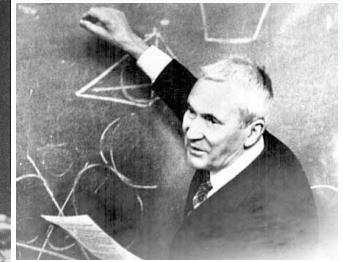
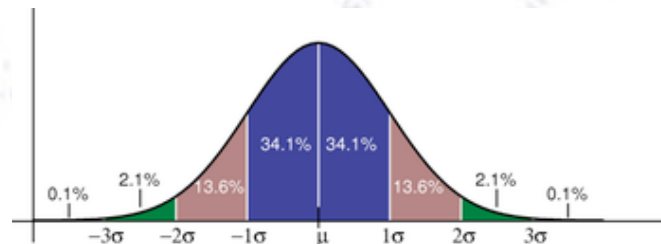


Applied Statistics

Correlations



Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

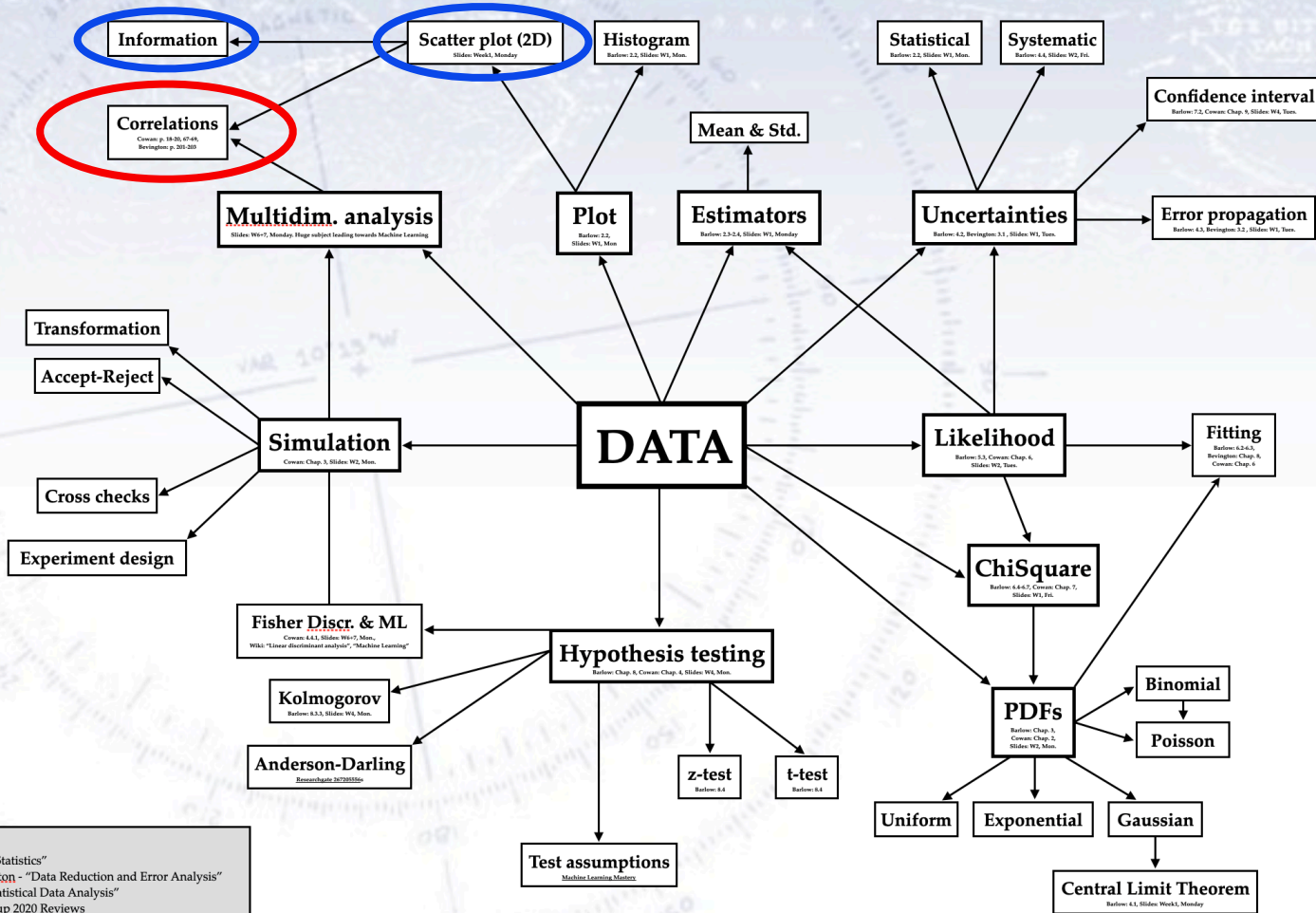
Correlations

Applied Statistics

Describe data (Quantify & Visualise)

Overview of subjects

Version 1.2, 6. Nov. 2020



Simulate data (Design & Cross Check)

