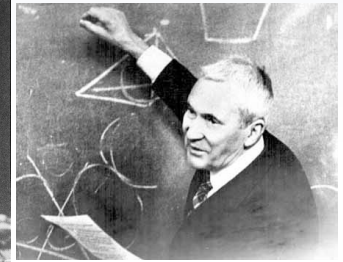
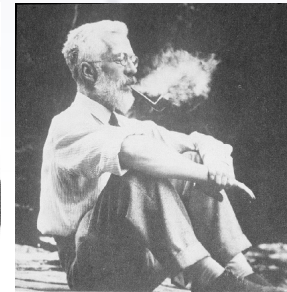
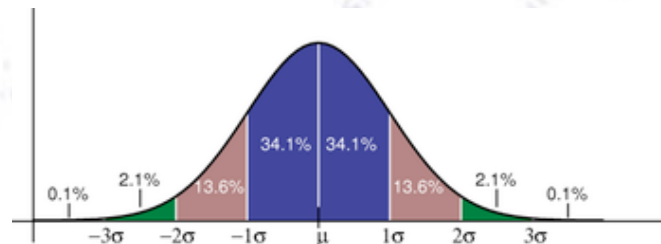


Applied Statistics

Project evaluation



Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Project evaluation

Pendulum:

- Did you measure $T \pm \sigma(T)$ correctly? Combine with Chi2 and comments?
- Did you measure $L \pm \sigma(L)$ correctly?
- Did you provide the individual precisions?

Ball on incline:

- $T \pm \sigma(T)$
 - $L \pm \sigma(L)$
- } $\Rightarrow a \pm \sigma(a)$, with Chi2 and comments.
- $\theta, \Delta\theta$ obtained correctly and
 - d, R and errors propagated correctly?

Generally:

- **Correctly propagated uncertainties, showing individual contributions.**
- All necessary figures and tables there? 2-3 essential figures needed.
- Text enough to understand results? Clear and fitting captions?
- Comment on result, especially inconsistencies.
- Significant digits.

Comments to your projects

One of the reasons for asking for a table with all final measurements to be combined in a value for g was, that we wanted to cross check them!

I'm sorry to say, that several failed to propagate errors correctly!

Another general comment that seems to transpire is, that doing a project requires **organisation of data, scripts, text, etc.**

Reproducibility is a stringent requirement of science. Not all reports would allow others (us?) to reproduce/recalculate the final results.

Several had missed the point about significant digits.

In the end, each experiment was to produce **only one value for g** , but as I didn't state this clearly (I will next year), we have not considered it a mistake.

Reporting the individual error contributions to g was not easy, probably again on my part. **"State the uncertainty, as if this was the only source of error"**.

Comments to your projects

One of the reasons for asking for a table with all final measurements to be combined in a value for g was, that we wanted to cross check them!

I'm sorry to say, that several failed to propagate errors correctly!

Another general comment that seems to transpire is, that doing a project requires **organisation of data, scripts, text, etc.**

Reproducibility is a stringent requirement of science. Not all reports would allow others (us?) to reproduce/recalculate the final results.

Several had missed the point about significant digits.

$$g = (9.842 \pm 0.096_T \pm 0.003_L) \text{ m/s}^2$$

Reporting the individual error contributions to g was not easy, probably again on my part. **"State the uncertainty, as if this was the only source of error"**.

General comments

Tests and cross checks:

- **For weighted average, calculate Chi2 and p-values (to test consistency).**
- Never combine inconsistent numbers (then you are SURE to make a mistake!).
- Comment (heavily) on inconsistent numbers.

Correlations:

- **Careful in not combining correlated numbers.**
- Careful not to repeat measurements that are small or systematically dominated.

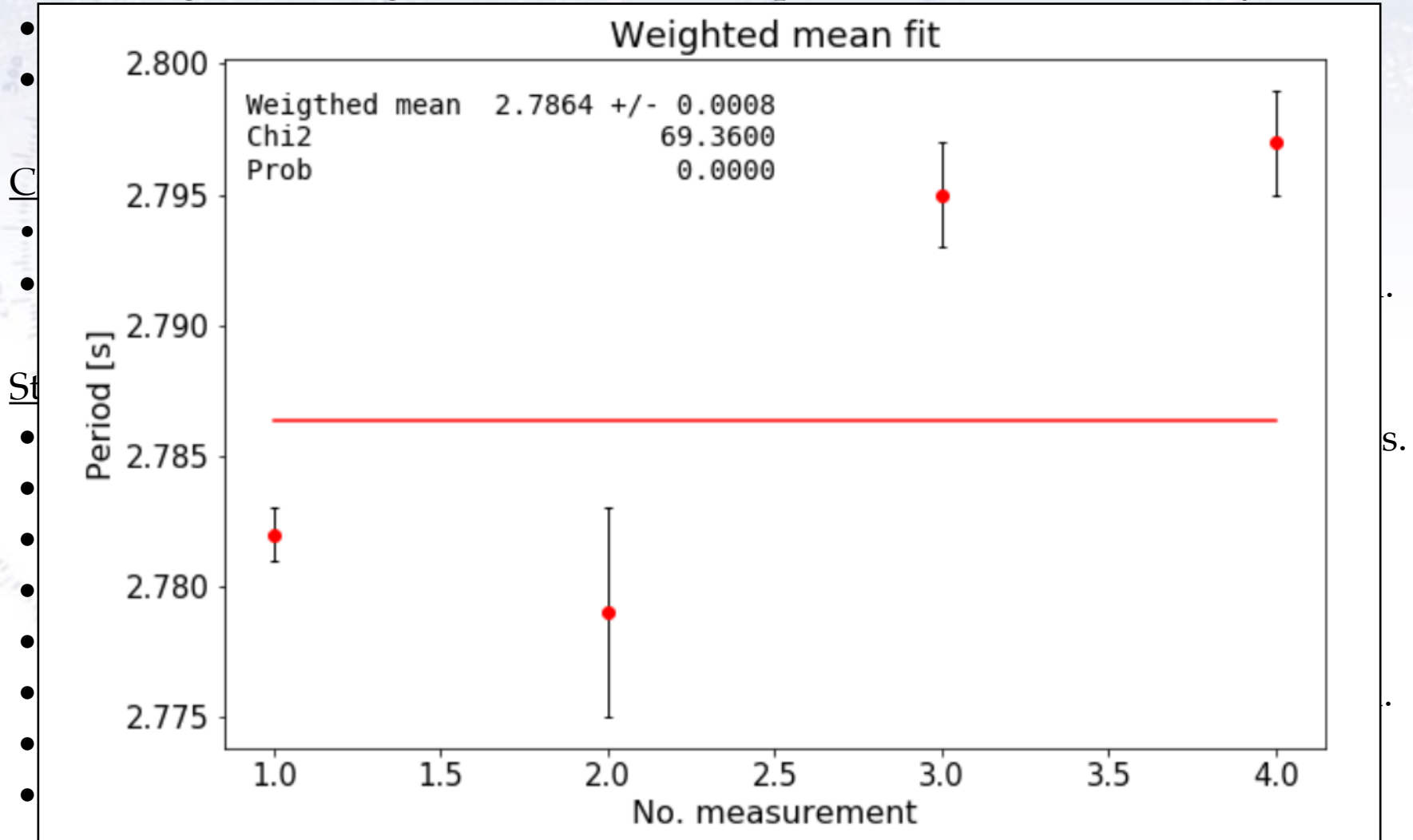
Structure of report:

- **Plan your plots.** Here 3-4 is fitting. At least length vs. time for both experiments.
- **Plan your tables.** Make sure they are readable and complete.
- Put results and conclusions in abstract. Meant for saving readers time.
- Be short and precise. Label carefully and refer to these labels.
- Always remember correct number of significant digits.
- **Write IMPACT on g from each input variable.** Each term in error prop. formula.
- Don't put definition of mean, RMS, etc. in your text. Considered known to all.
- Be VERY detailed about removing data. Show explicitly why. Give p-values.

General comments

Tests and cross checks:

- For weighted average, calculate Chi2 and p-values (to test consistency).



Pendulum - comments

Time measurement:

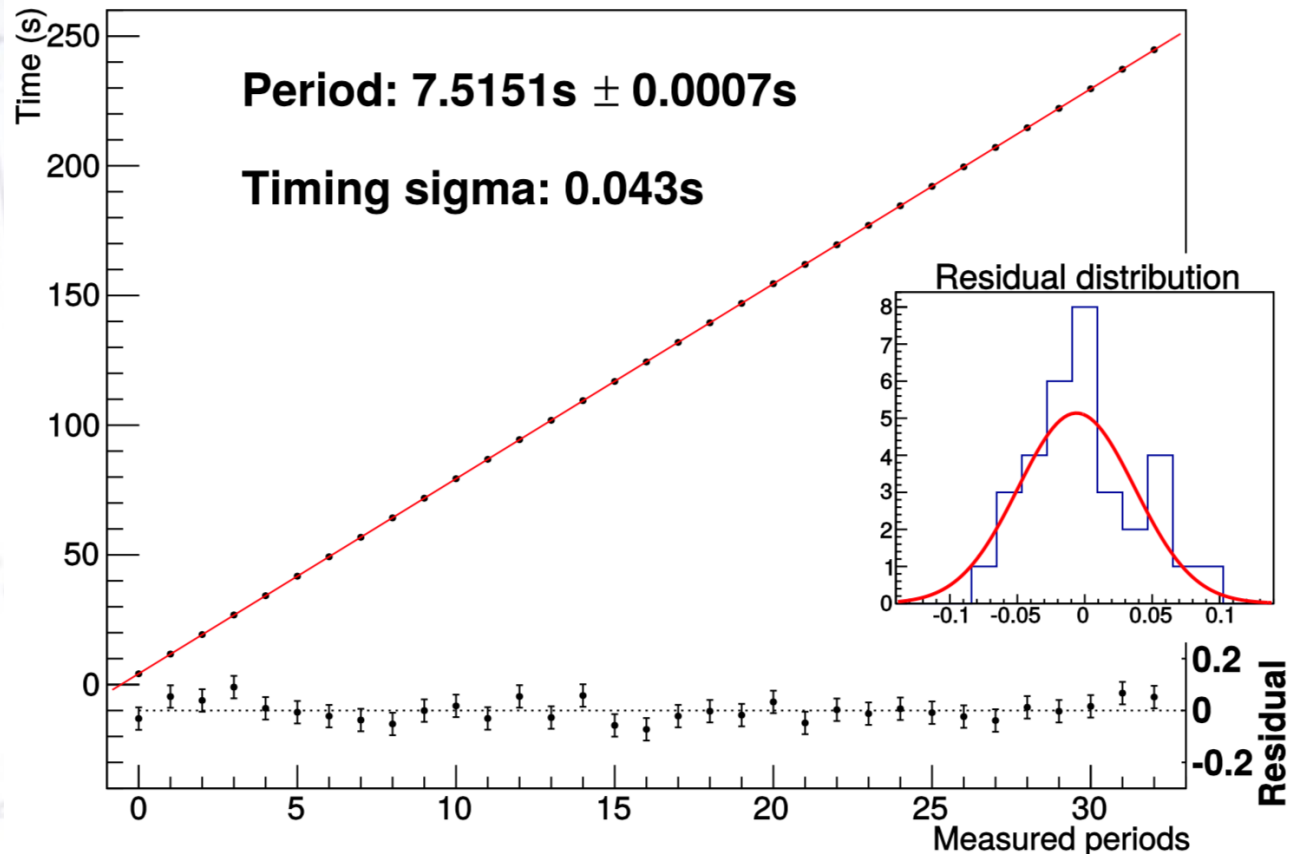
Many independent measurements, little systematic \Rightarrow Good error estimate

Length measurement:

Some independent measurements but also some systematics \Rightarrow check difference between instruments.

You can not reduce the uncertainty by multiple measurements, if the main limitation is some inherent systematic!

Several groups managed to get uncertainties below 0.1%.



Pendulum - comments

Time measurement:

Many independent

Very importantly:

RMS of residuals = **your single measurement** precision.

Length measurement:

Some independent

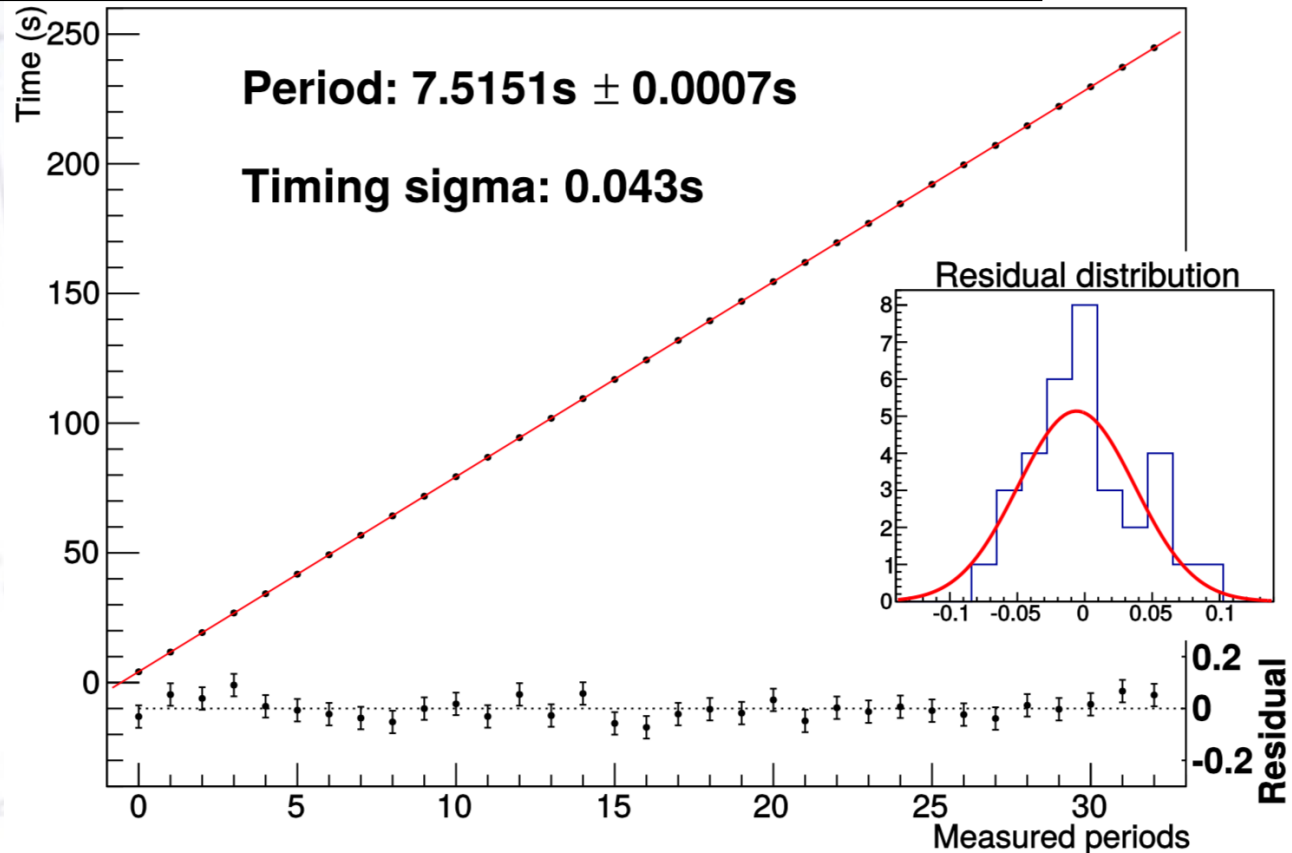
Uncertainty on fit slope = **pendulum period** precision.

between instruments.

The latter is MUCH smaller!

You can not reduce the uncertainty by multiple measurements, if the main limitation is some inherent systematic!

Several groups managed to get uncertainties below 0.1%.



Pendulum - comments

Time measurement:

Many independent

Very importantly:

RMS of residuals = **your single measurement** precision.

Length measurement:

Uncertainty on fit slope = **pendulum period** precision.

Some independent

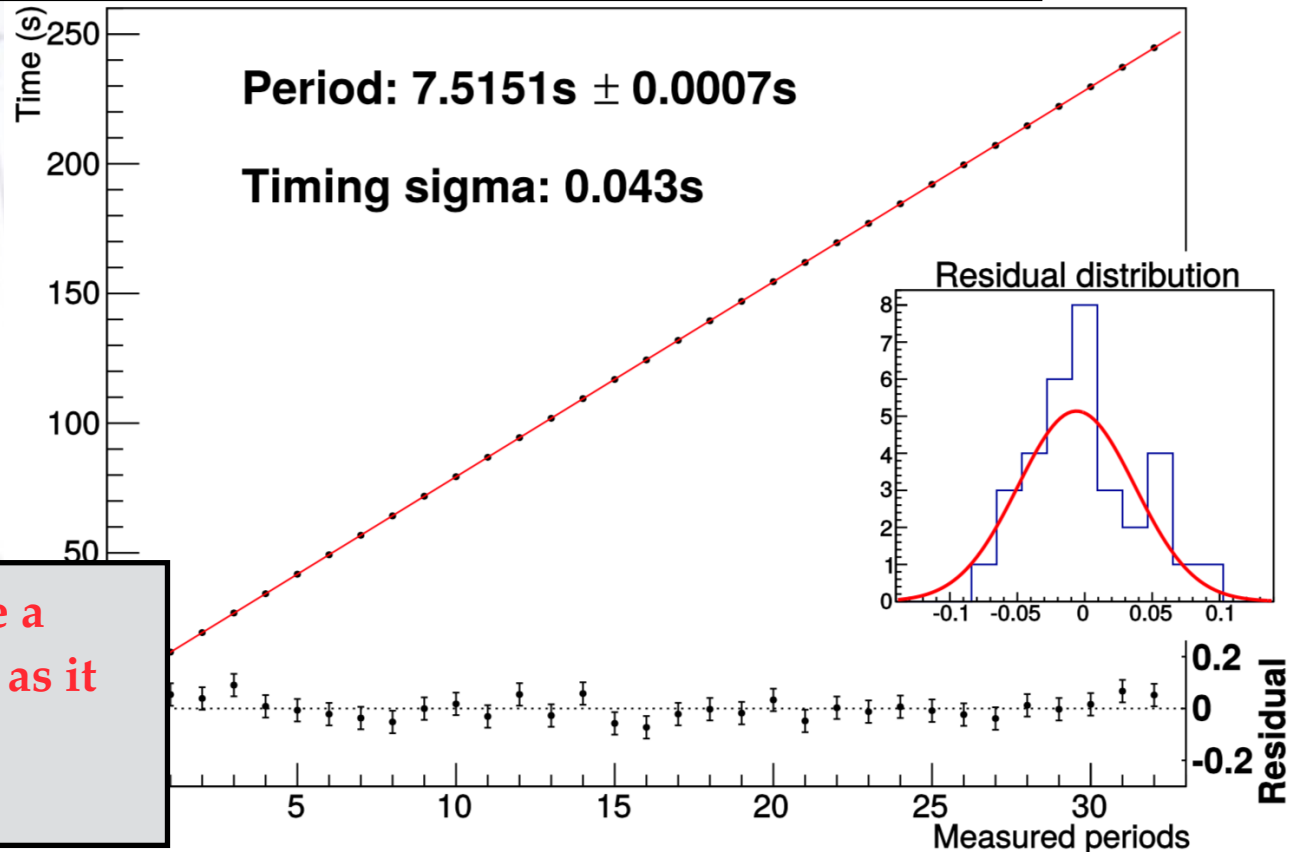
The latter is MUCH smaller!

between instruments.

