Statistical mechanics

- Slowly decreasing temperature

  → Stronger alloy
- Minimize a cost function by slowly reducing the ‘temperature’
- Proposed in “Optimization by Simulated Annealing”, 1983



Annealing

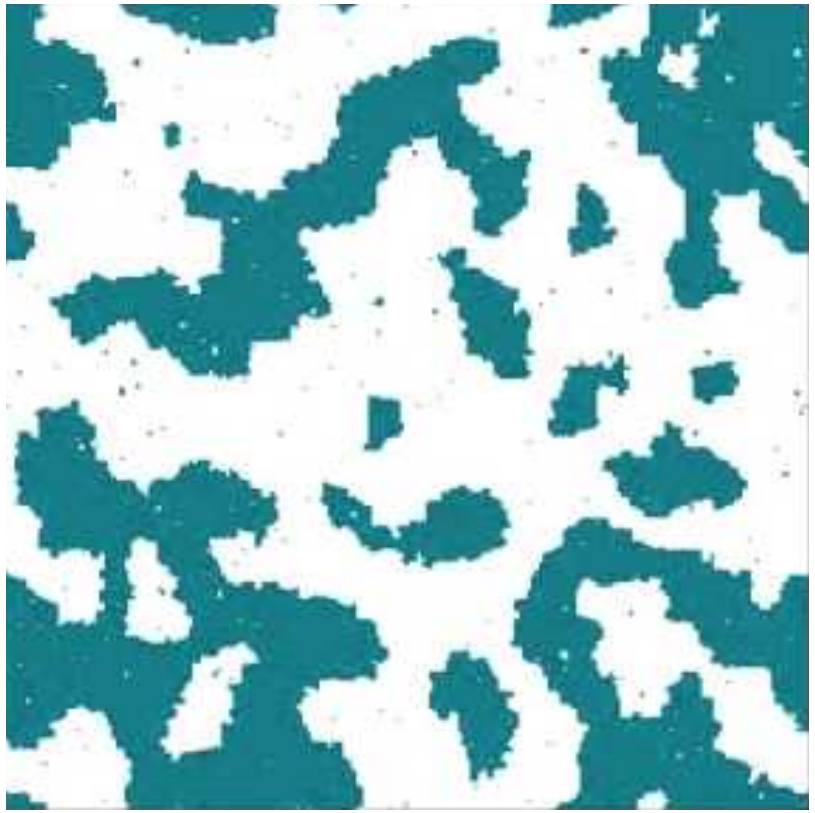
http://www.turingfinance.com/simulated-annealing-for-portfolio-optimization/

Statistical Mechanics

Anti-aligned spins → higher energy

Probability of finding system in certain state is proportional to its Boltzmann factor

$$P_i = \frac{1}{Z} \exp\left(-\frac{E_i}{k_{\mathrm{B}}T}\right)$$



Magnetization in a metal
https://www.youtube.com/watch?v=PWUTBnvGegg

The Algorithm

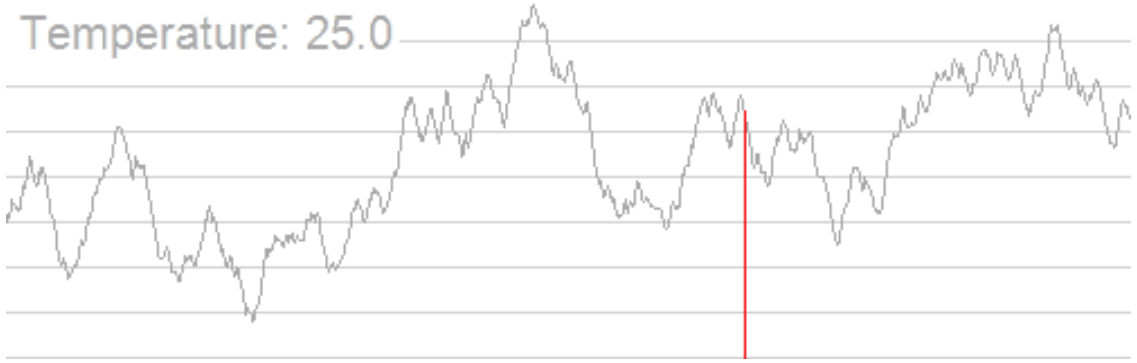
Define cost and temperature functions E and T