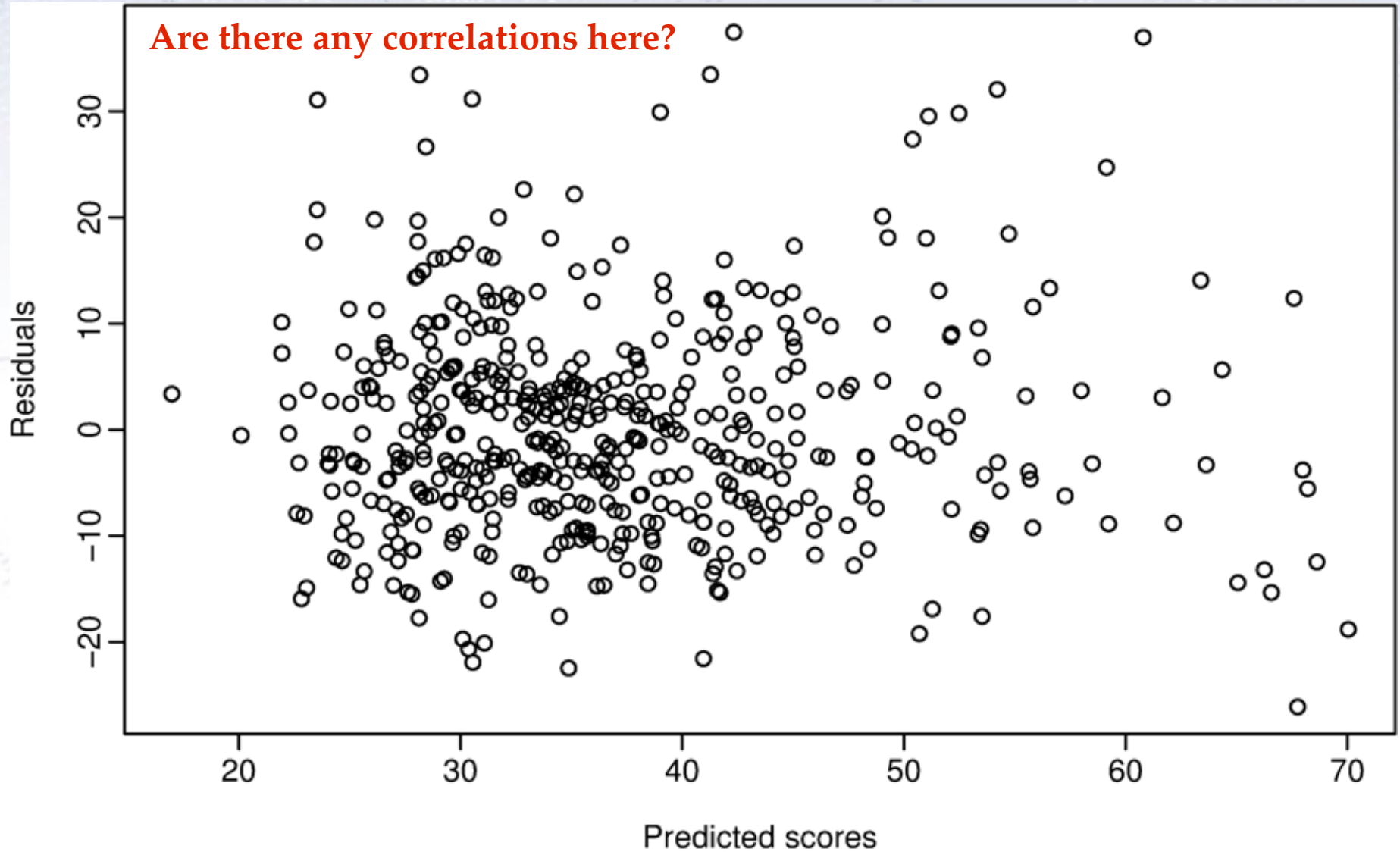
Model data (Predict & Understand)

References:
 Barlow: R. J. Barlow - "Statistics"
 Bevington: P. H. Bevington - "Data Reduction and Error Analysis"
 Cowan: G. Cowan - "Statistical Data Analysis"
 PDG: Particle Data Group 2020 Reviews
 Slides: T. C. Petersen - "Applied Statistics 2020" course (W = Week)
 Wiki: Good reference for ALL subjects (only specified when essential)
 SciPy: SciPy Statistical Functions and (very brief) documentation

Test hypotheses on data (Decide)