You can not reduce the uncertainty by multiple measurements, if the main limitation is some inherent systematic!

Several groups managed

Note: This is a case, where a ChiSquare is not in place, as it will by construction yield around 50%.



Pendulum - comments

Time measurement:

Many independent

Very importantly:

RMS of residuals = **your single measurement** precision.

Length measurement:

Some independent

Uncertainty on fit slope = **pendulum period** precision.

between instruments.

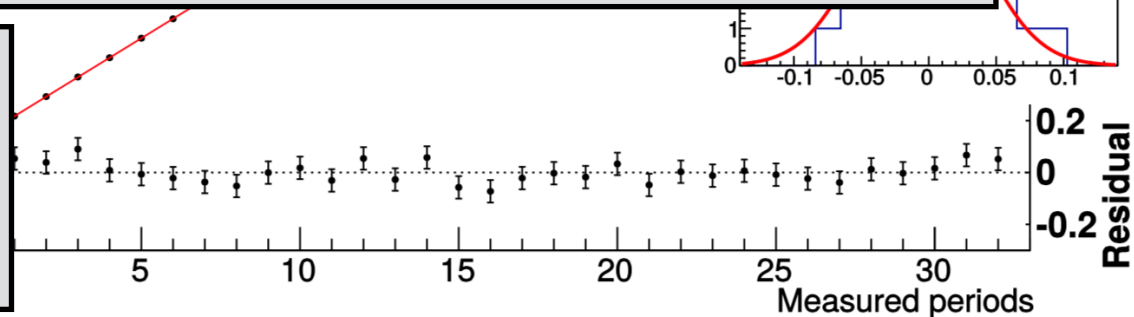
The latter is MUCH smaller!

$$\chi^2 = \sum_i \frac{(y_i - f_i)^2}{\sigma_i^2} \quad RMS = \sqrt{\frac{1}{N-1} \sum_i (y_i - f_i)^2}$$

Defining the uncertainty to be the RMS, one gets a good Chi2 value:

$$\chi^2 = (N - 1) \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - f_i)^2} = N - 1$$

Note: This is a case, where a ChiSquare is not in place, as it will by construction yield around 50%.



Pendulum - comments

Time measurement:

Many independent

Very importantly:

RMS of residuals = **your single measurement** precision.

Length measurement:

Some independent

Uncertainty on fit slope = **pendulum period** precision.

between instruments.

The latter is MUCH smaller!

$$\chi^2 = \sum_i \frac{(y_i - f_i)^2}{\sigma_i^2} \quad RMS = \sqrt{\frac{1}{N-1} \sum_i (y_i - f_i)^2}$$

Defining the uncertainty to be the RMS, one gets a good Chi2 value:

$$\chi^2 = (N - 1) \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - f_i)^2} = N - 1$$

Note: This is a case, where a **ChiSquare** is not in place, as it **will by construction yield around 50%.**

N single measurements (t1 to t2) does not rival the precision of N consecutive measurements.

OK... no more grey boxes here!

5 10 15 20 25 30
Measured periods

Residual

Ball on incline - comments

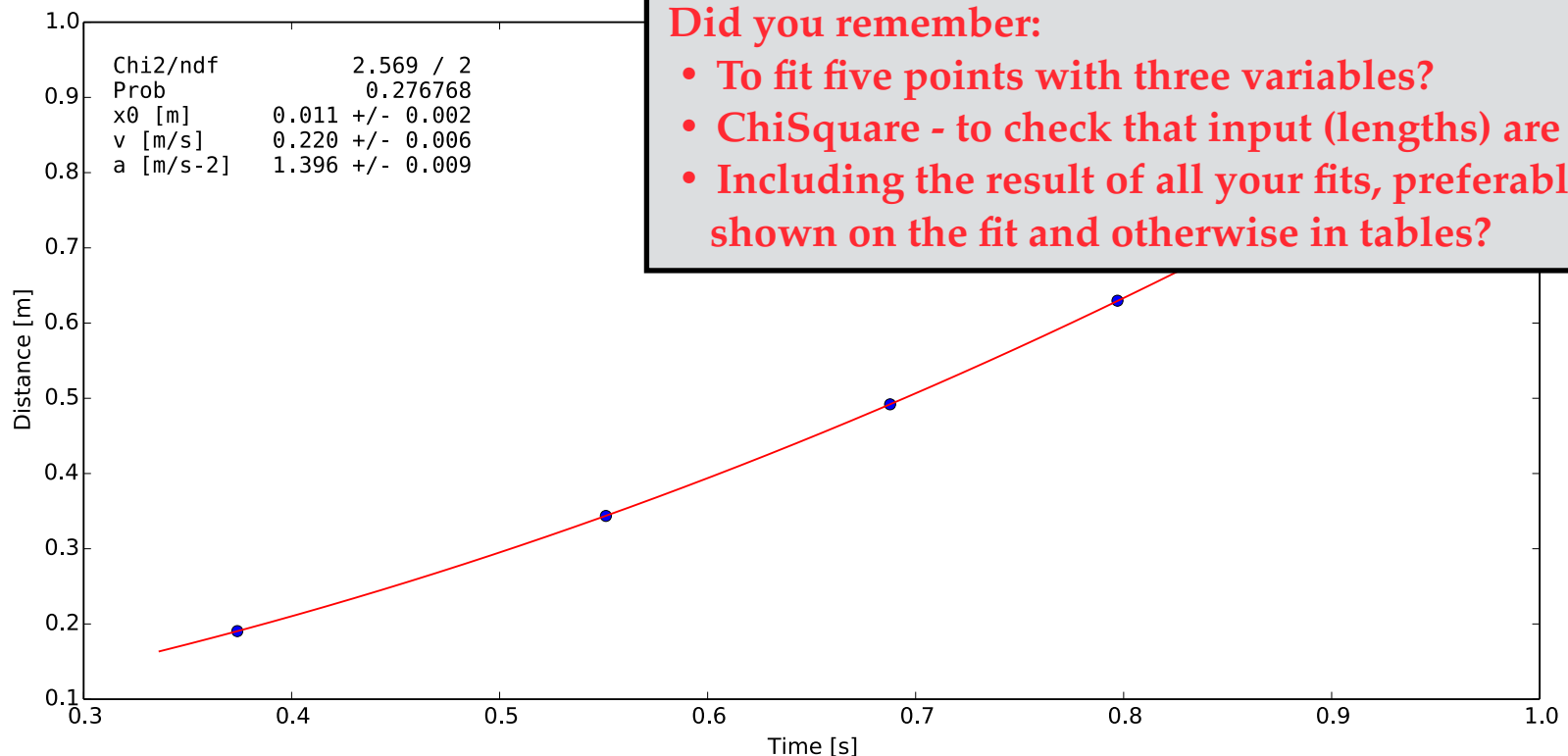
Time measurement: Insignificant uncertainties, so repetition doesn't help!

However, it is helpful to detect unknown systematics.

Length measurements: Some uncertainties, some systematics, but OK errors.

Note that diode position is not central!

Angle measurement: This is the real challenge! And angle vs. lengths have very different systematics, so they are good to combine!



Ball on incline - comments

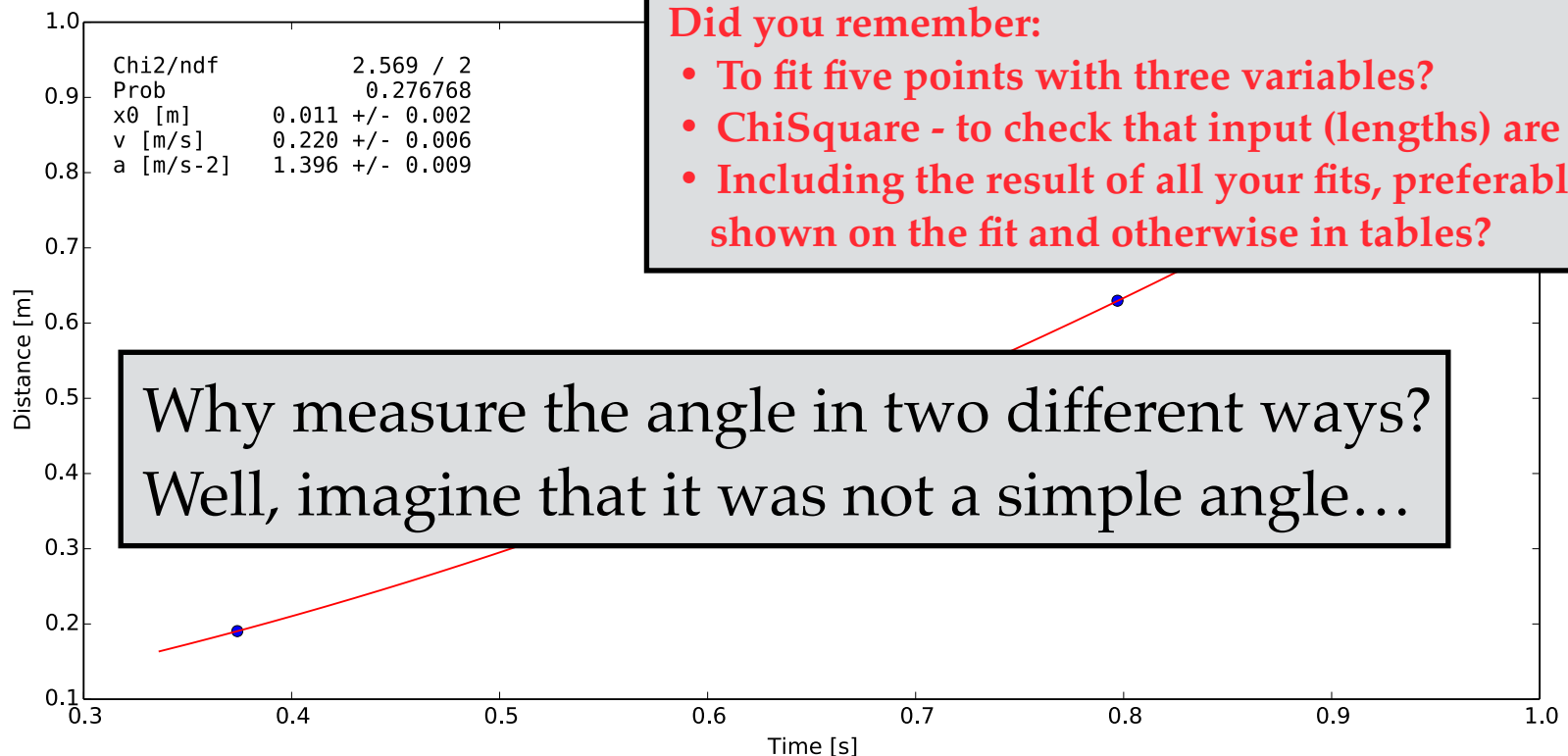
Time measurement: Insignificant uncertainties, so repetition doesn't help!

However, it is helpful to detect unknown systematics.

Length measurements: Some uncertainties, some systematics, but OK errors.

Note that diode position is not central!

Angle measurement: This is the real challenge! And angle vs. lengths have very different systematics, so they are good to combine!



Ball on incline - comments



Measurement situation

There are four possible situations in experimental measurements of a quantity:

One measurement, no error:

$$X = 3.14$$

Situation: You are f***ed!

You have no clue about uncertainty, and you can not obtain it!

Several measurements, no errors:

$$X_1 = 3.14$$

$$X_2 = 3.21$$

$$X_3 = \dots$$

Situation: You are OK

You can combine the measurements, and from RMS get error on mean.

One measurement, with error:

$$X = 3.14 \pm 0.13$$

Situation: You are OK

You have a number with error, which you can continue with.

Several measurements, with errors:

$$X_1 = 3.14 \pm 0.13$$

$$X_2 = 3.21 \pm 0.09$$

$$X_3 = \dots$$

Situation: You are on top of things!

You can both combine to a weighted, average and check with a chi-square.

Specific comments

Pendulum experiment:

- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Pendulum line may be stretching. Requires times and lengths for individual g.
- Swinging pendulum at 90 degrees is a good check of impact of hook.
- Combine lengths (L) with normal mean, and get uncertainty from RMS.
- Combine periods (T) with weighted mean and use Chi2 to check consistency.
- Large swings (e.g. 7 degrees) result in 1/1000 violation of formula assumption!

$$T = 2\pi\sqrt{\frac{\ell}{g}} \left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \frac{173}{737280}\theta_0^6 + \frac{22931}{1321205760}\theta_0^8 + \frac{1319183}{951268147200}\theta_0^{10} + \frac{233526463}{2009078326886400}\theta_0^{12} + \dots \right),$$

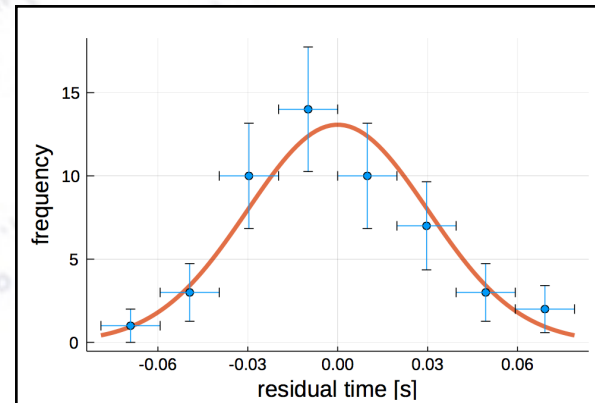
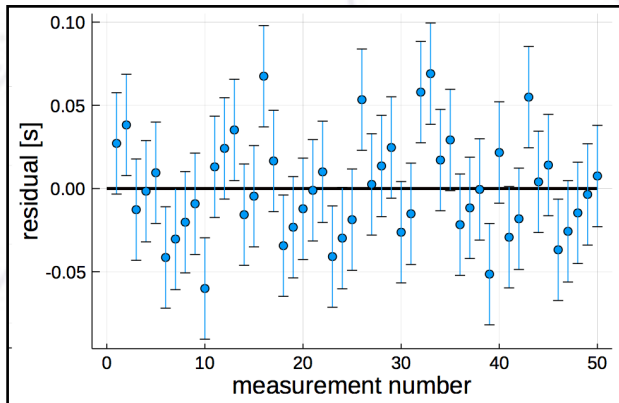
Specific comments

Pendulum experiment:

- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Pendulum line may be stretching. Requires times and lengths for individual g .
- Swinging pendulum at 90 degrees is a good check of impact of hook.
- Combine lengths (L) with normal mean, and get uncertainty from RMS.
- Combine periods (T) with weighted mean and use Chi2 to check consistency.
- Large swings (e.g. 7 degrees) result in 1/1000 violation of formula assumption!

$$T = 2\pi\sqrt{\frac{\ell}{g}} \left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \frac{173}{737280}\theta_0^6 + \frac{22931}{1321205760}\theta_0^8 + \frac{1319183}{951268147200}\theta_0^{10} + \frac{233526463}{2009078326886400}\theta_0^{12} + \dots \right),$$

Note: Getting precise timing from a camera (16 f/s) is actually not that easy:



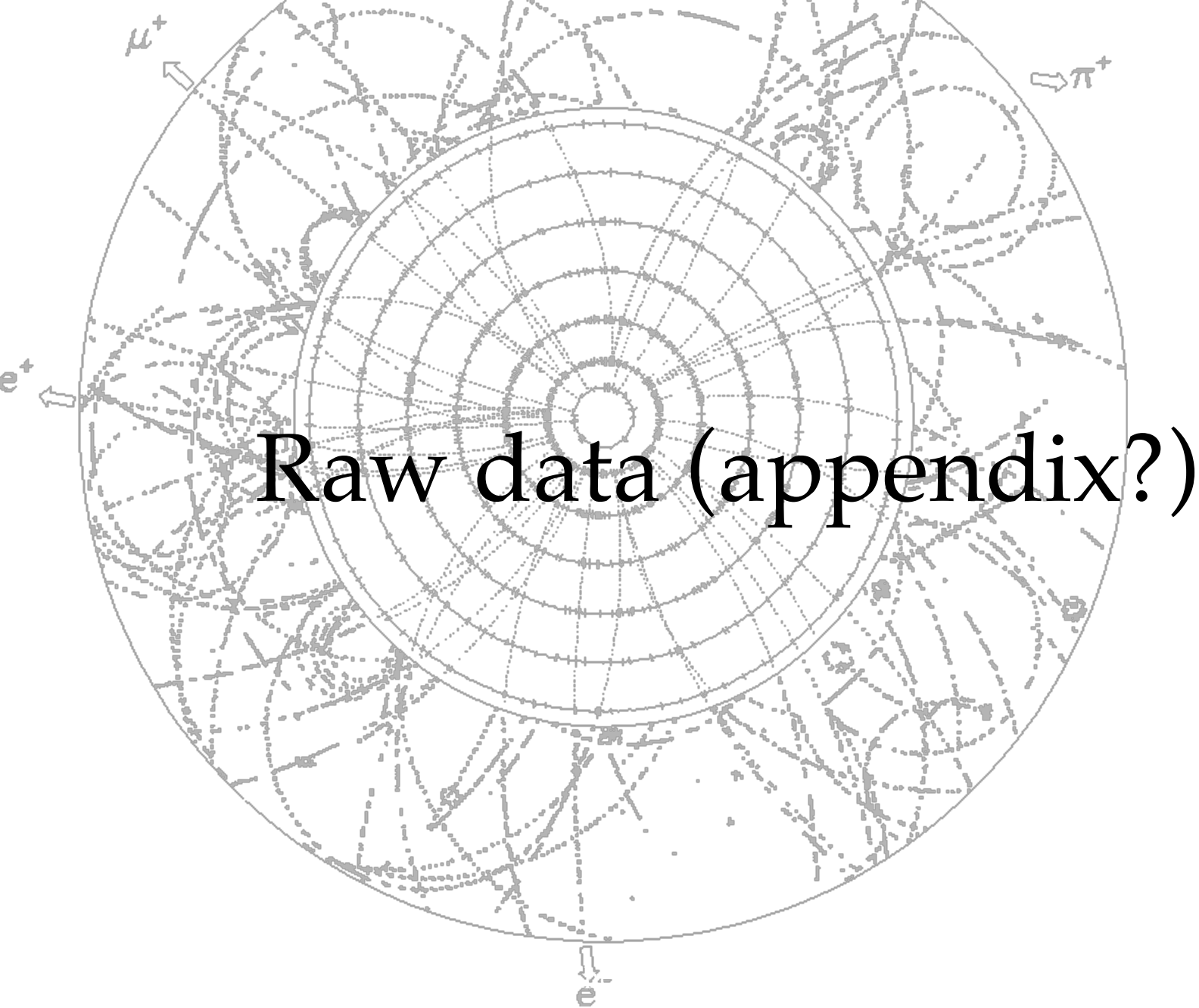
Specific comments

Pendulum experiment:

- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Pendulum line may be stretching. Requires times and lengths for individual g .
- Swinging pendulum at 90 degrees is a good check of impact of hook.
- Combine lengths (L) with normal mean, and get uncertainty from RMS.
- Combine periods (T) with weighted mean and use Chi2 to check consistency.
- Large swings (e.g. 7 degrees) result in 1/1000 violation of formula assumption!