Propose a candidate change to system

Accept change with probability

$$P_{\text{transition}} = \begin{cases} 1, & \text{if } E_{\text{new}} < E_{\text{old}} \\ \exp(-\Delta E/T), & \text{if } E_{\text{new}} \geq E_{\text{old}} \end{cases}$$

Lower temperature and repeat



From: https://en.wikipedia.org/wiki/Simulated_annealing

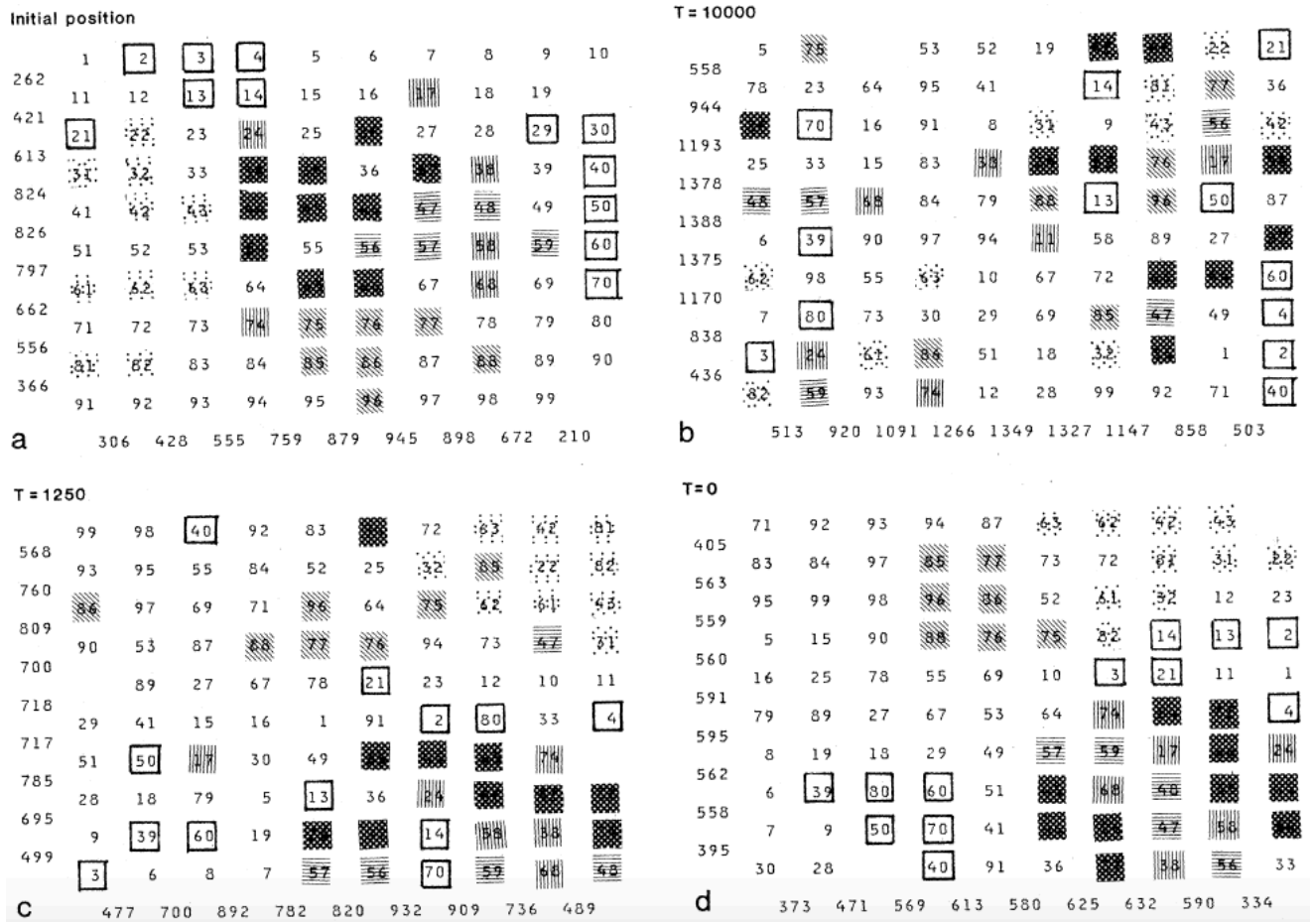
Applications

Placement of chips on module

Minimize signal propagation time without creating too congested regions

Swapping two chips

Constraints on distance for low temperatures

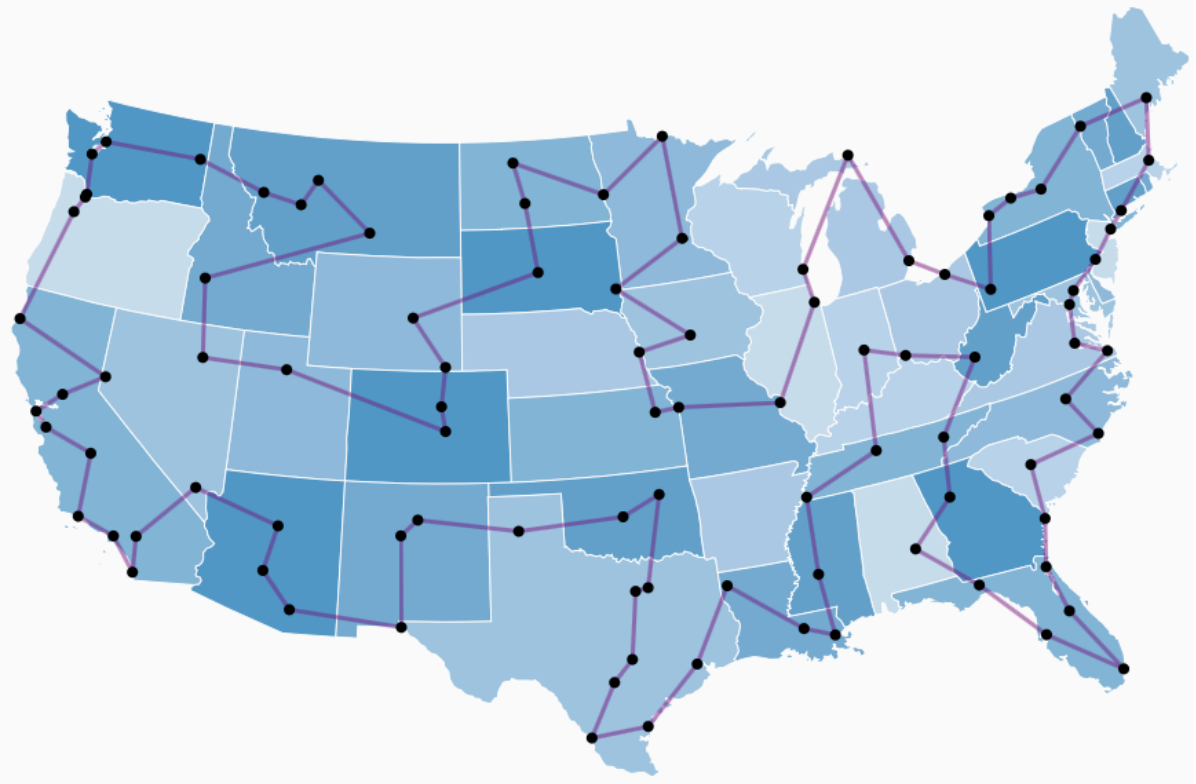


From: Optimization by Simulated Annealing, S. Kirkpatrick, C. D. Gelatt, M. P. Vecchi