Correlation

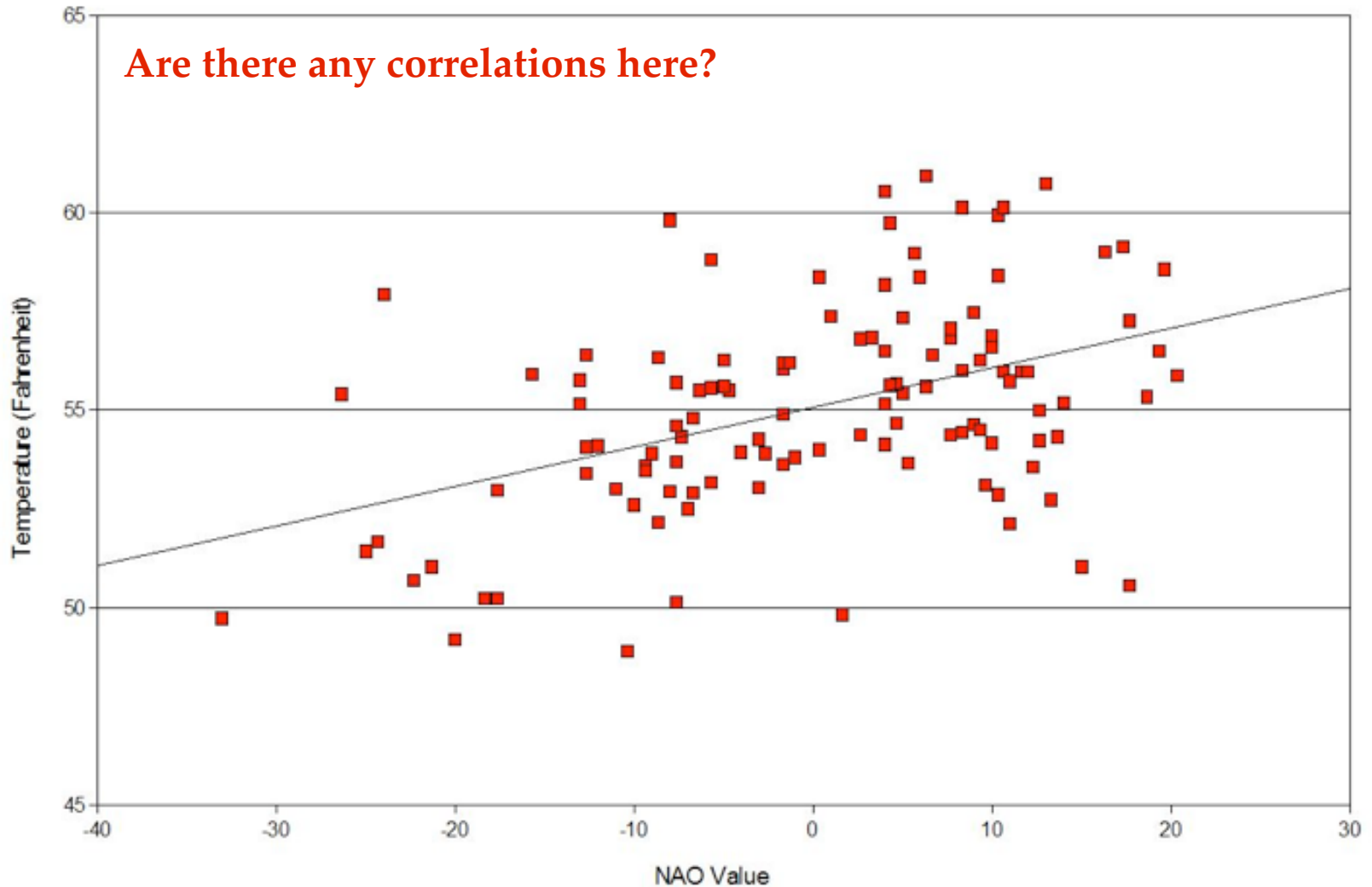
Are there any correlations here?



Correlation

North Atlantic Oscillation (NAO) Effects

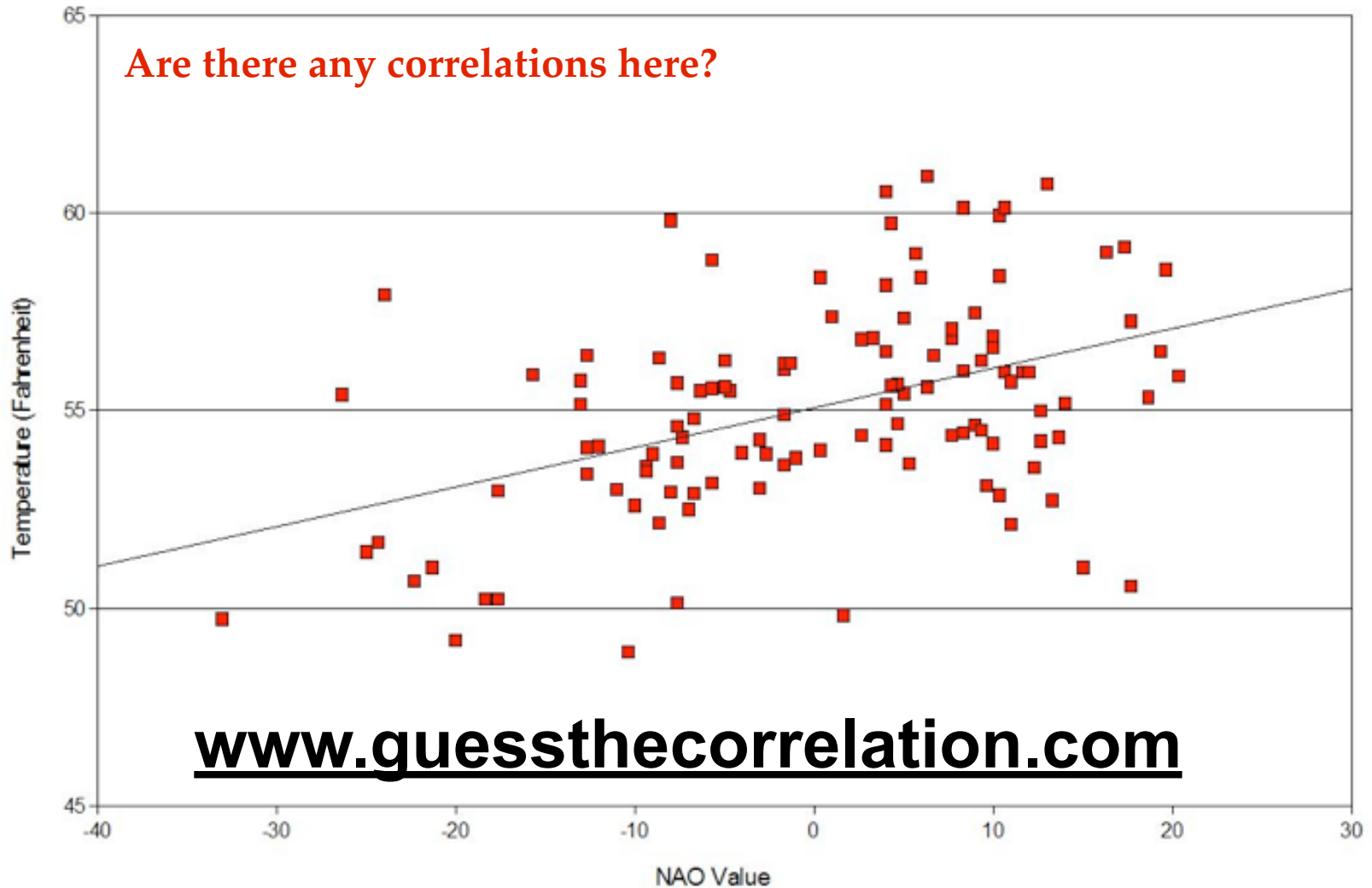
Upper Texas Coast Temperature



Correlation

North Atlantic Oscillation (NAO) Effects

Upper Texas Coast Temperature



Correlation

Recall the definition of the Variance, V :

$$V = \sigma^2 = \frac{1}{N} \sum_i^n (x_i - \mu)^2 = E[(x - \mu)^2] = E[x^2] - \mu^2$$