Ball-on-incline experiment:

- Rerunning ball a few times to get $\sigma(t_{\text{gate}})$ is hard, as x_0 and v_0 changes.
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Write fit function with constants in front, i.e. $0.5 * a * t^2$. Gives correct errors!
- Different ball size give different results. 1-2% variations (10-15mm).
- No, you can't measure the angle with 0.001 degree precision!!!



Raw data (appendix?)

Table V: Measurements as originally performed by the observers.

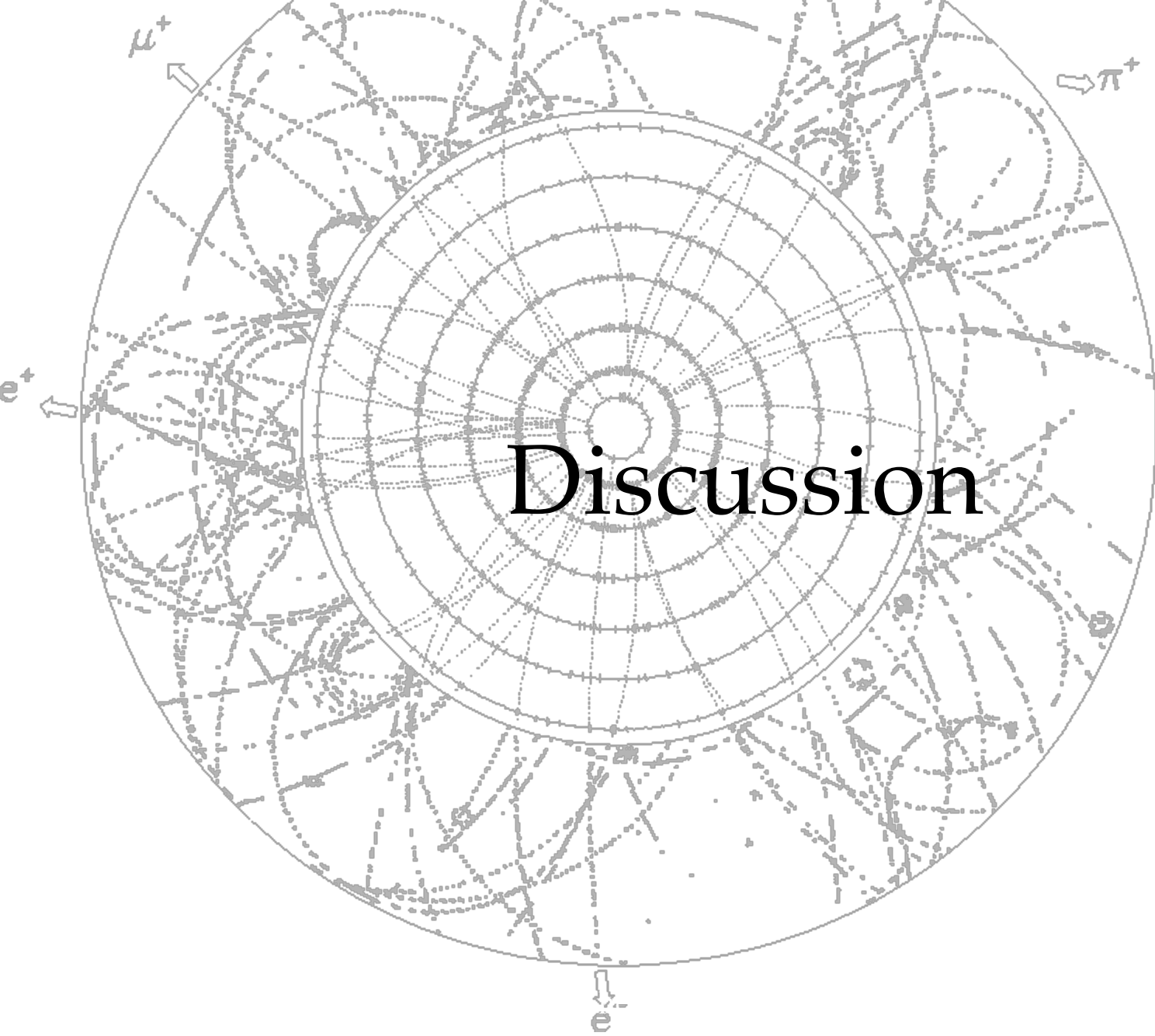
| Pendulum measurements | | | | | | | | | | |
|-----------------------|------------|---------------------|----------|-----------------|------------|---------------------|----------|-----------------|----------|-----------------|
| Observer | Ruler | | Lead | | Laser | | Hook | | Floor | |
| | d_r [cm] | σ_{d_r} [cm] | l [cm] | σ_l [cm] | d_l [cm] | σ_{d_l} [cm] | h [cm] | σ_h [cm] | f [cm] | σ_f [cm] |
| Sebastien | 176.8 | 0.5 | 14.85 | 0.05 | 210.2 | 0.1 | 3.630 | 0.05 | 14.9 | 0.1 |
| Giulia | 177.0 | 0.5 | 14.90 | 0.05 | 210.1 | 0.1 | 3.660 | 0.05 | 15.0 | 0.1 |

| Inclined rail measurements | | | | | | | | | |
|----------------------------|--------------|---------------------|--------------|---------------------|--------------|---------------------|--------------|---------------------|--|
| Troels | | | Vojtech | | | Sebastien | | Giulia | |
| Gates positions | | | | | | | | | |
| N_{gate} | x [cm] | σ_x [cm] | x [cm] | σ_x [cm] | x [cm] | σ_x [cm] | x [cm] | σ_x [cm] | |
| 0 | 26.95 | 0.05 | 26.90 | 0.05 | 26.90 | 0.05 | 26.95 | 0.05 | |
| 1 | 40.82 | 0.05 | 40.80 | 0.05 | 40.80 | 0.05 | 40.88 | 0.05 | |
| 2 | 55.03 | 0.05 | 55.00 | 0.05 | 55.15 | 0.05 | 55.03 | 0.05 | |
| 3 | 69.04 | 0.05 | 69.10 | 0.05 | 69.00 | 0.05 | 69.05 | 0.05 | |
| 4 | 83.65 | 0.05 | 83.65 | 0.05 | 83.65 | 0.05 | 83.62 | 0.05 | |
| Inclination Angle | | | | | | | | | |
| Orientation | θ [°] | σ_θ [°] | θ [°] | σ_θ [°] | θ [°] | σ_θ [°] | θ [°] | σ_θ [°] | |
| front | 77.2 | 0.3 | 77.0 | 0.5 | 76.80 | 0.25 | 77.2 | 0.5 | |
| back | 76.1 | 0.3 | 76.5 | 0.5 | 76.20 | 0.25 | 76.5 | 0.5 | |
| | | | | | | | 76.3 | 0.3 | |
| | | | | | | | 76.3 | 0.3 | |

| $N_{measure}$ | r_b [mm] | σ_{r_b} [mm] | r_b [mm] | σ_{r_b} [mm] | r_b [mm] | σ_{r_b} [mm] | r_b [mm] | σ_{r_b} [mm] | |
|--|--------------------------|---------------------------------|------------|-------------------------|--------------------------|---------------------------------|------------|-------------------------|--|
| | | | 14.95 | 0.05 | 14.95 | 0.05 | 15.00 | 0.05 | |
| Rail distance | | | | | | | | | |
| $N_{measure}$ | d [mm] | σ_d [mm] | d [mm] | σ_d [mm] | d [mm] | σ_d [mm] | d [mm] | σ_d [mm] | |
| 1 | | | 6.10 | 0.05 | 6.10 | 0.05 | 6.05 | 0.05 | |
| 2 | | | | | 6.05 | 0.05 | 6.10 | 0.05 | |
| 3 | | | | | 6.10 | 0.05 | 6.10 | 0.05 | |
| 4 | | | | | | | 6.15 | 0.05 | |
| Acceleration Fit and gravitational pull estimation | | | | | | | | | |
| Observer | Giulia | | | | | Sebastien | | | |
| Run | a [m s ⁻²] | σ_a [m s ⁻²] | χ^2 | $P_{N_{dof}=2}(\chi^2)$ | a [m s ⁻²] | σ_a [m s ⁻²] | χ^2 | $P_{N_{dof}=2}(\chi^2)$ | |
| 1 (0°) | 1.475 | 0.030 | 1.923 | 0.382 | 1.484 | 0.024 | 0.632 | 0.729 | |
| 2 (0°) | 1.481 | 0.030 | 1.273 | 0.529 | 1.481 | 0.024 | 0.606 | 0.739 | |
| 3 (0°) | 1.472 | 0.024 | 1.381 | 0.501 | 1.486 | 0.023 | 0.689 | 0.709 | |
| 4 (0°) | 1.488 | 0.024 | 1.715 | 0.424 | 1.488 | 0.024 | 0.603 | 0.740 | |
| 5 (0°) | 1.488 | 0.018 | 0.265 | 0.876 | 1.485 | 0.024 | 0.416 | 0.812 | |
| 1 (180°) | 1.575 | 0.025 | 0.101 | 0.951 | 1.555 | 0.025 | 0.447 | 0.800 | |
| 2 (180°) | 1.569 | 0.024 | 0.061 | 0.970 | 1.559 | 0.025 | 0.397 | 0.820 | |
| 3 (180°) | 1.562 | 0.024 | 0.105 | 0.949 | 1.559 | 0.025 | 0.629 | 0.730 | |
| 4 (180°) | 1.559 | 0.023 | 1.225 | 0.542 | 1.560 | 0.025 | 0.608 | 0.738 | |
| 5 (180°) | 1.569 | 0.024 | 1.174 | 0.556 | 1.559 | 0.025 | 0.455 | 0.797 | |

Essentially all of you did well here - thanks.

Example table



Discussion

Correlated measurements

If measurements are independent, then they can be combined to decrease the uncertainty.

When are measurements correlated?

- When they are (partly) based on the same sub-measurement / input?
- When they are measured with the same instrument repeatedly?
- When they are from different methods for measuring the same quantity?
- When they involve a commonly extracted quantity?

Examples from the experiments:

- The gate positions in the Ball-on-Incline experiment, when they are measured
 - a) as positions on one ruler placed in a fixed position?
 - b) as distances between each adjacent gate?
- The pendulum times as measured by different people at the same time?
- The accelerations when repeating the balls roll?
- Two measurements of g based on turning setup around and measuring angle
 - a) with a goniometer (i.e. angle measuring device)?
 - b) using trigonometry?

Correlated measurements

If measurements are independent, then they can be combined to decrease the uncertainty.

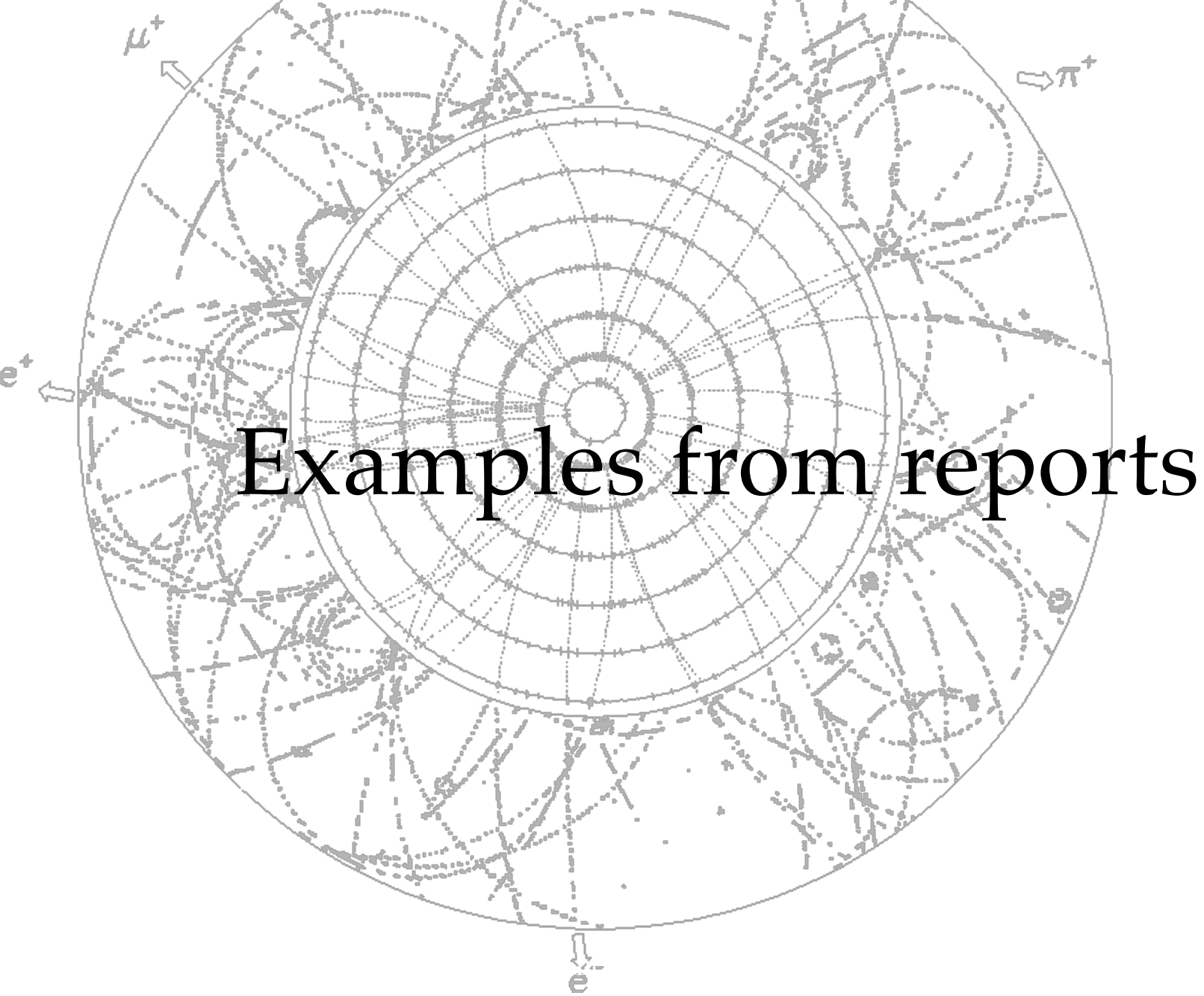
Note that correlations can be used to your advantage!

When are

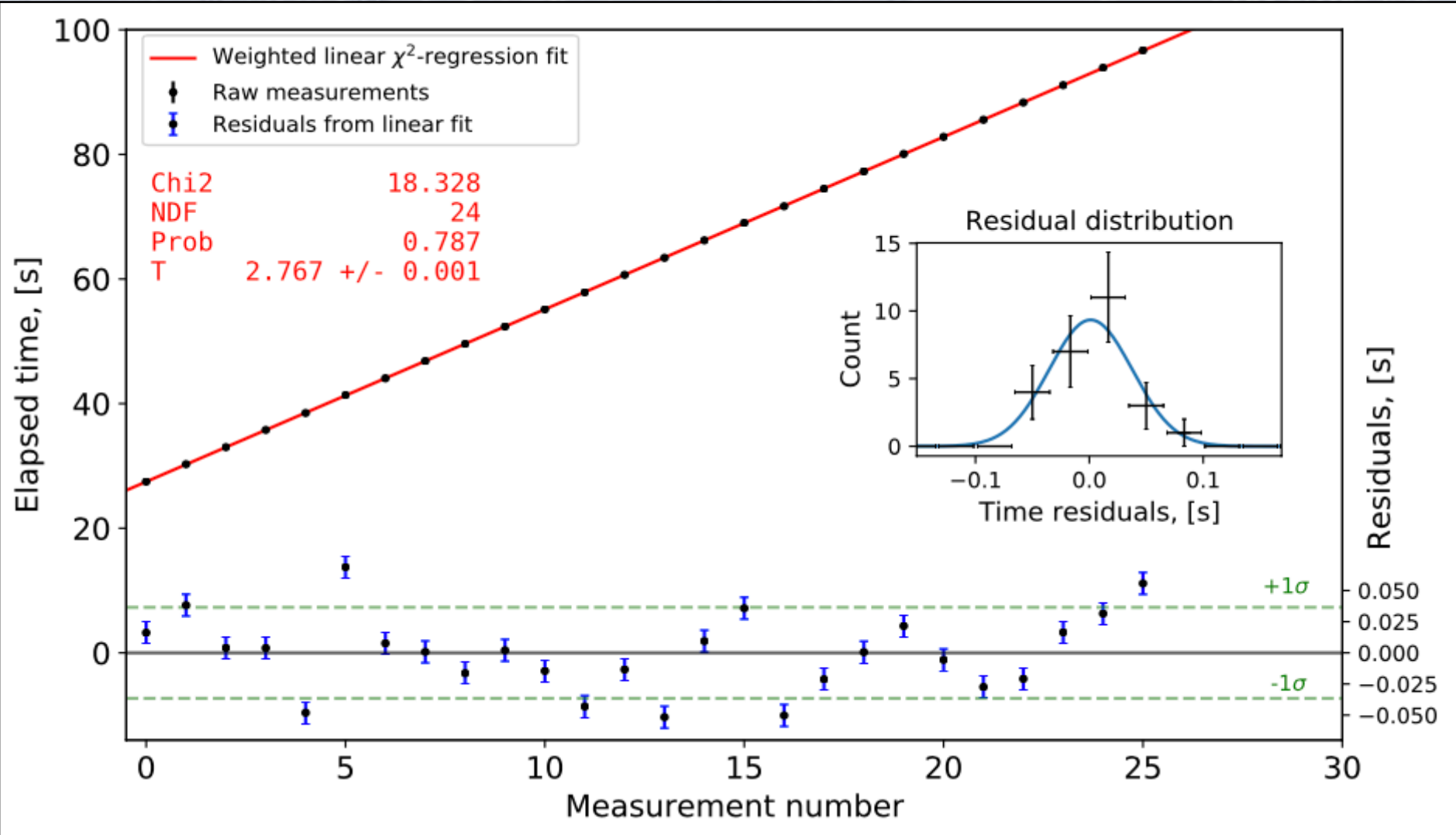
- When the
 - When the
 - When the
 - When the
- Imagine measuring the pendulum length with the laser. You have 2mm precision written on the instrument, but you actually don't know, and it is hard to claim anything better.