Travelling Salesman

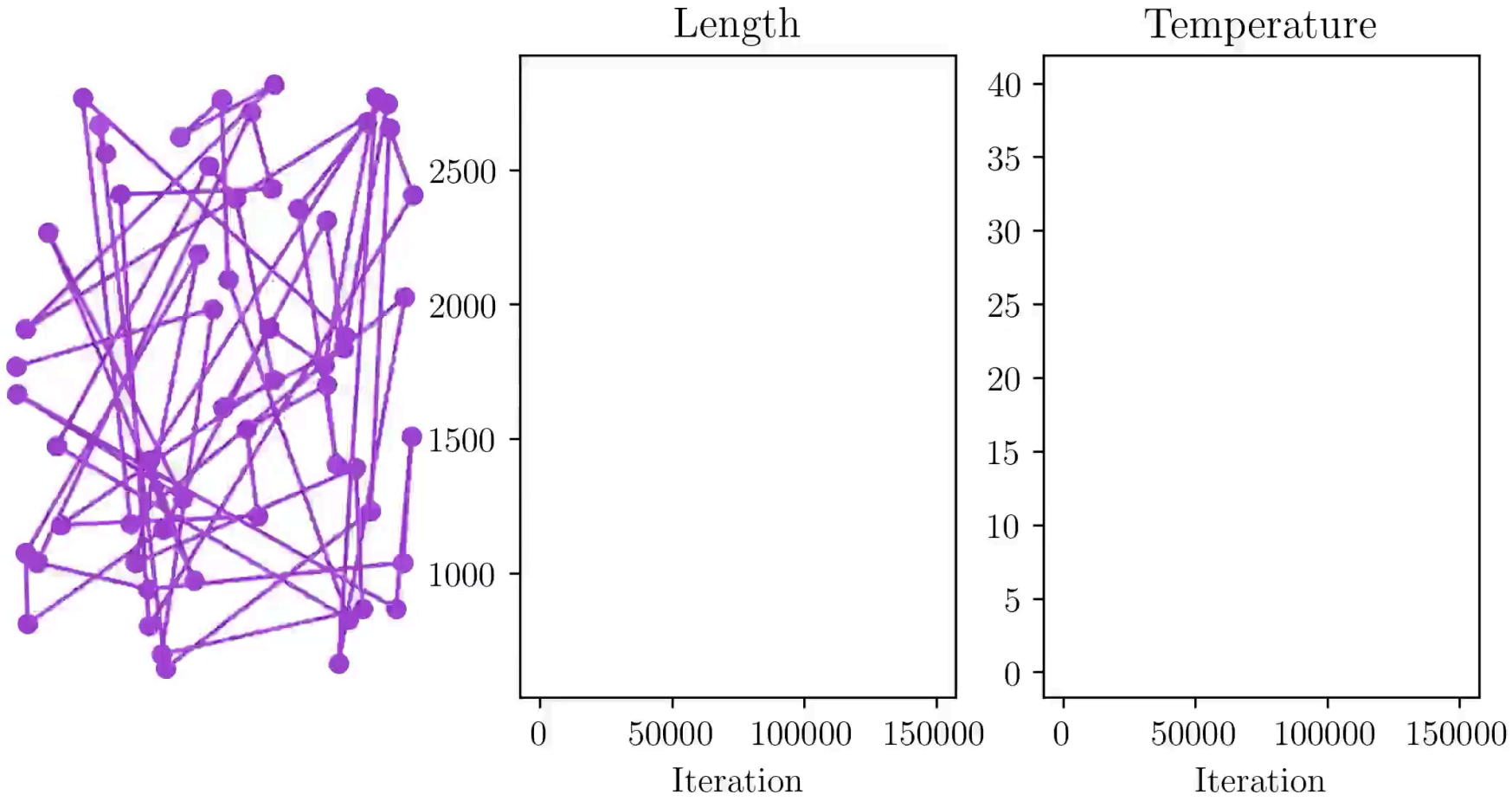
Given a list of cities, what is the shortest route that visits each city once before returning home?

Brute force solution has time complexity $\mathcal{O}(n!)$



From: http://examples.gurobi.com/traveling-salesman-problem/

Travelling Salesman



Travelling Salesman

Initial random route

60 cities randomly distributed

2500 temperatures

60 swaps at each temperature

Near optimal final route