Correlation

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Likewise, one defines the **Covariance**, V_{xy} :

$$V_{xy} = \frac{1}{N} \sum_i^n (x_i - \mu_x)(y_i - \mu_y) = E[(x_i - \mu_x)(y_i - \mu_y)]$$

Correlation

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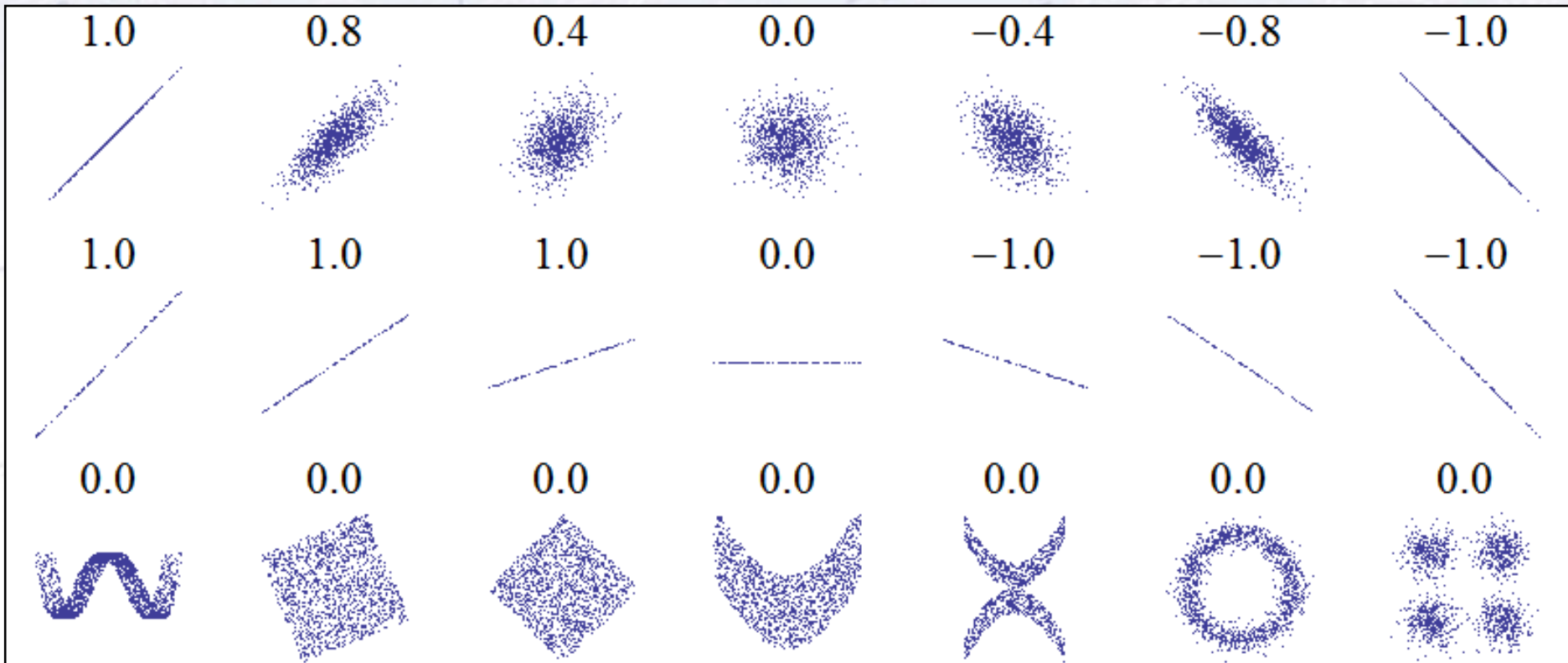
$$V_{xy} = \frac{1}{N} \sum_i^n (x_i - \mu_x)(y_i - \mu_y) = E[(x_i - \mu_x)(y_i - \mu_y)]$$

“Normalising” by the widths, gives Pearson’s (linear) correlation coefficient:

$$\rho_{xy} = \frac{V_{xy}}{\sigma_x \sigma_y} \quad -1 < \rho_{xy} < 1$$
$$\sigma(\rho) \simeq \sqrt{\frac{1}{n}(1 - \rho^2)^2 + O(n^{-2})}$$

Correlation

Correlations in 2D are in the Gaussian case the “degree of ovalness”!



Note how ALL of the bottom distributions have $\rho = 0$, despite obvious correlations!