Examples

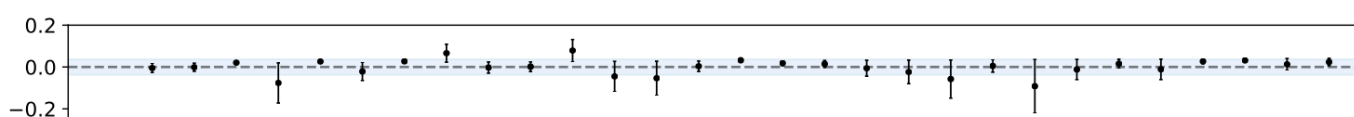
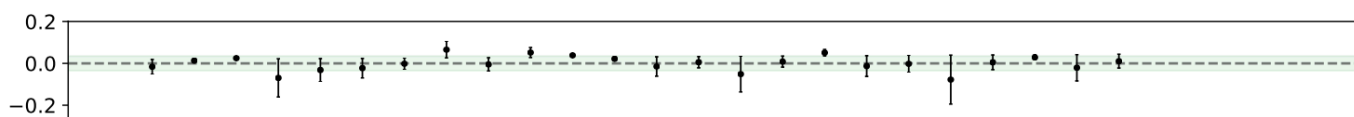
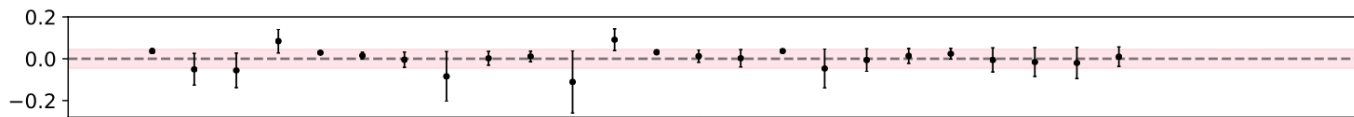
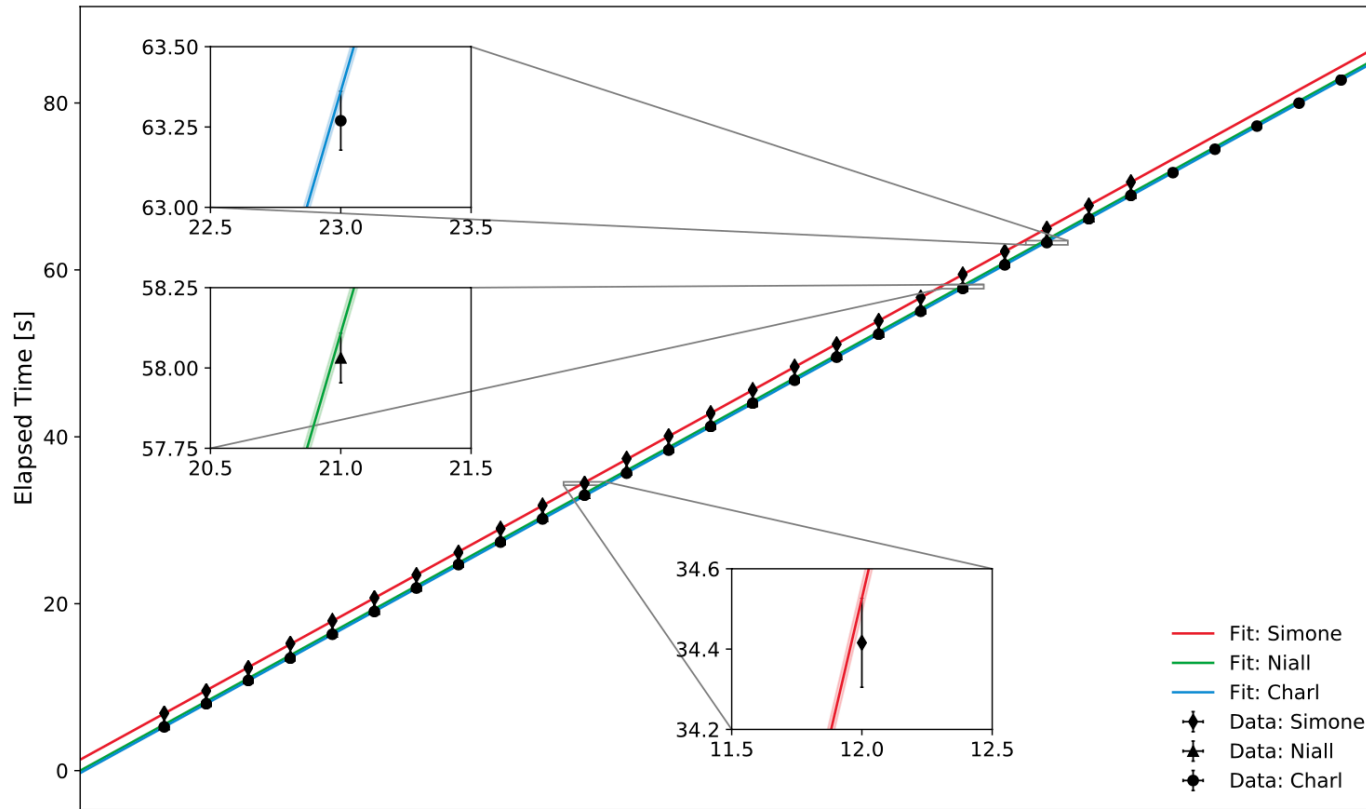
- The gate the floor and up to the pendulum, and from the floor to the ceiling, then any bias will cancel, and repeated measurements can improve the precision beyond 2mm!
- The pendulum times as measured by different people at the same time?
- The accelerations when repeating the balls roll?
- Two measurements of g based on turning setup around and measuring angle
 - a) with a goniometer (i.e. angle measuring device)?
 - b) using trigonometry?



Great drawings & plotting

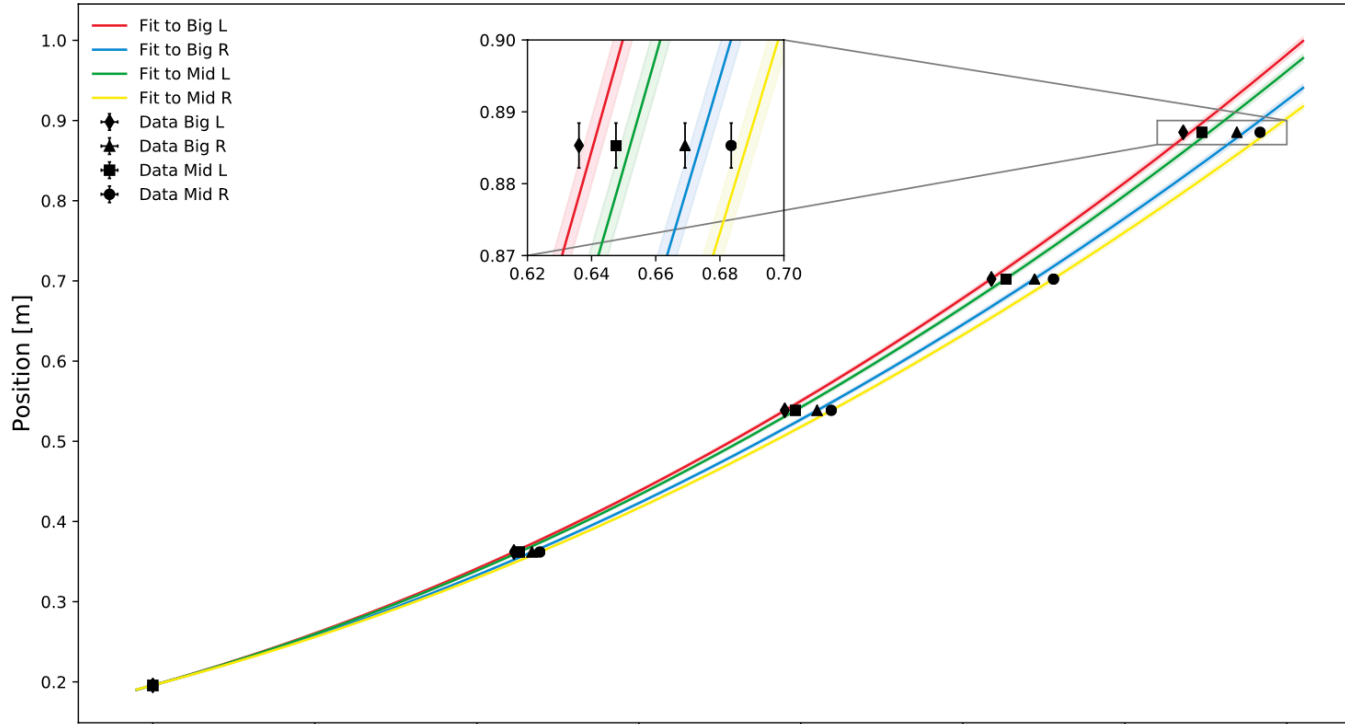


Pendulum



Measurement Number

Ball on Incline



Fit parameters: Big L
 $a = 1.58 \pm 0.01$
 $b = 0.57 \pm 0.0$
 $c = 0.2 \pm 0.0$
 $p(\text{Chi}^2=5.91, \text{N dof}=2)=0.02$

Fit parameters: Big R
 $a = 1.42 \pm 0.01$
 $b = 0.54 \pm 0.0$
 $c = 0.2 \pm 0.0$
 $p(\text{Chi}^2=5.68, \text{N dof}=2)=0.02$

Fit parameters: Mid L
 $a = 1.52 \pm 0.01$
 $b = 0.56 \pm 0.0$
 $c = 0.2 \pm 0.0$
 $p(\text{Chi}^2=5.66, \text{N dof}=2)=0.02$

Fit parameters: Mid R
 $a = 1.36 \pm 0.01$
 $b = 0.53 \pm 0.0$
 $c = 0.2 \pm 0.0$
 $p(\text{Chi}^2=5.71, \text{N dof}=2)=0.02$



Residuals: Big L
 Shaded region correspond to $\pm\sigma$



Residuals: Big R
 Shaded region correspond to $\pm\sigma$



Residuals: Mid L
 Shaded region correspond to $\pm\sigma$



Residuals: Mid R
 Shaded region correspond to $\pm\sigma$

Time [s]

Great drawings & plotting

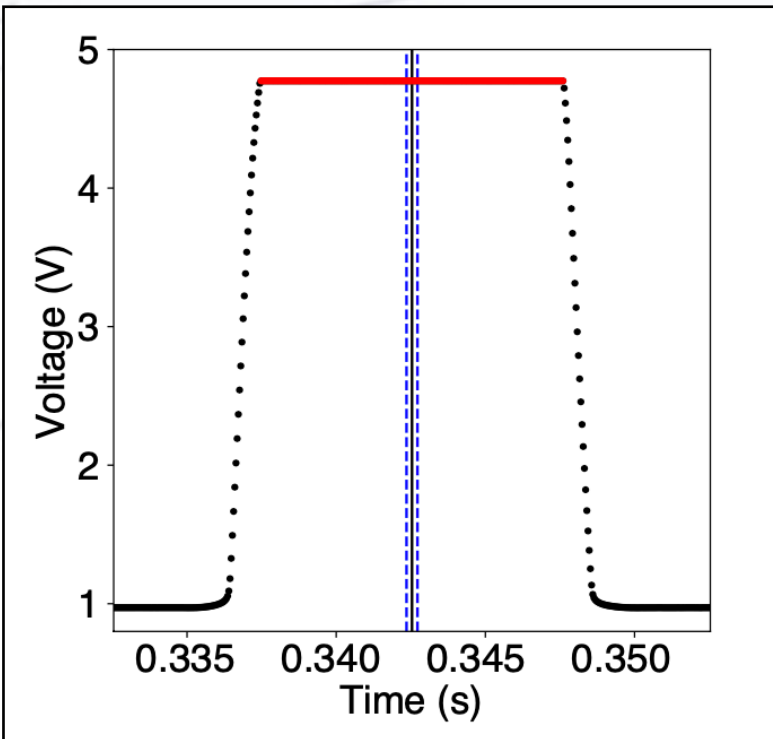
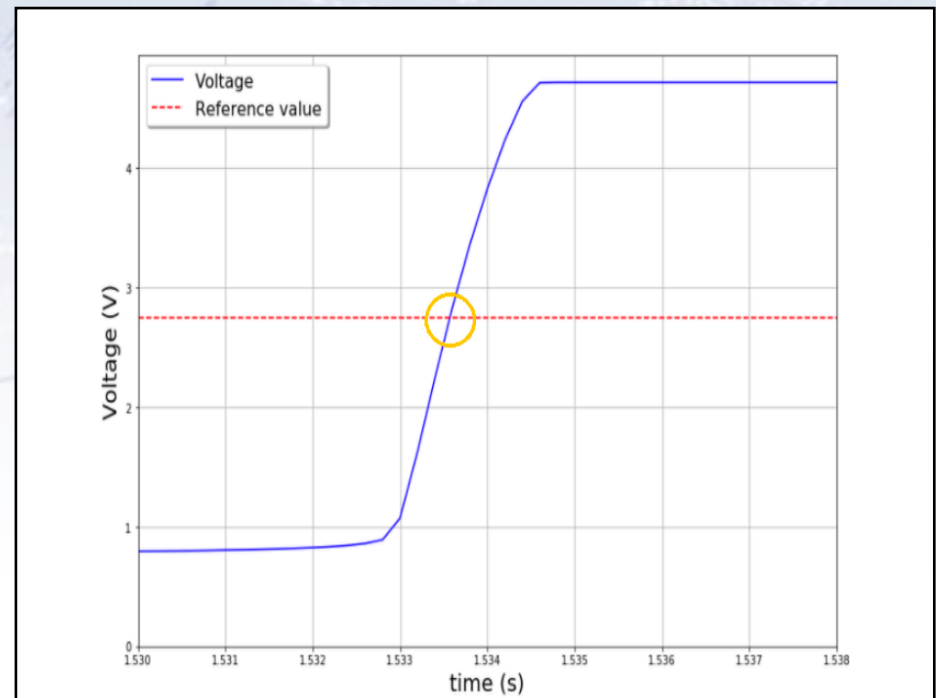
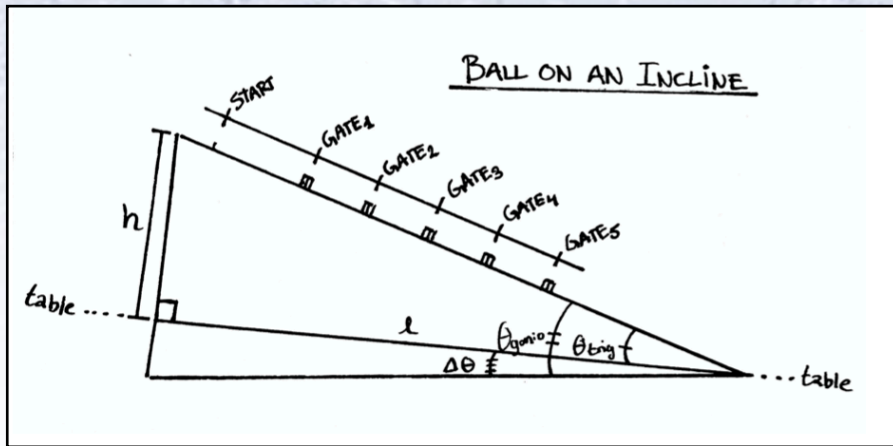


FIG. 5. The criterion followed for selecting the characteristic time of each peak: the central value of the rising ramp in each peak (in circle), i.e. when the ball arrives at the laser beam. More precisely, it was determined as the closest measurement from above to $(t_{V,max}-t_{V,min})/2$.