Correlation Matrix

The correlation matrix V_{xy} explicitly looks as:

$$V_{xy} = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 & \cdots & \sigma_{1N}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2N}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_N^2 & \sigma_{N2}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix}$$

Very specifically, the calculations behind are:

$$V = \begin{bmatrix} E[(X_1 - \mu_1)(X_1 - \mu_1)] & E[(X_1 - \mu_1)(X_2 - \mu_2)] & \cdots & E[(X_1 - \mu_1)(X_n - \mu_n)] \\ E[(X_2 - \mu_2)(X_1 - \mu_1)] & E[(X_2 - \mu_2)(X_2 - \mu_2)] & \cdots & E[(X_2 - \mu_2)(X_n - \mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ E[(X_n - \mu_n)(X_1 - \mu_1)] & E[(X_n - \mu_n)(X_2 - \mu_2)] & \cdots & E[(X_n - \mu_n)(X_n - \mu_n)] \end{bmatrix}.$$

Correlation and Information

Correlations influence results in complex ways!

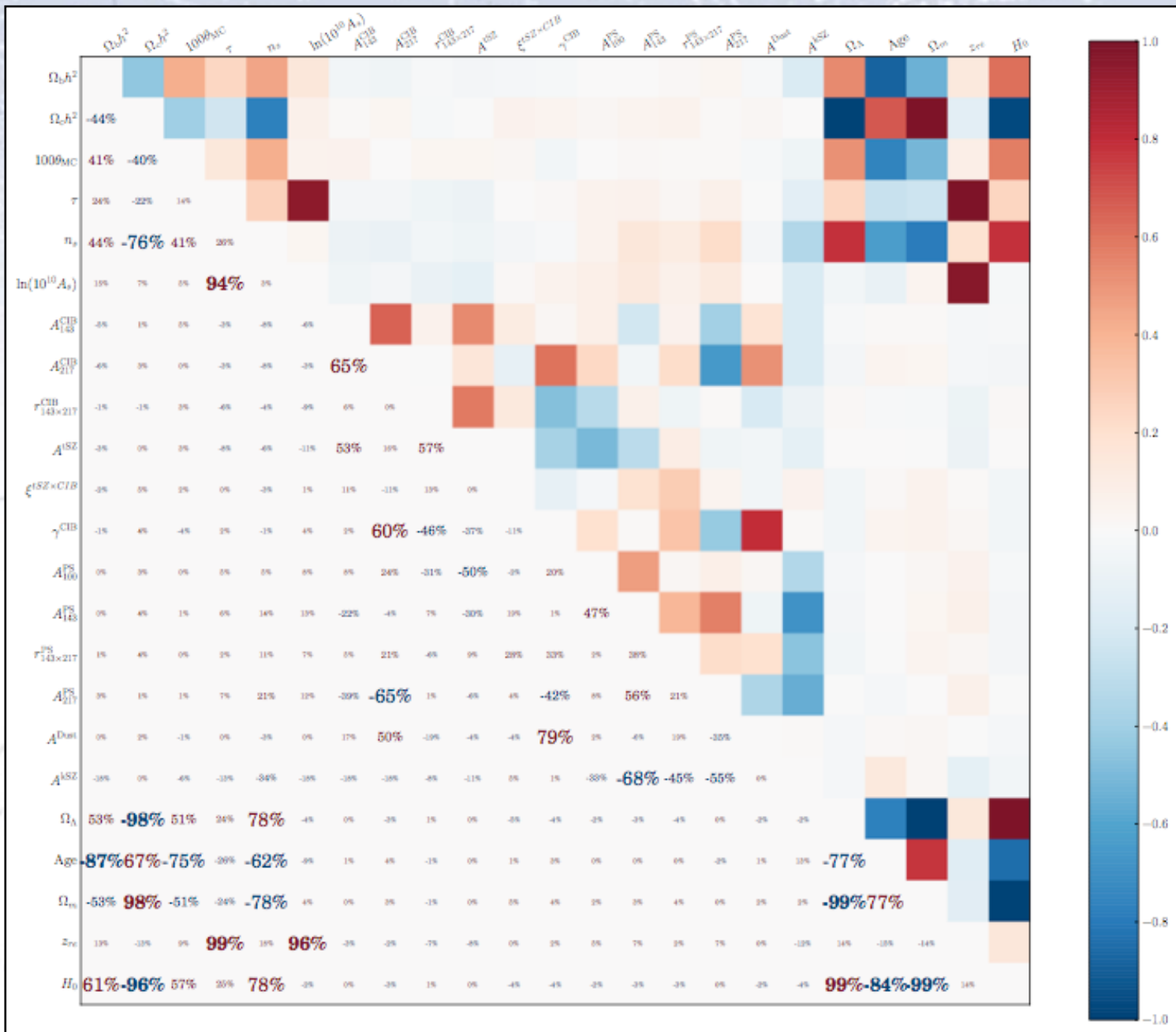
They need to be taken into account, for example in **Error Propagation!**

Correlations may contain a significant amount of information.

We will consider this more when we play with multivariate analysis.



Planck example



Rank correlations

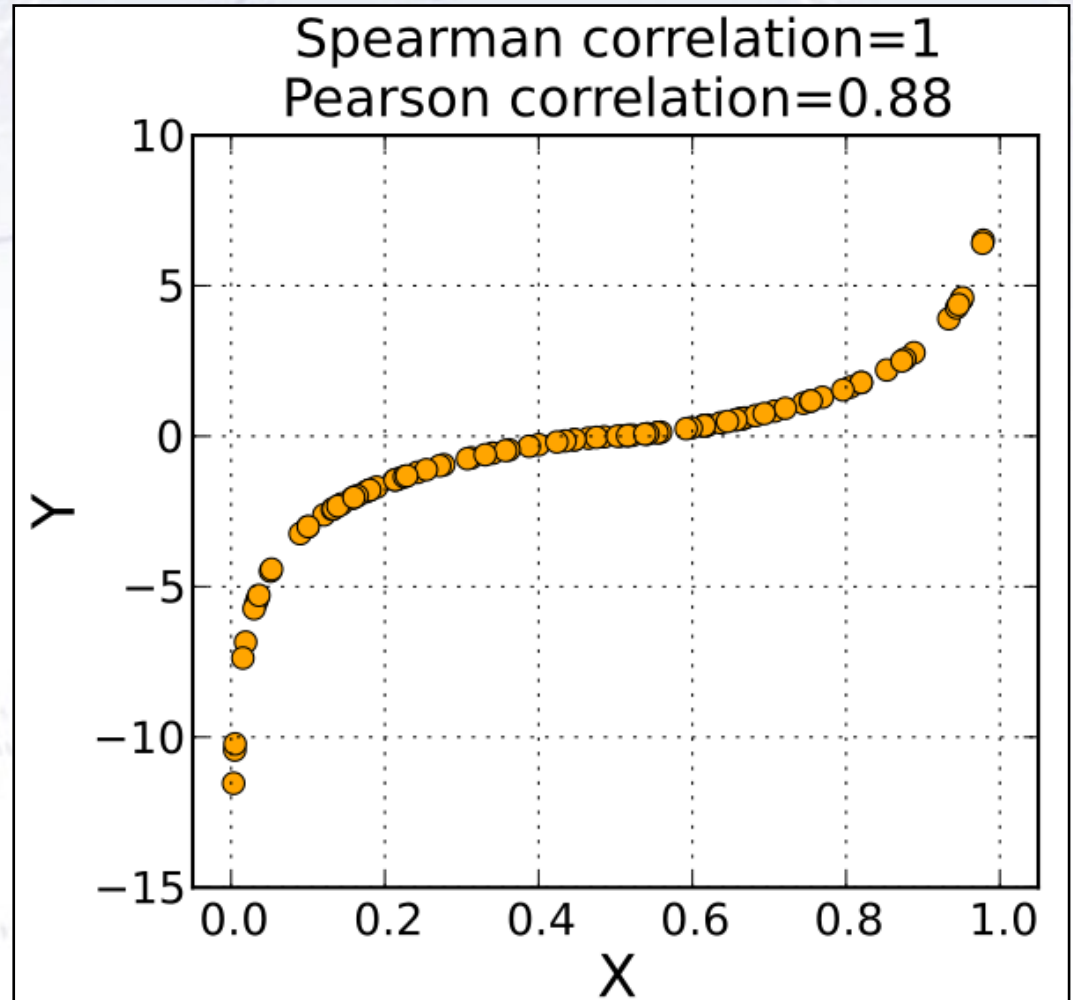
Sometimes, variables are perfectly correlated, just not linearly:

In this case the Pearson correlation is not the best measure.

Rank correlation compares the ranking between the two sets, and therefore gets a good measure of the correlation (see figure).

The two main cases of rank correlations are:

- Spearman's rho
- Kendall's tau



Rank correlations

An additional advantage is, that the rank correlation is less sensitive to outliers:

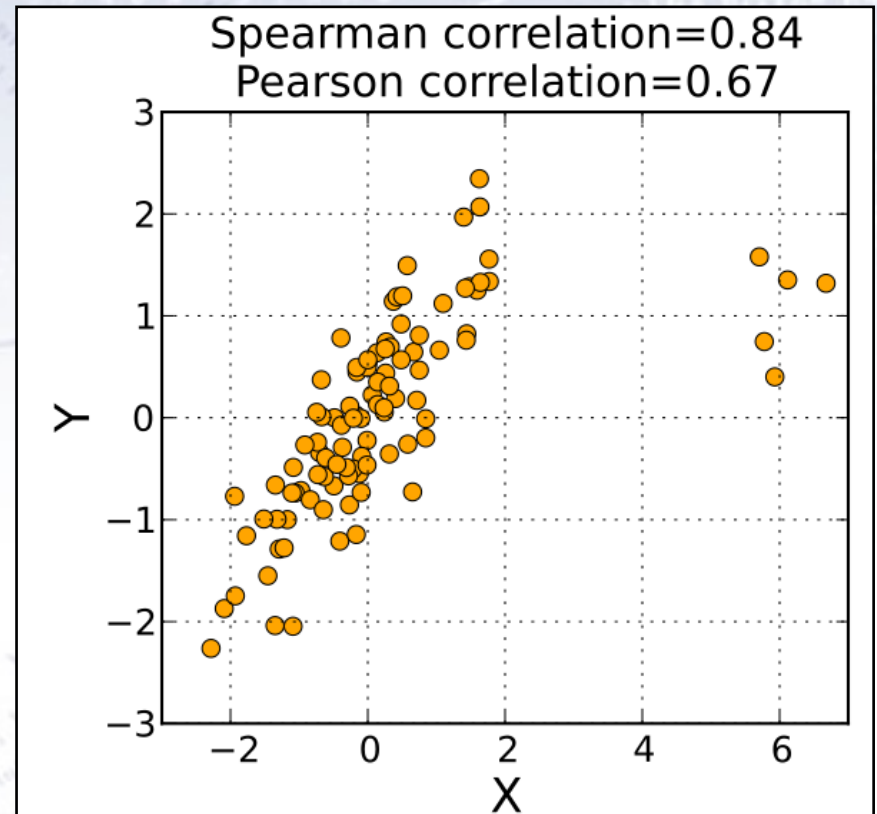
The two rank correlations are special cases of a more general rank correlation.

Typically, Spearman's rank correlation is used.