Great drawings & plotting

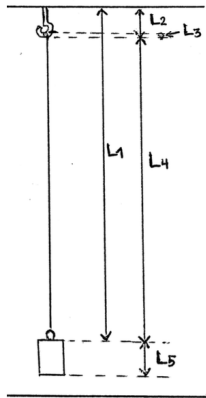


Figure 1: Experimental setup for the pendulum showing which lengths were measured and combined to a final estimate of the length of the pendulum. L_1 , L_2 and L_5 were measured with a measuring tape, L_3 with a caliper and L_4 with a laser distance meter

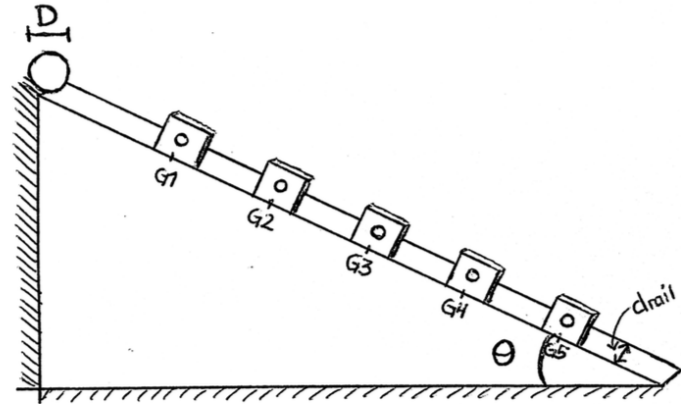
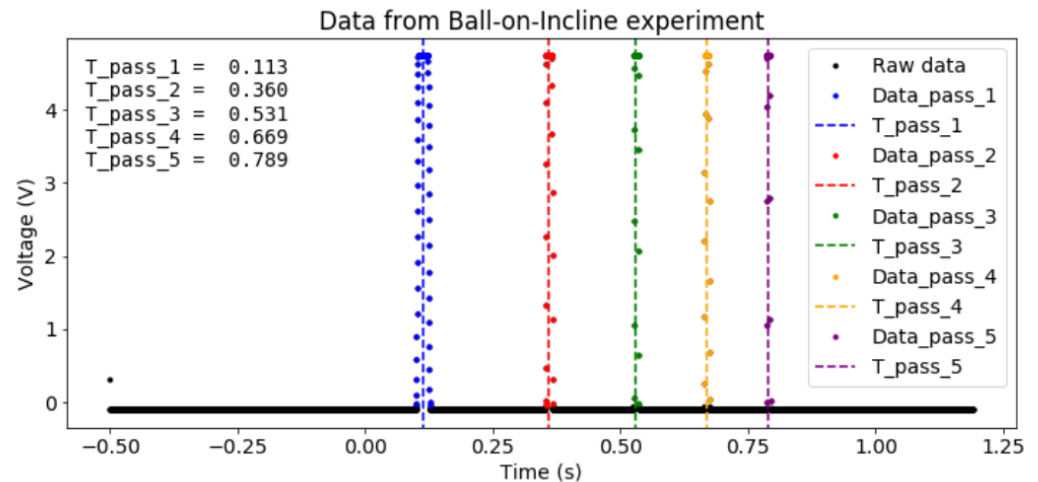
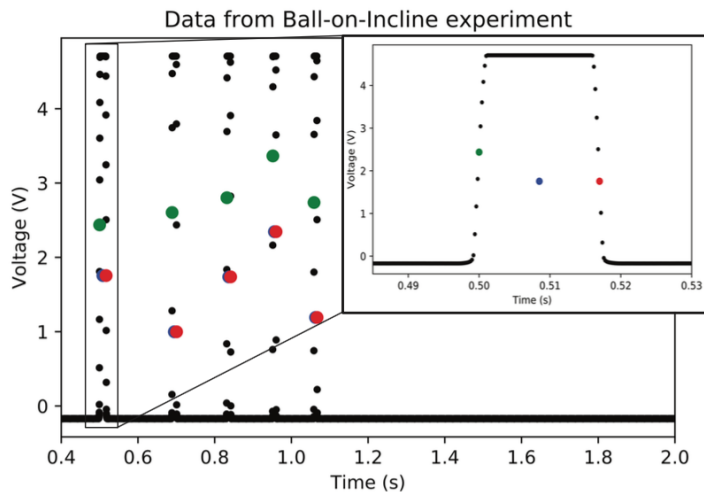


Figure 2: Experimental setup for the incline showing which measurements were made and where the gate positions G_1 - G_5 were measured.



Residuals and plotting

Good illustration of why plotting is useful!

Residuals for Group members

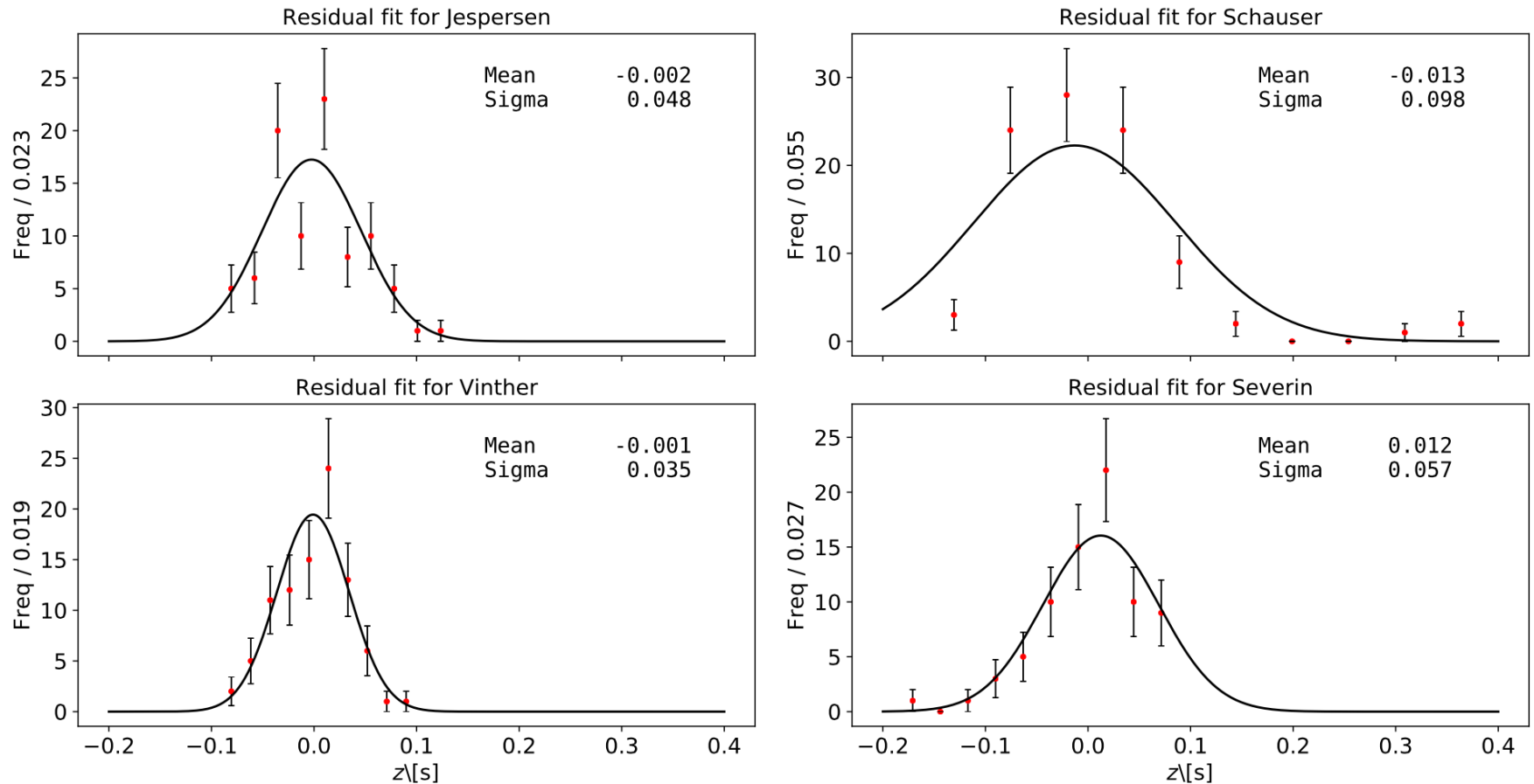
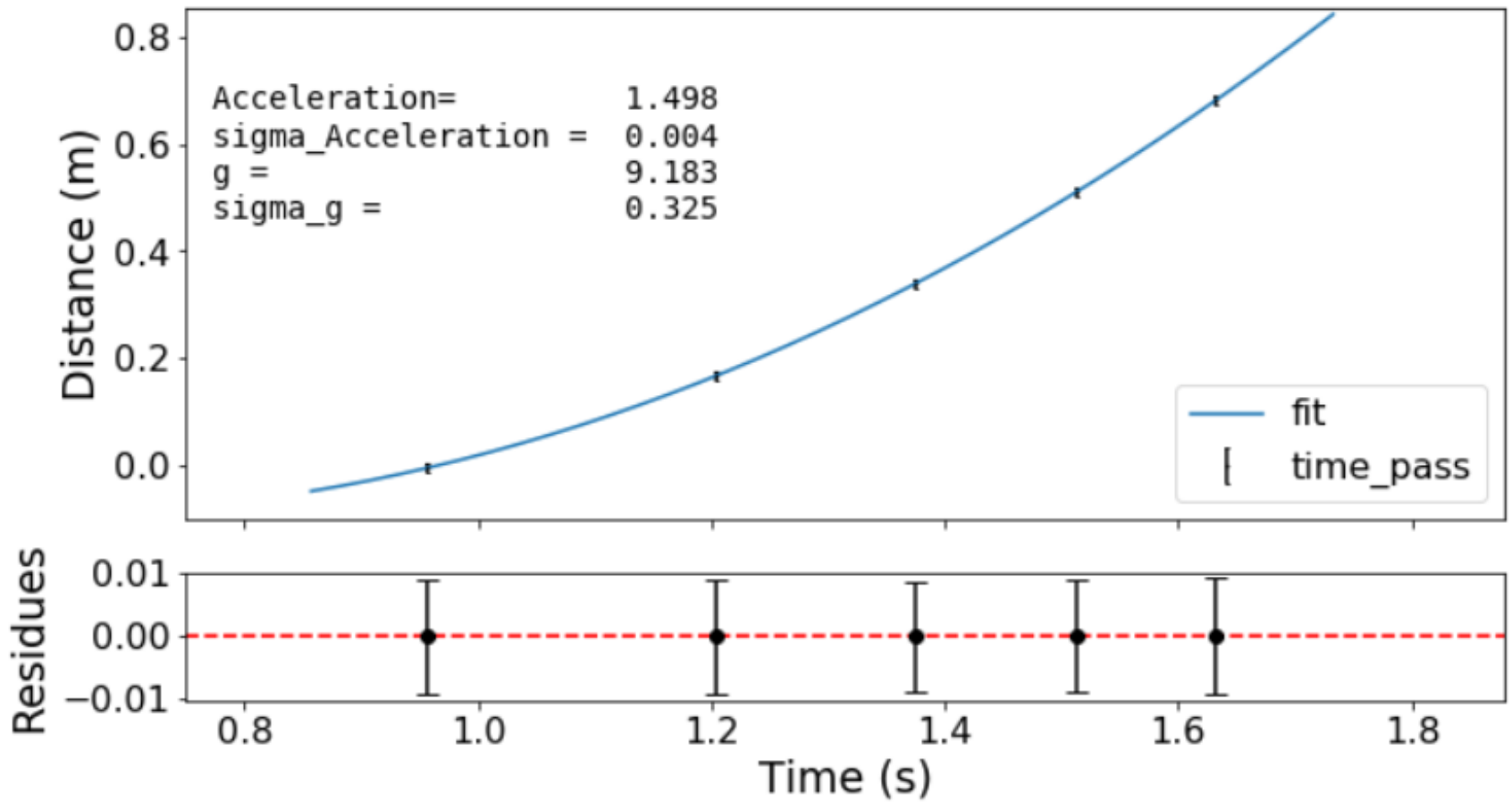


Figure 6: Normal distribution of the residuals fitted with a binned likelihood fit.

Nice fit, but...



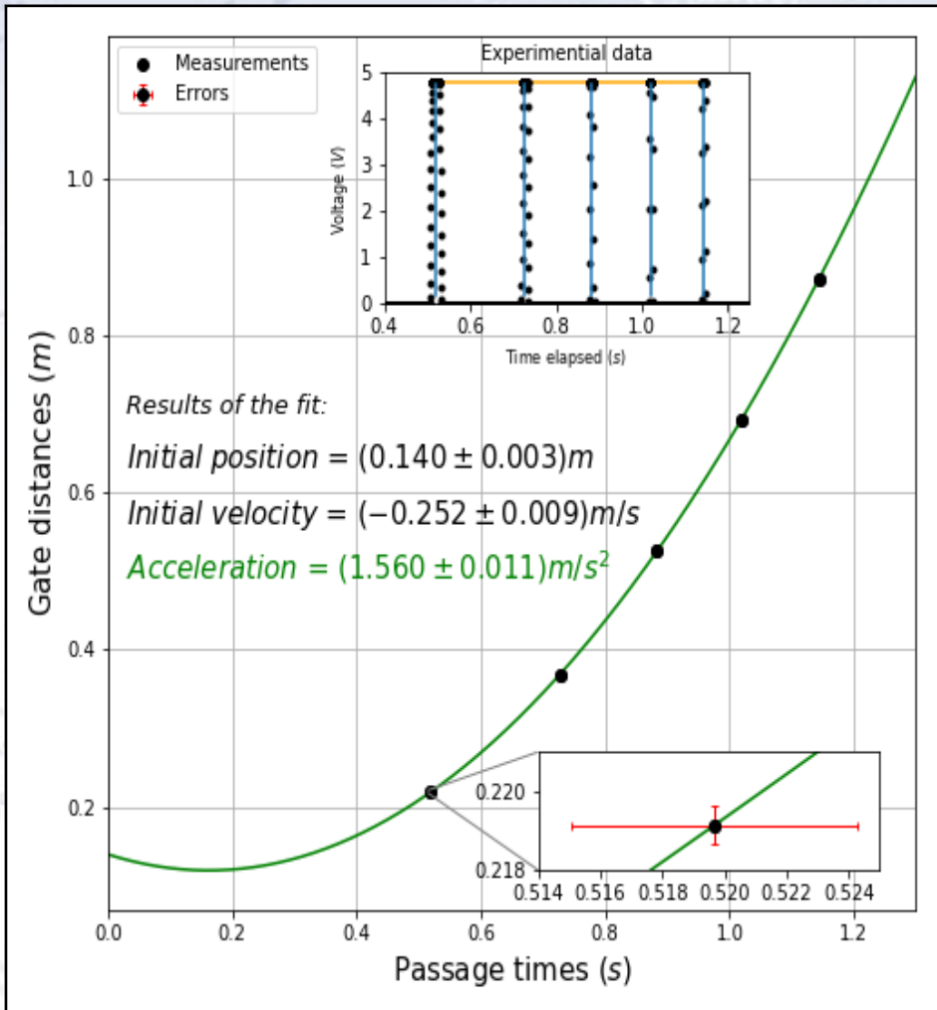
Nice figure/zoom and text

The acceleration for each of the four setups is found in four steps: *Firstly* the individual sets of photogate measurements are fitted to a parabola model, Eq (1.4), using a χ^2 fit including uncertainties on positions of the photogates. *Secondly* these fit results are used to propagate the time uncertainties on the photogate measurements onto the position axis (i.e. $\sigma_{s,tot}^2 = \sigma_s^2 + \left(\frac{\partial s_{fit}}{\partial t}\right)^2 \sigma_t^2$). *Thirdly* the individual data measurements are fitted anew with the propagated uncertainty, and *fourthly* a weighted mean is calculated for the collected fit acceleration parameters for each of the four setups according to Eq. (1.5). For an illustration see Figure 2 and Table II for the found values.

The figure shows nicely what the points fitted looks like up close. An alternative is to do a residual plot. I'm surprised by the (very) large error in time.

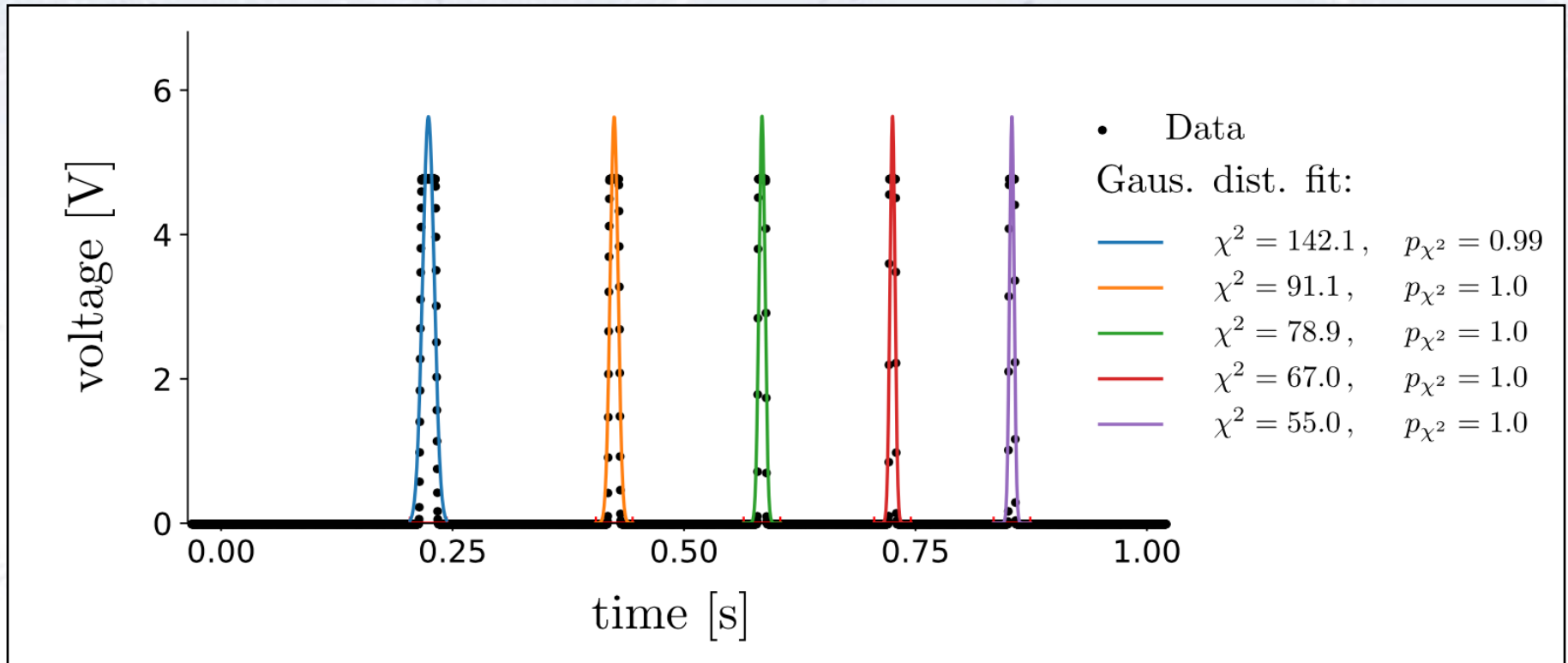
Text explains principle very nicely.

NOTE: Figure and text does not match!