The definition is:

$$\rho = 1 - 6 \sum_i (r_i - s_i)^2 / (n^3 - n)$$

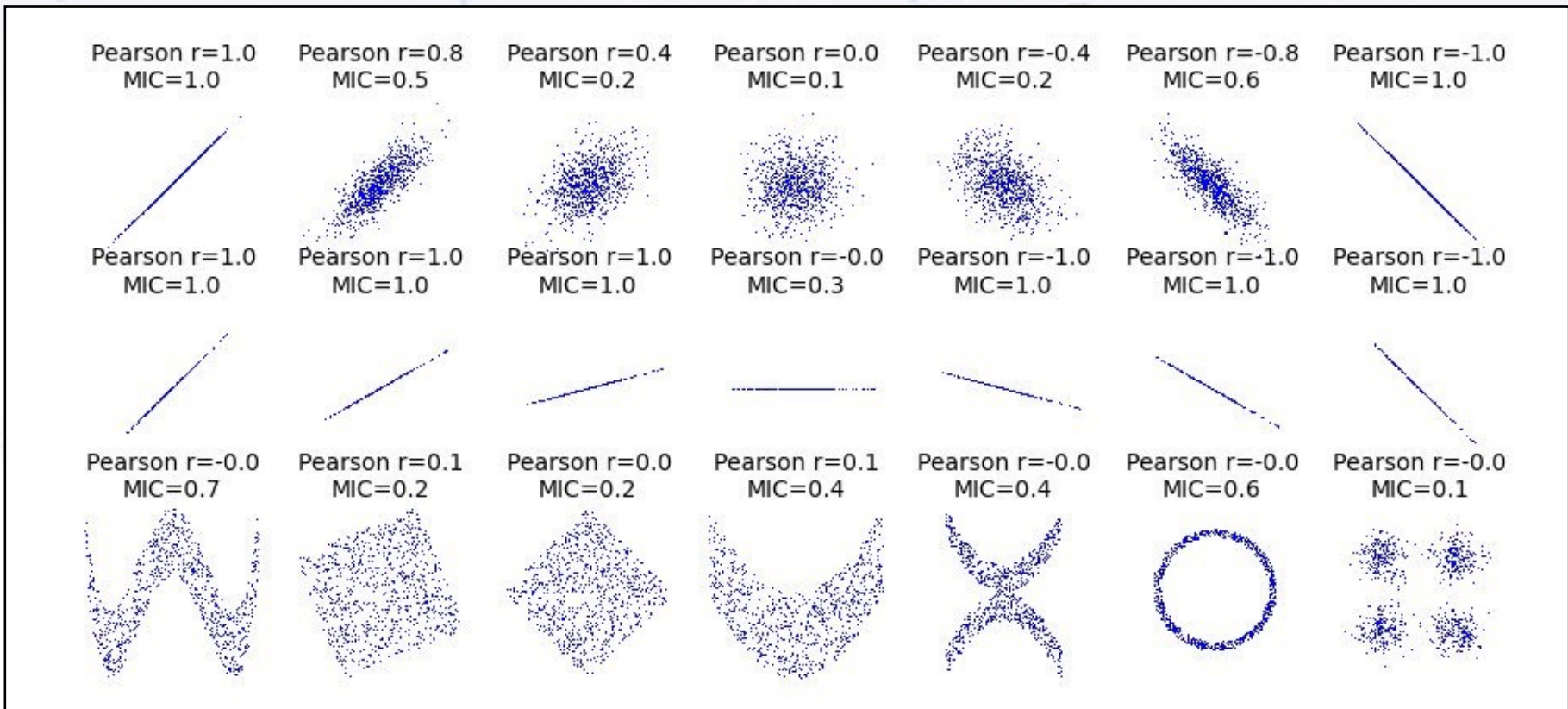
where r_i and s_i is the rank of the i 'th element.



Non-linear correlations

Non-linear correlations (associations) are harder to measure, but possible:

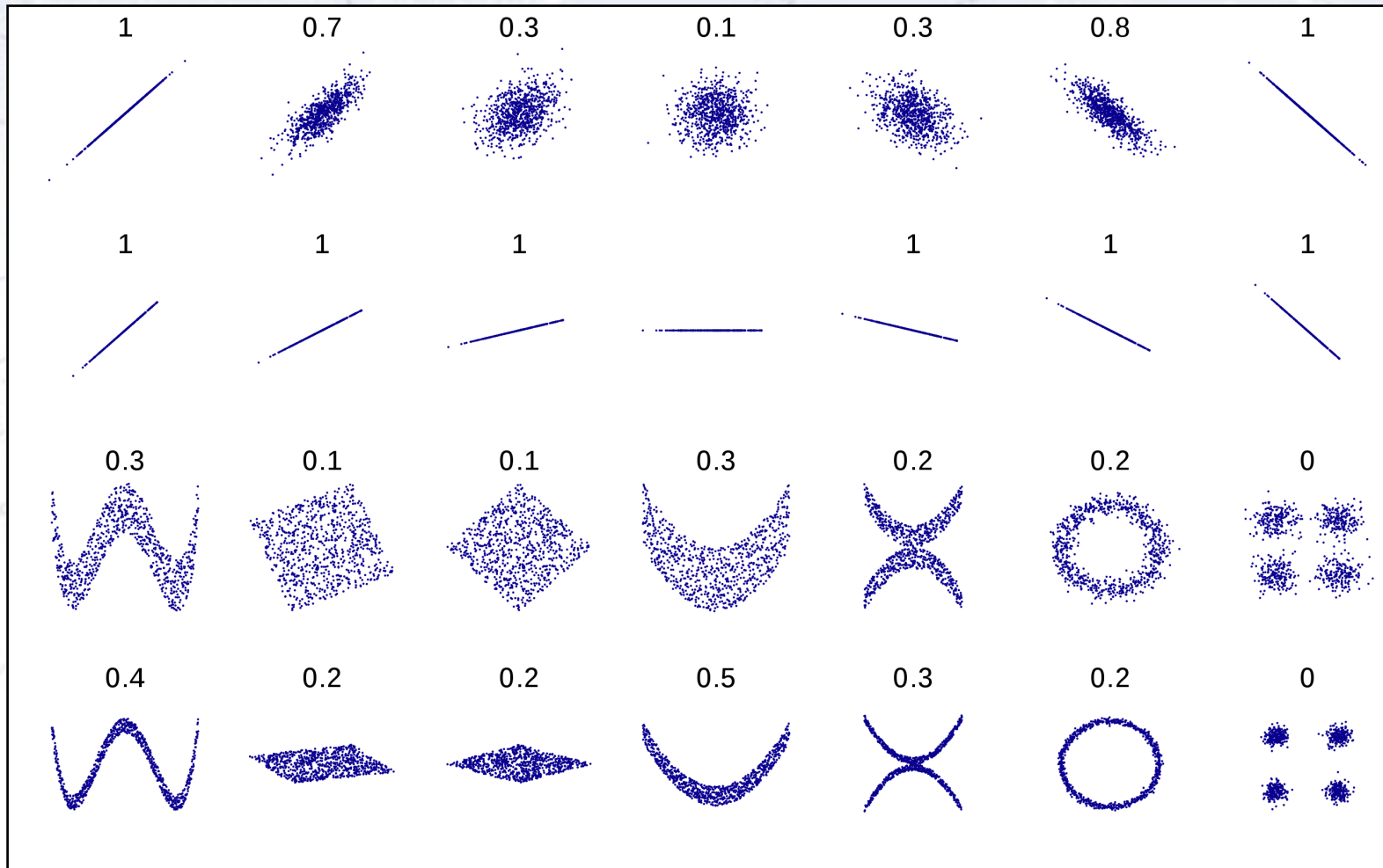
- Maximal Information Coefficient (MIC), see reference and [Wikipedia on MIC](#).
- Mutual Information (MI), linked to entropy, see [Wikipedia on MI](#) and [SKLearn](#).
- Distance Correlation (DC) between paired vectors, see [Wikipedia on DC](#).



Non-linear correlations

Non-linear correlations (associations) are harder to measure, but possible:

- Distance Correlation (DC) between paired vectors, see [Wikipedia on DC](#).



Correlation Vs. Causation

“Com hoc ergo propter hoc”

(with this, therefore because of this)

Fig. 1
IS FACEBOOK DRIVING
THE GREEK DEBT CRISIS?

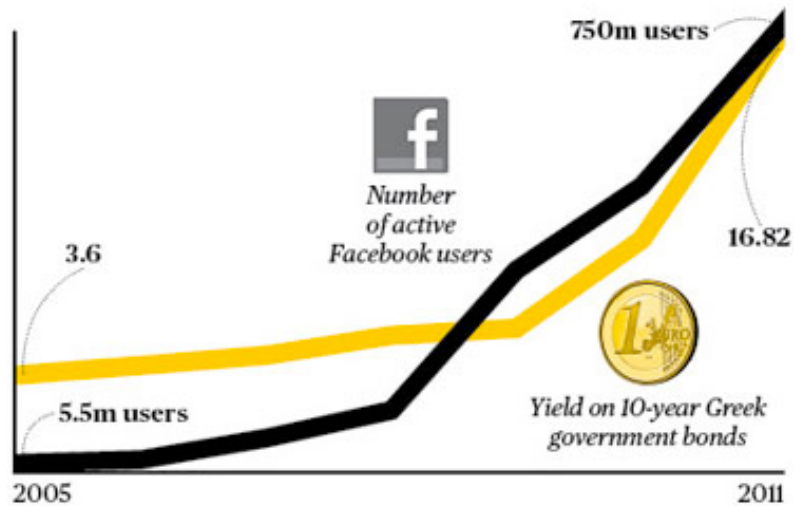
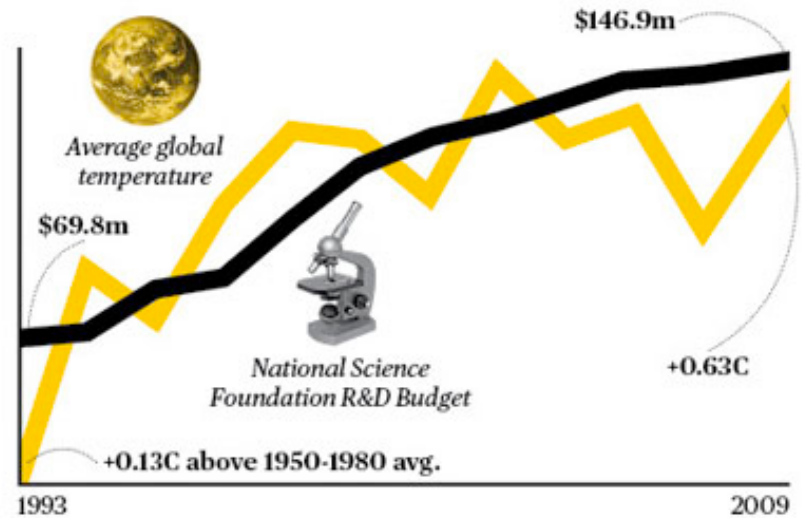


Fig. 2
IS GLOBAL WARMING A HOAX
PROPAGATED BY SCIENTISTS?



It is a common mistake to think that correlation proves causation...

Correlation Vs. Causation

“Com hoc ergo propter hoc”

(with this, therefore because of this)