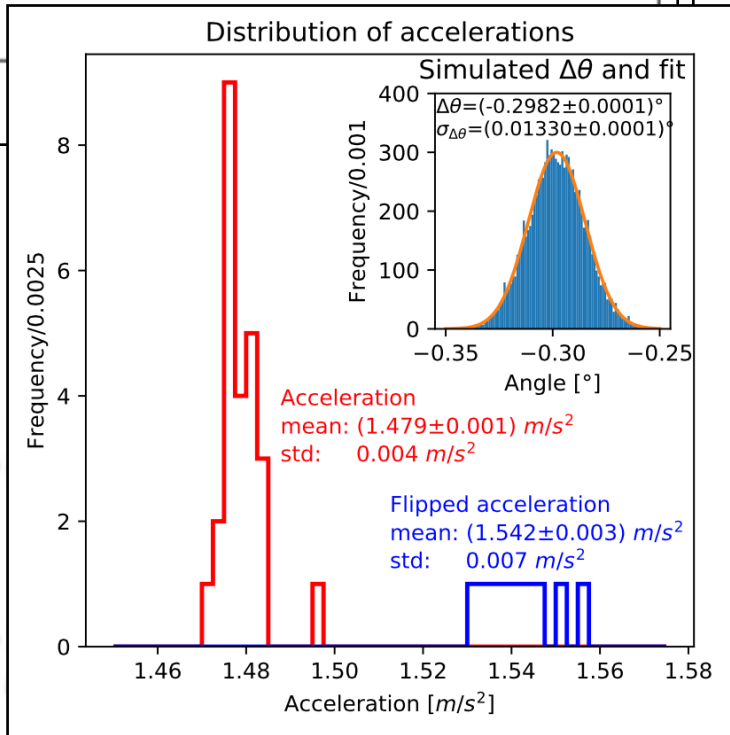
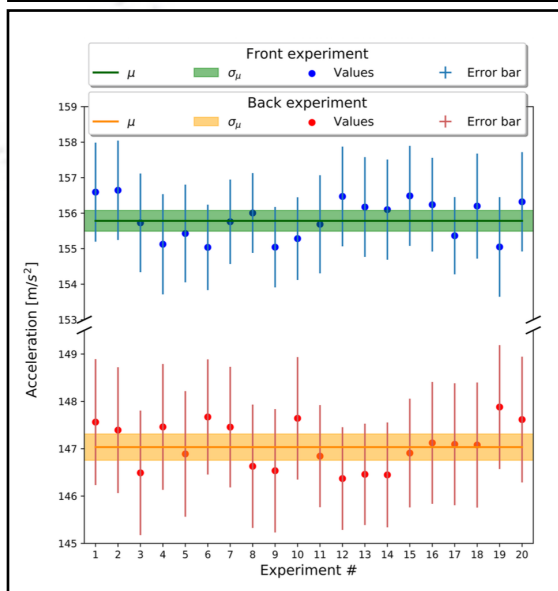
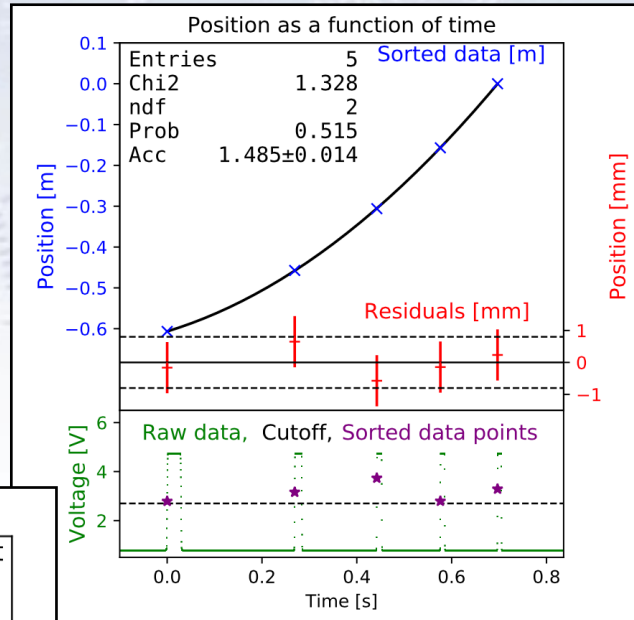
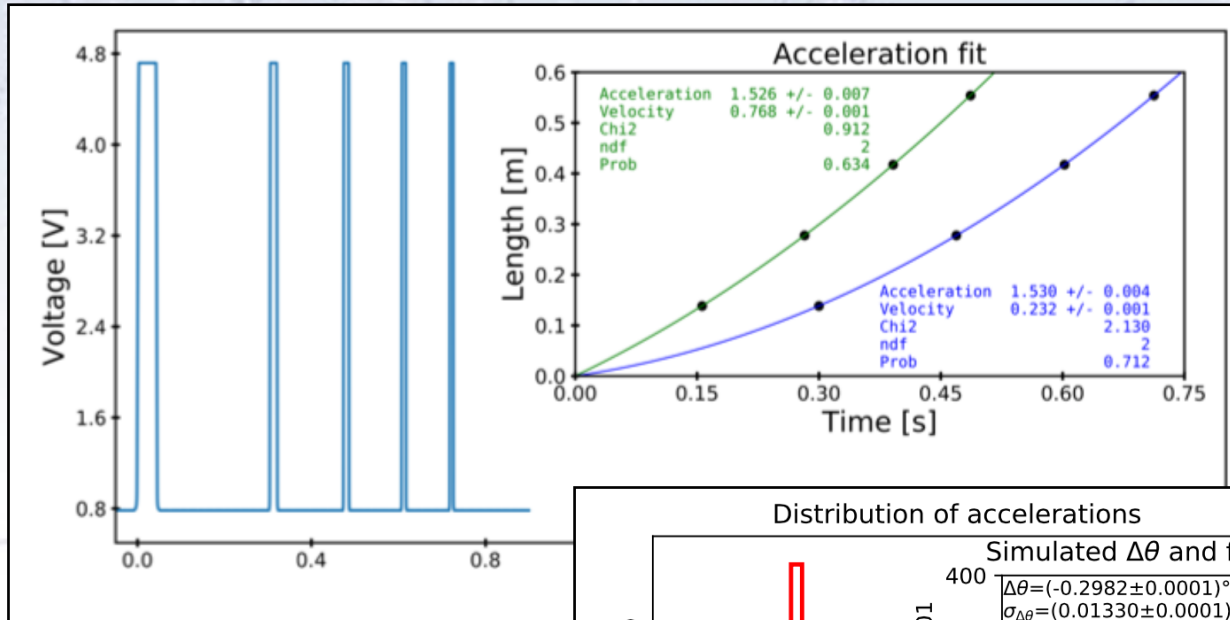
Careful...

Not everything is Gaussian, for example voltage peaks!



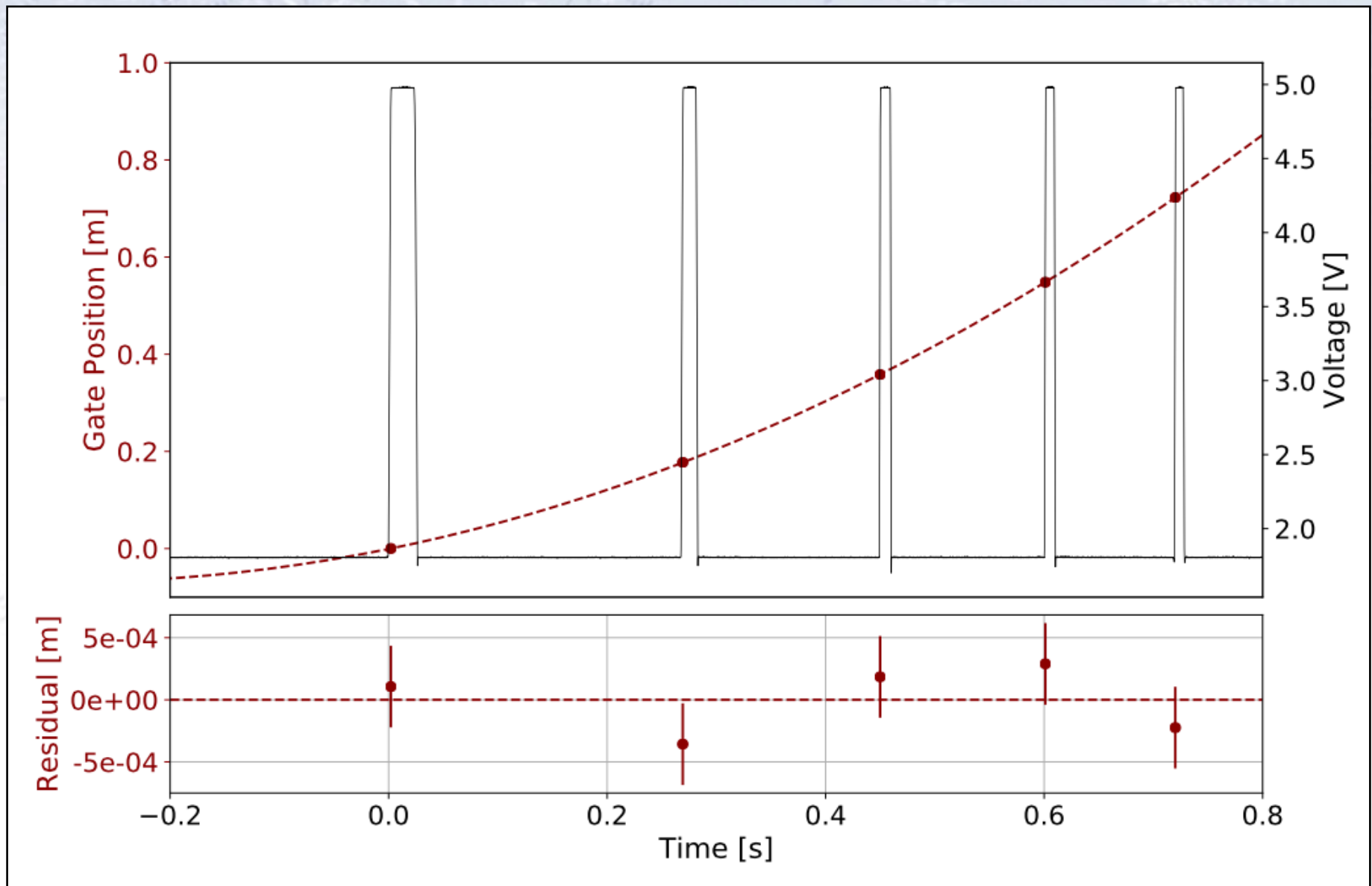
Some also tried to apply “peak finder” software. This might be OK, but you have to know, what they exactly do, as there is no guarantee, that it fits your purpose! In all cases, this requires a **reference**.

Great figures from the past



These figures are just a selection of the many great figures produced in your reports. Thanks a lot for that.

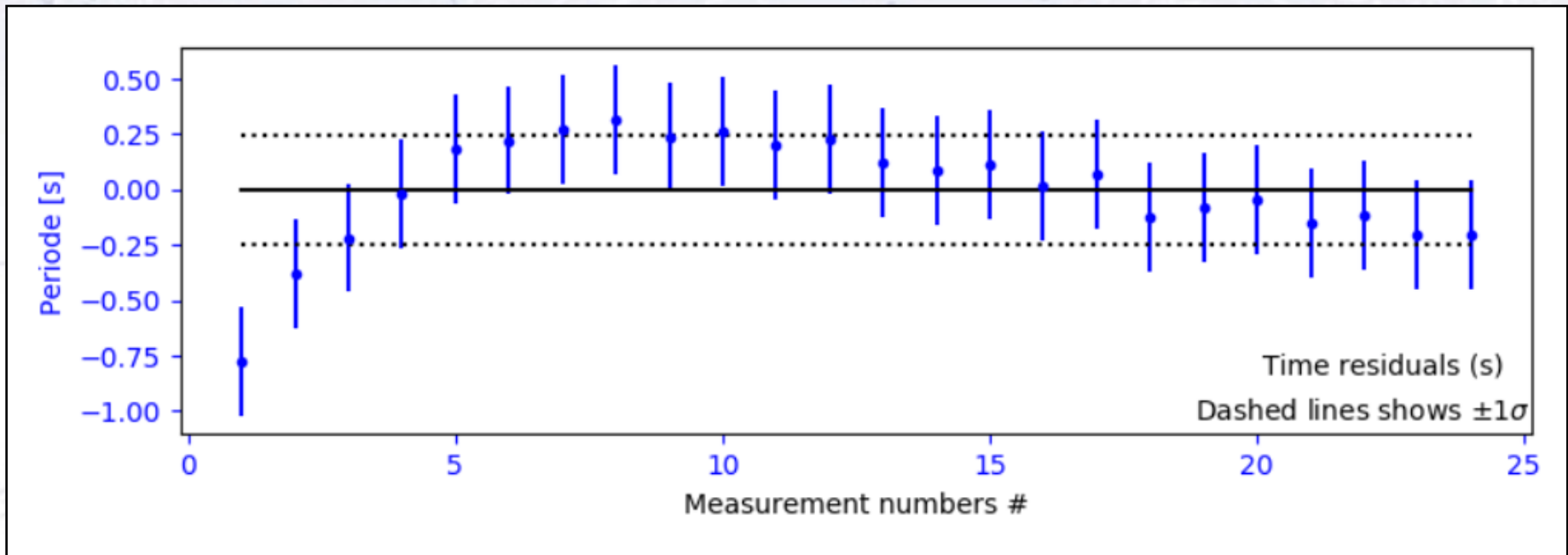
Great figures



Great figures

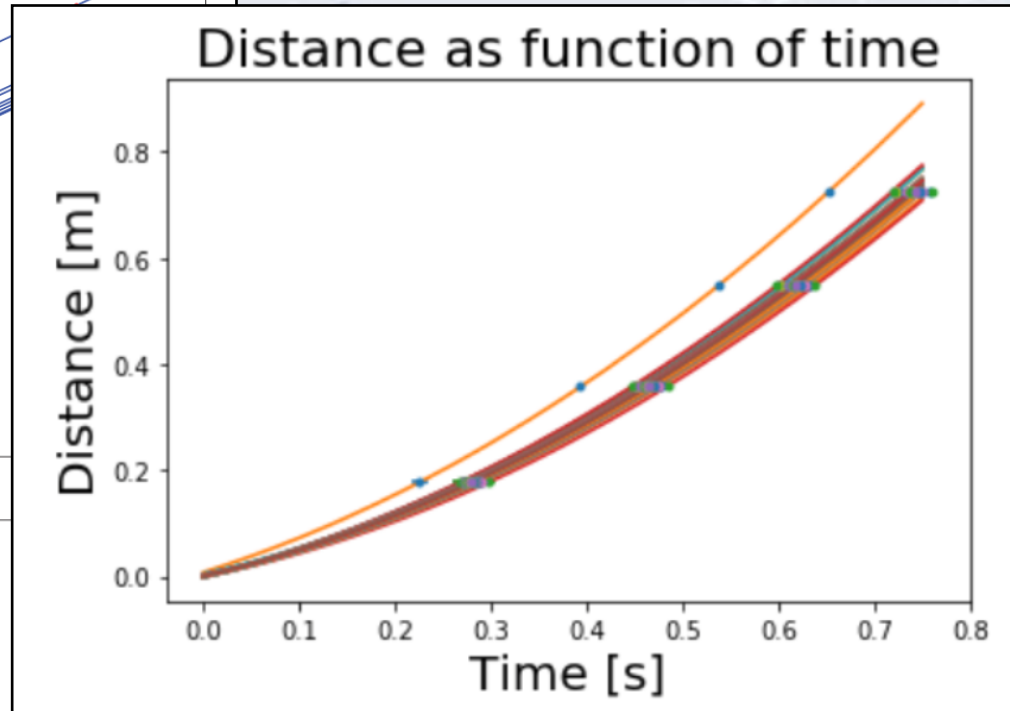
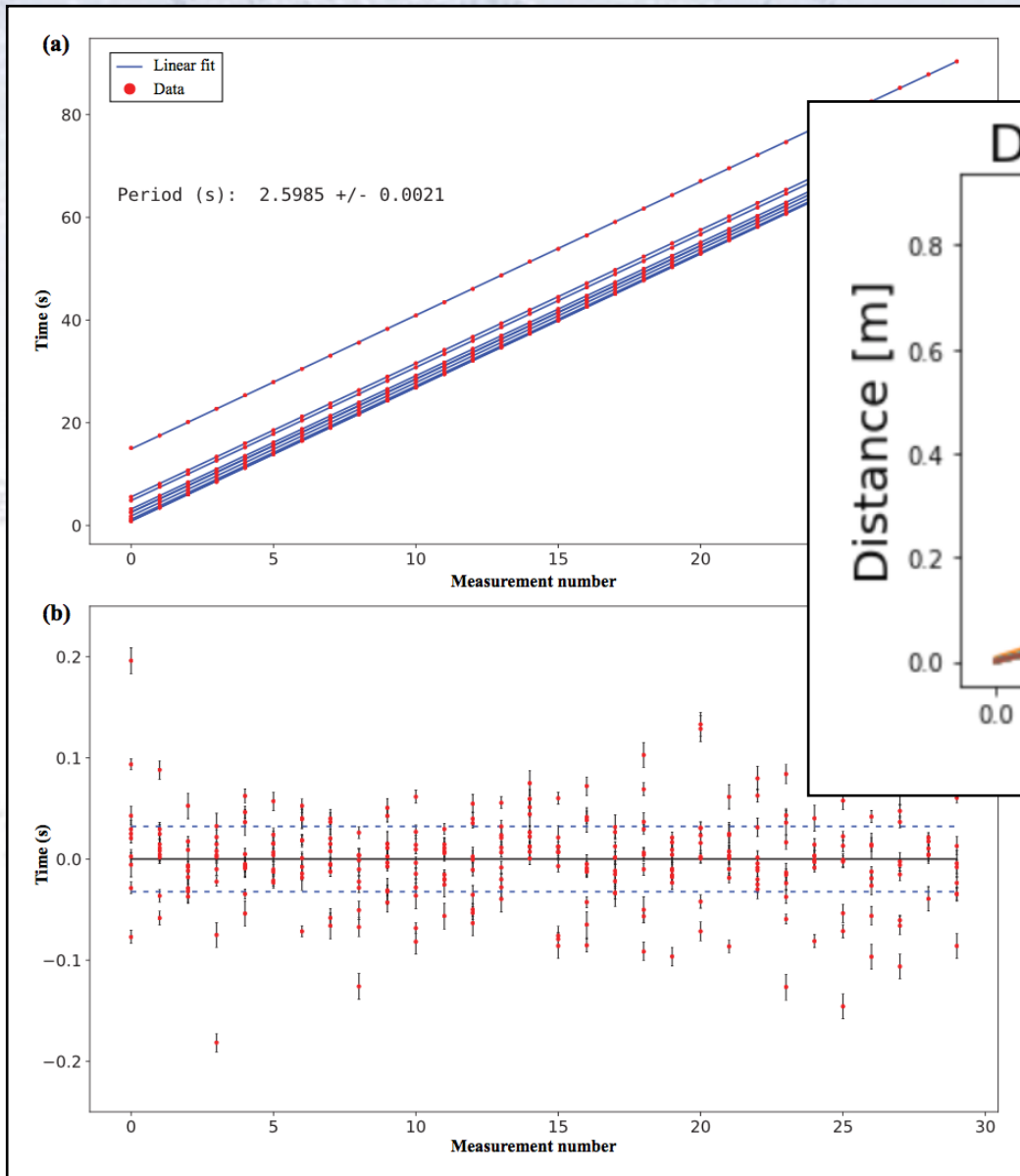
I like this figure....

It argues very strongly, why this data was discarded.

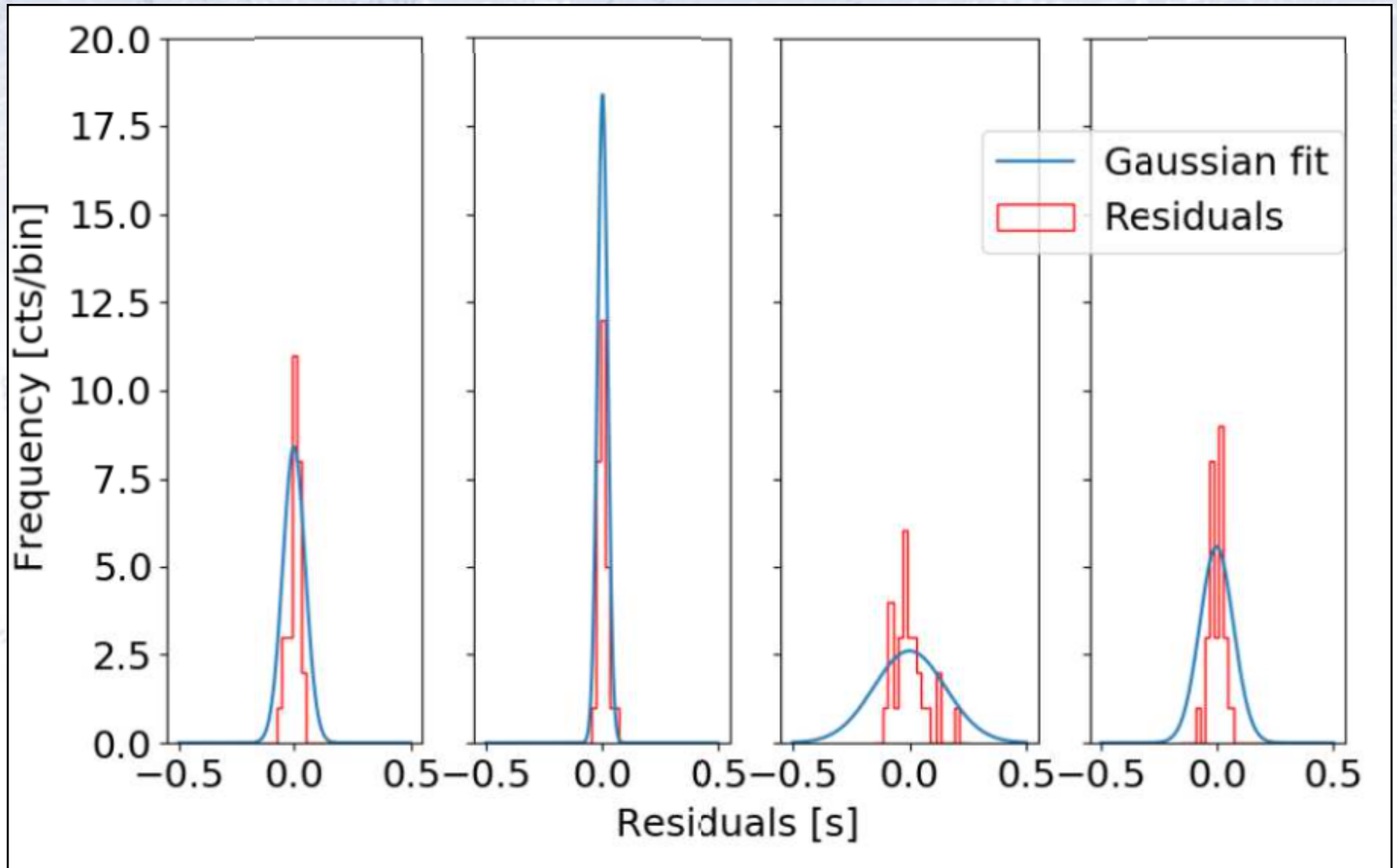


While such plots are not easy to put into publications, they server very well in the appendix of a thesis.

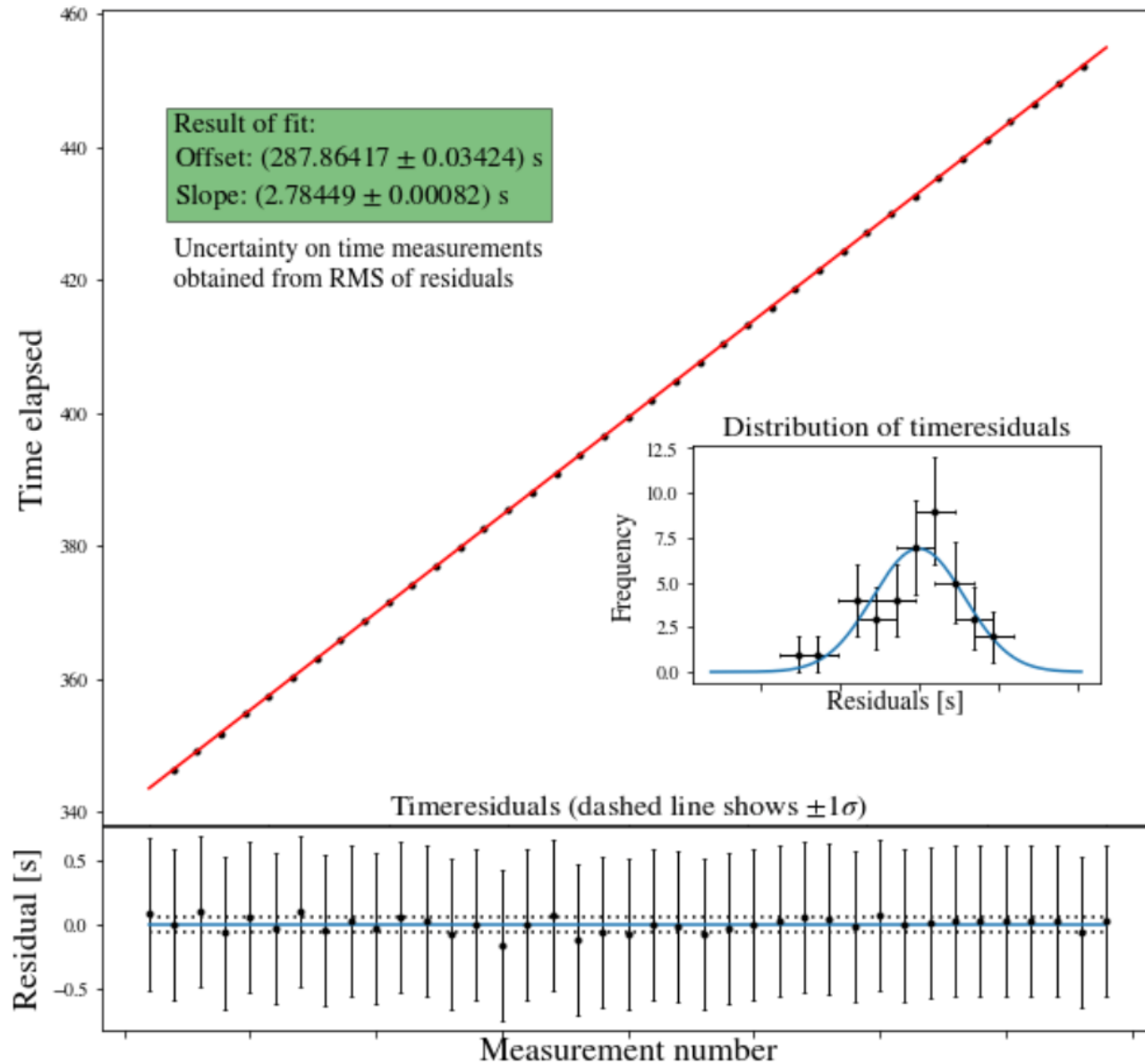
Great figures, but...



Great figure, but...



Great figure, but...



Great tables

| Pendulum | | |
|--------------|----------|-------------------|
| Variable | Value | Statistical error |
| Length L_1 | 2.6907 m | 0.0017 m |
| L_2 | 2.6902 m | 0.0017 m |
| L_3 | 2.6922 m | 0.0017 m |
| L_4 | 2.6921 m | 0.0017 m |
| Period T_1 | 3.2879 s | 0.0012 s |
| T_2 | 3.2881 s | 0.0012 s |
| T_3 | 3.2872 s | 0.0004 s |
| T_4 | 3.2900 s | 0.0011 s |

| Variable | Mean | Statistical error | RMS | χ^2 | χ^2_{prob} |
|----------|---------|-------------------|---------|----------|-----------------|
| T | 3.2876s | 0.0004s | 0.0006s | 5.85 | 0.12 |
| L | 2.6913m | 0.0009m | 0.0005m | 1.02 | 0.80 |

Great tables

Pendulum

g - Pendulum

| Variable | Value | Statistical error |
|----------------|-----------------------|-----------------------|
| g | 9.830m/s ² | 0.004m/s ² |
| Err. cont. L | 0.003m/s ² | |
| Err. cont. T | 0.002m/s ² | |

| Variable | Mean | Statistical error | RMS | χ^2 | χ^2_{prob} |
|----------|---------|-------------------|---------|----------|-----------------|
| T | 3.2876s | 0.0004s | 0.0006s | 5.85 | 0.12 |
| L | 2.6913m | 0.0009m | 0.0005m | 1.02 | 0.80 |

Great table, but...

The two bottom values are probably correlated, and thus not fit for a weighted mean.

| | Value | Con. σ_g | Con. σ_g , flipped |
|----------------------|--------------------|---------------------|---------------------------|
| a [m/s^2] | 1.479 ± 0.001 | 0.008 [m/s^2] | - |
| a_F [m/s^2] | 1.542 ± 0.003 | - | 0.015 [m/s^2] |
| θ [deg] | 13.99 ± 0.02 | 0.015 [m/s^2] | 0.014 [m/s^2] |
| $\Delta\theta$ [deg] | -0.298 ± 0.013 | 0.008 [m/s^2] | 0.008 [m/s^2] |
| r [mm] | 6.374 ± 0.005 | 0.001 [m/s^2] | 0.001 [m/s^2] |
| d [mm] | 6.025 ± 0.005 | 0.002 [m/s^2] | 0.002 [m/s^2] |
| Result | Value | Value, flipped | Weighted mean |
| g [m/s^2] | 9.470 ± 0.019 | 9.47 ± 0.02 | 9.470 ± 0.014 |

Even great equations!

$$\begin{aligned} \sigma_g^2 = & \left. \sigma_{\Delta\theta}^2 \frac{\left(\frac{8R^2}{4R^2-d^2} + 5\right)^2 a^2 \cos(\theta + \Delta\theta)^2}{25 \sin(\theta + \Delta\theta)^4} \right\} I_{\Delta\theta}^2 \\ & + \left. \sigma_{\theta}^2 \frac{\left(\frac{8R^2}{4R^2-d^2} + 5\right)^2 a^2 \cos(\theta + \Delta\theta)^2}{25 \sin(\theta + \Delta\theta)^4} \right\} I_{\theta}^2 \\ & + \left. \sigma_R^2 \frac{256 \left(\frac{4R^3}{(4R^2-d^2)^2} - \frac{R}{4R^2-d^2}\right)^2 a^2}{25 \sin(\theta + \Delta\theta)^2} \right\} I_R^2 \\ & + \left. \sigma_d^2 \frac{256 R^4 a^2 d^2}{25 (4R^2 - d^2)^4 \sin(\theta + \Delta\theta)^2} \right\} I_d^2 \\ & + \left. \sigma_a^2 \frac{\left(\frac{8R^2}{4R^2-d^2} + 5\right)^2}{25 \sin(\theta + \Delta\theta)^2} \right\} I_a^2 \end{aligned}$$

Examples from other reports

Using different ball sizes gave variations in the result (10, 12.7, and 15 mm).

$$g_{Big} = (9.865 \pm 0.040_{stat} \pm 0.001_{sys}) \text{ m/s}^2,$$

$$g_{Medium} = (9.822 \pm 0.042_{stat} \pm 0.001_{sys}) \text{ m/s}^2,$$

$$g_{Small} = (9.741 \pm 0.055_{stat} \pm 0.001_{sys}) \text{ m/s}^2,$$

Table of (changing) impact is GREAT:

| Parameter | Big Ball | Medium Ball | Small Ball |
|----------------|----------|-------------|------------|
| a | 0.024 | 0.024 | 0.022 |
| θ | 0.030 | 0.030 | 0.030 |
| $\Delta\theta$ | 0.0014 | 0.0014 | 0.0013 |
| D | 0.003 | 0.007 | 0.019 |
| d | 0.010 | 0.016 | 0.035 |
| Total | 0.040 | 0.042 | 0.055 |

Examples from other reports

The table shows a great example of reporting the inputs for the measurement of g , along with their impact on the final precision.

The figure shows the result of 10 period measurements and their weighted average, where the ChiSquare reveals, that one measurement seems flawed.

| Variable | Value | Stat. error | Impact on g |
|---------------------------------|------------------------|--------------------------------|-----------------------------------|
| Pendulum | | | |
| Period T | 8.5373 s | 0.0005 s | 0.0011 $\frac{m}{s^2}$ |
| Length L | 18.1363 m | 0.0002 m | 0.0001 $\frac{m}{s^2}$ |
| Gravity g | 9.8319 $\frac{m}{s^2}$ | 0.0011 $\frac{m}{s^2}$ | |
| Ball on Incline | | | |
| Acc. a_{LR} | 0.7193 $\frac{m}{s^2}$ | 0.0018 $\frac{m}{s^2}$ | 0.02 $\frac{m}{s^2}$ |
| Acc. a_{RL} | 0.7851 $\frac{m}{s^2}$ | 0.0019 $\frac{m}{s^2}$ | 0.02 $\frac{m}{s^2}$ |
| Angle θ | 7.303° | 0.002° | * |
| $\Delta\theta$ | 0.321° | 0.013° | * |
| Radius R | 0.0071 m | 0.0002 m | 0.04 $\frac{m}{s^2}$ |
| Width d | 0.0064 m | 0.0002 m | 0.05 $\frac{m}{s^2}$ |
| Gravity g | 8.89 $\frac{m}{s^2}$ | 0.06 $\frac{m}{s^2}$ | |
| Resulting g | 9.8316 | ± 0.0011 | $\frac{m}{s^2}$ |

Table III: Final results from both experiments. θ and $\Delta\theta$ have a combined impact on g , which is calculated as 0.02° .

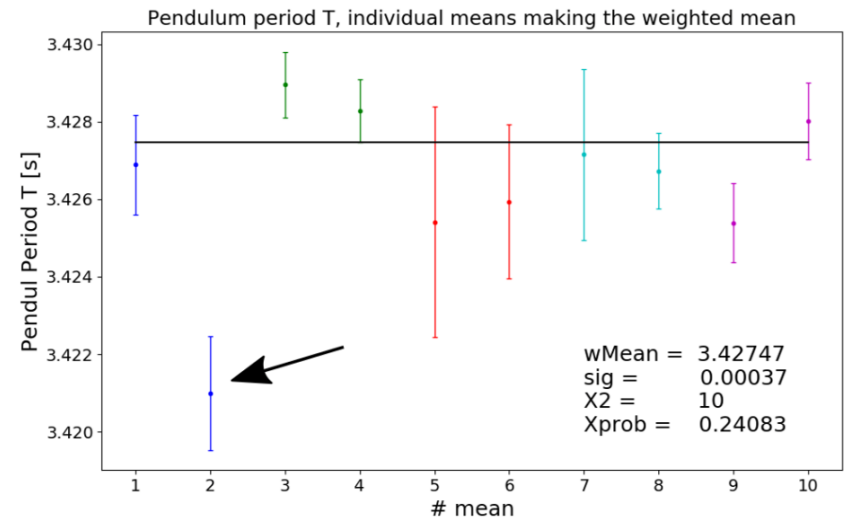
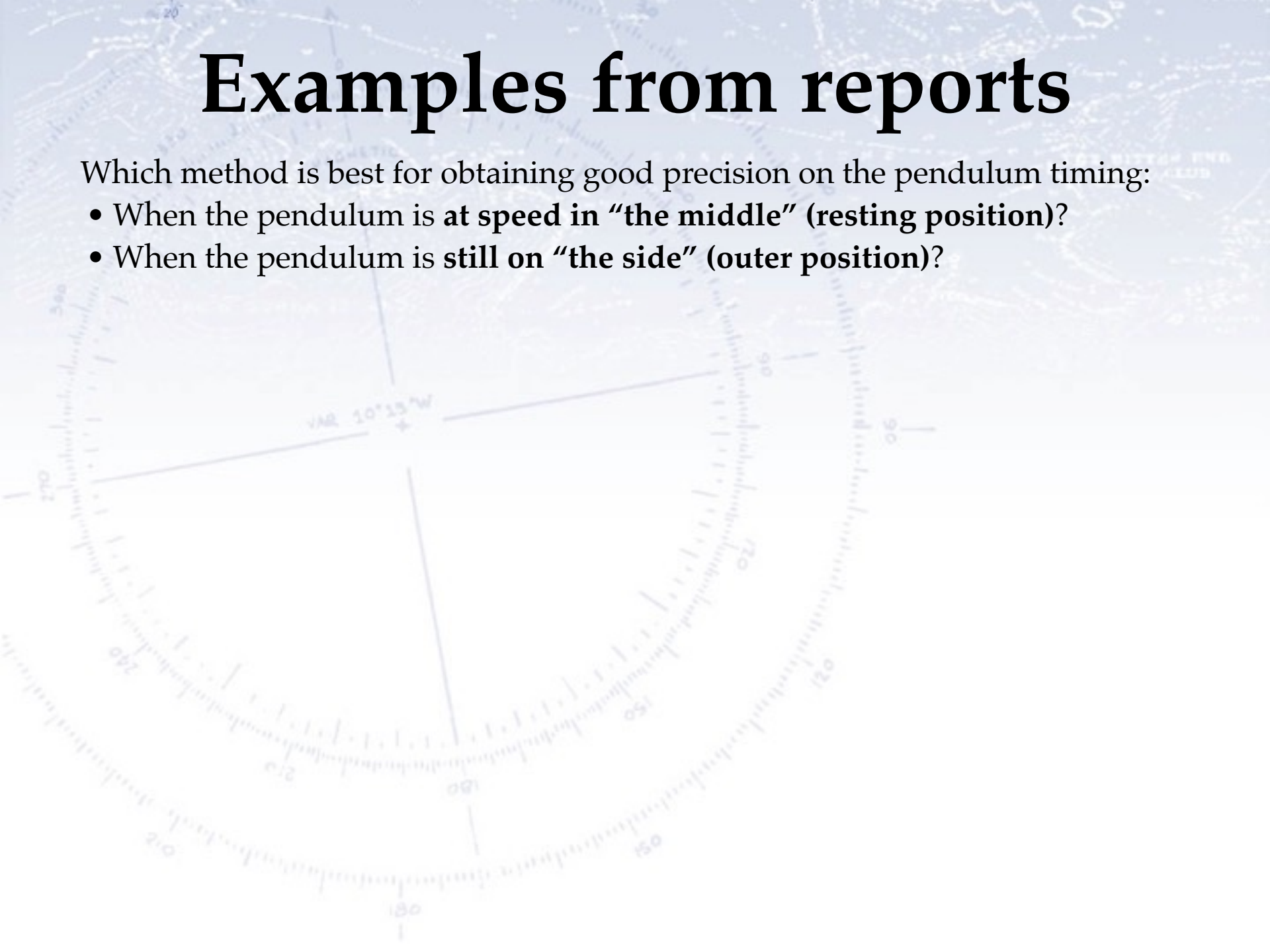


Figure 3: Period T , colours indicate different peoples individual mean, and the black line is the weighted mean of $T=3.4275 \pm 0.0004$. The highlighted data point is 4.4σ away from the mean, we excluded this from our data analysis. We note the χ^2 is within our 9 Degrees of Freedom. The χ^2 prob. is the probability of getting a worse fit, 24% is somewhat low but not low enough to raise any alarms

Examples from reports

Which method is best for obtaining good precision on the pendulum timing:

- When the pendulum is at speed in “the middle” (resting position)?
- When the pendulum is still on “the side” (outer position)?



Examples from reports

Which method is best for obtaining good precision on the pendulum timing:

- When the pendulum is at speed in “the middle” (resting position)?
- When the pendulum is still on “the side” (outer position)?

That can be measured:

| Variable | Value | Stat. error |
|------------------------------|--------|-------------|
| First approach: Resting pos. | | |
| μ_T [s] | 3.4306 | 0.0019 |
| g [m/s ²] | 9.8098 | 0.0035 |
| Second approach: Outer pos. | | |
| μ_T [s] | 3.4345 | 0.0071 |
| g [m/s ²] | 9.7826 | 0.0113 |

Examples from reports

Remember to only put 1-2 significant digits on the error, and then the SAME number of digits on the result.

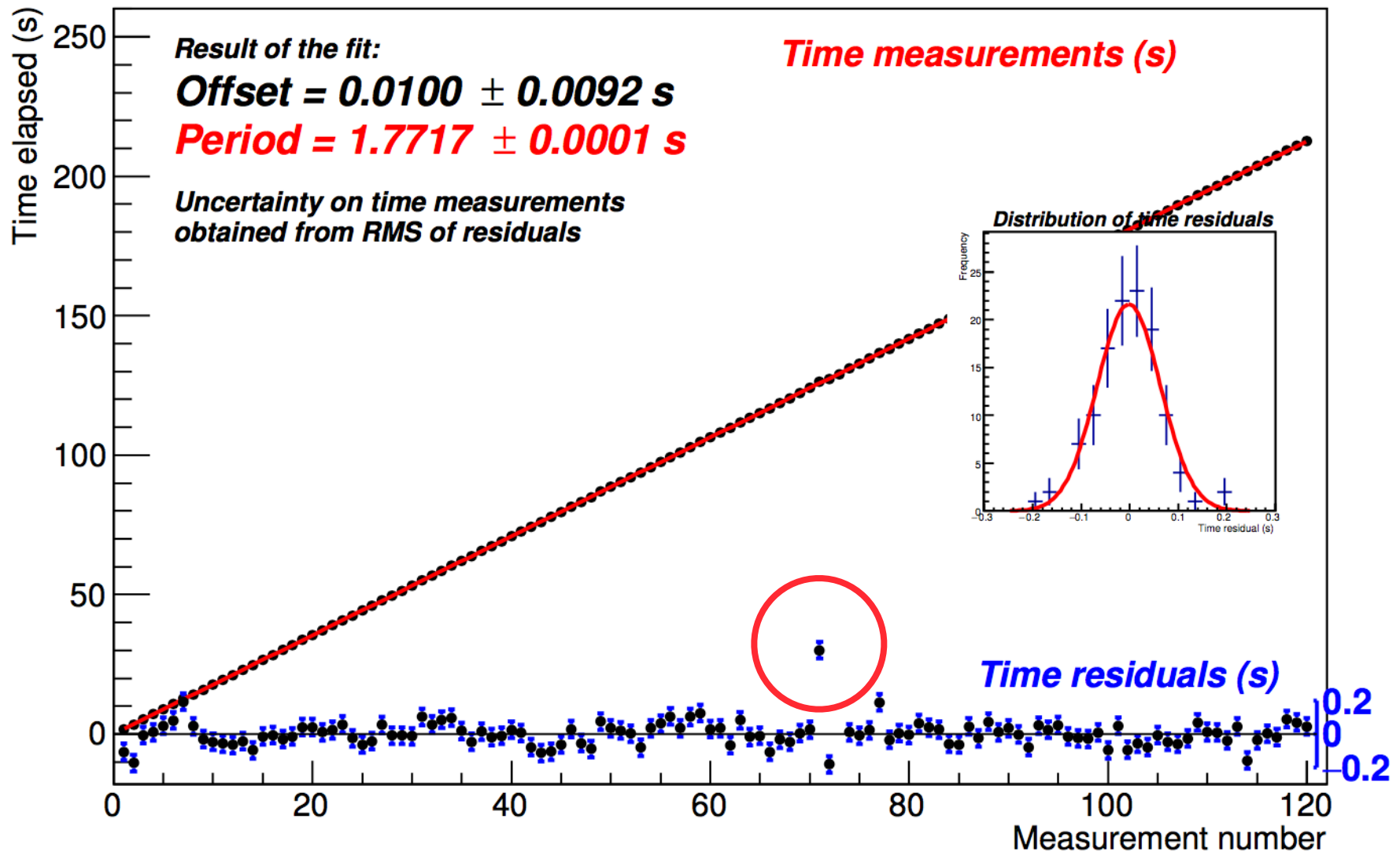
| Exp. | a_h | σ_{ah} | a_v | σ_{av} |
|---------------|------------------------------|--------------------------------|------------------------------|--------------------------------|
| 1 | 1.541 m/s ² | 0.01170 m/s ² | 1.473 m/s ² | 0.008515 m/s ² |
| 2 | 1.543 m/s ² | 0.01176 m/s ² | 1.472 m/s ² | 0.008517 m/s ² |
| 3 | 1.543 m/s ² | 0.01173 m/s ² | 1.475 m/s ² | 0.008550 m/s ² |
| 4 | 1.541 m/s ² | 0.01172 m/s ² | 1.474 m/s ² | 0.008533 m/s ² |
| 5 | 1.539 m/s ² | 0.01175 m/s ² | 1.474 m/s ² | 0.008544 m/s ² |
| 6 | 1.545 m/s ² | 0.01177 m/s ² | 1.473 m/s ² | 0.008521 m/s ² |
| 7 | 1.543 m/s ² | 0.01173 m/s ² | 1.471 m/s ² | 0.008541 m/s ² |
| 8 | 1.540 m/s ² | 0.01171 m/s ² | 1.474 m/s ² | 0.008513 m/s ² |
| 9 | 1.543 m/s ² | 0.01175 m/s ² | 1.471 m/s ² | 0.008498 m/s ² |
| 10 | discarded | discarded | 1.475 m/s ² | 0.008529 m/s ² |
| Result | 1.54 m/s ² | 0.0039 m/s ² | 1.47 m/s ² | 0.0027 m/s ² |
| χ^2/Ndf | 0.20/8 | | 0.27/9 | |
| Prob. | 1 | | 1 | |

| Tool | Height | Statistical error | Systematic error |
|---------------|------------------------|-------------------|------------------|
| Tomme | 313.29 cm | 0.1331 cm | 0.05 cm |
| Lineal | 311.88 cm | 0.1664 cm | 0.05 cm |
| Vinkel | 311.89 cm | 0.1464 cm | 0.05 cm |
| Result | 312.46 cm | 0.085 cm | 0.0162 cm |
| χ^2/Ndf | 66.19/2 | | |
| Prob. | $4.234 \cdot 10^{-15}$ | | |

Furthermore, remember to COMMENT on your Chi2 probabilities!
This is very important, because a test is worthless, if not acted upon.

Examples from reports

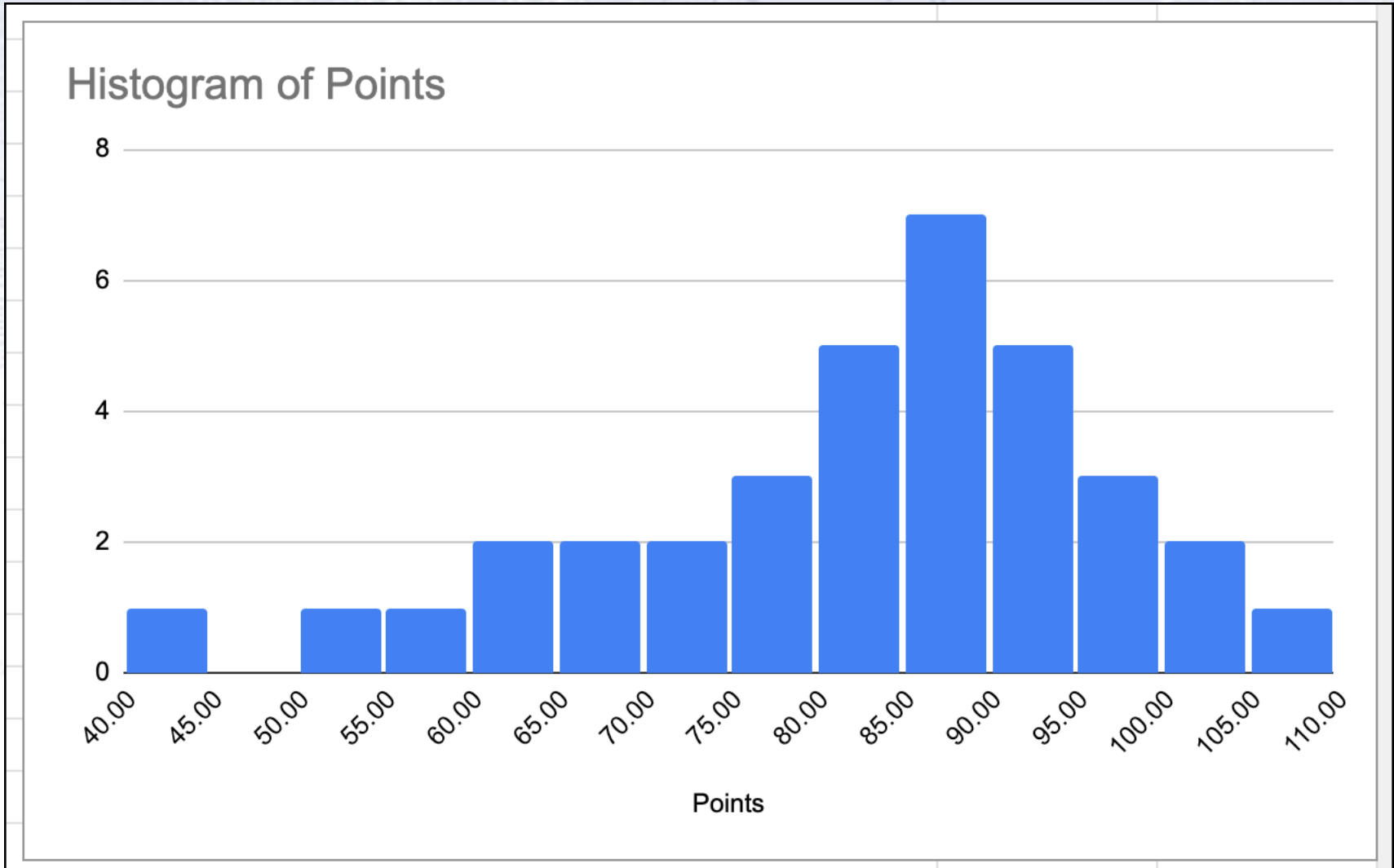
Sometimes, single measurement are clearly off... consider / comment on these!





Overview of results and errors

Your points



Various results

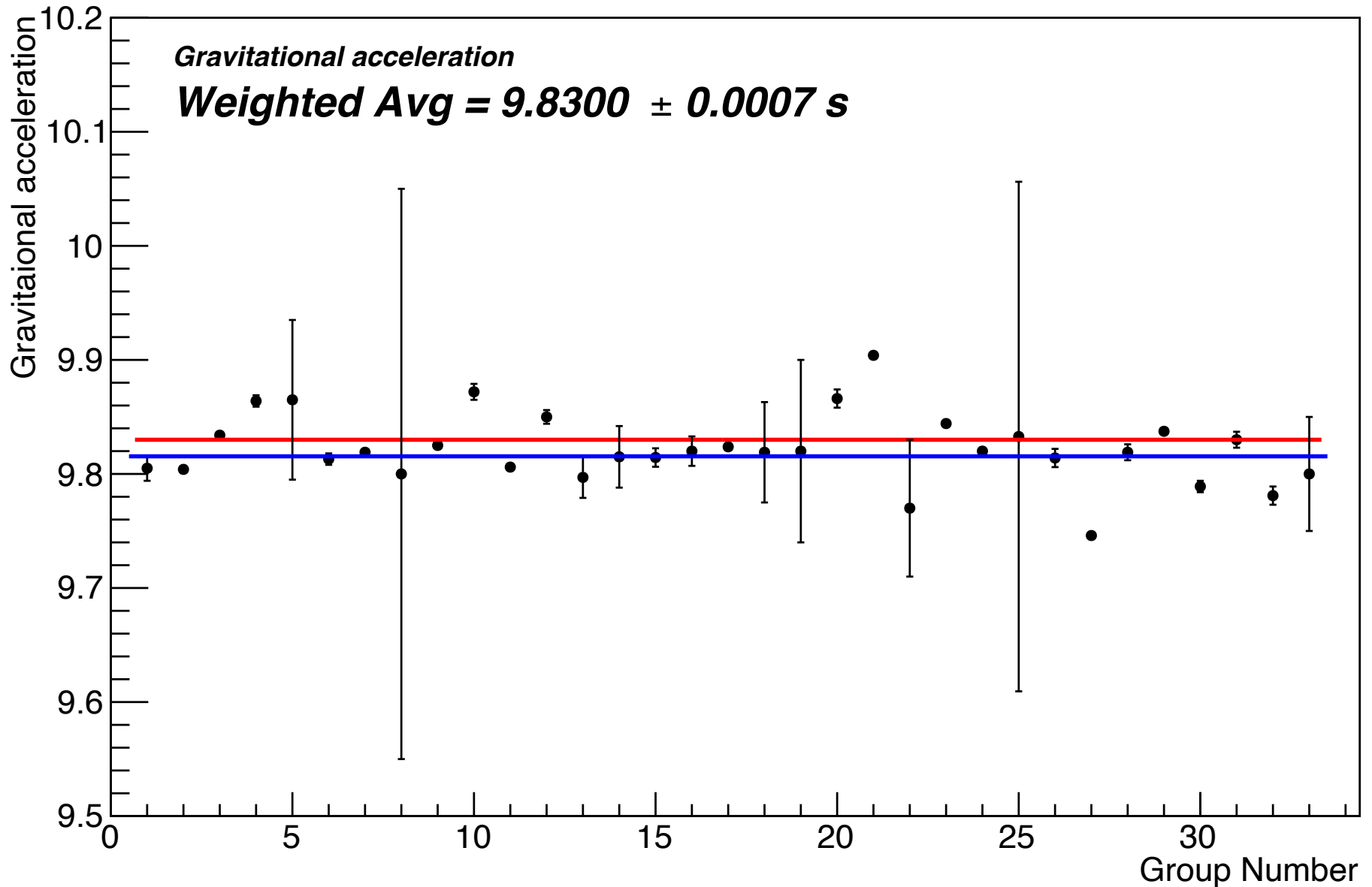
| g_Pend | sigma_g_Pend |
|---------|--------------|
| 9.82 | 0.06 |
| 9.819 | 0.003 |
| 9.826 | 0.0014 |
| 9.825 | 0.0044 |
| 9.82 | 0.13 |
| 9.828 | 0.002 |
| 9.8 | 0.1 |
| 9.793 | 0.011 |
| 9.901 | 0.003 |
| 10.0807 | 13.3317 |
| 11.8529 | 0.8 |
| 9.828 | 0.005 |
| 10.596 | 0.112 |
| 9.817 | 0.006 |
| 9.8 | 0.09 |
| 9.83 | 0.07 |
| 9.799 | 0.007 |
| 9.8 | 0.16 |
| 9.8143 | 0.0065 |
| 9.793 | 0.009 |
| 9.848 | 0.007 |
| 9.83 | 0.005 |

| | |
|--------|-------|
| 9.82 | 0.01 |
| 9.882 | 0.006 |
| 9.819 | 0.016 |
| 9.87 | 0.03 |
| 9.876 | 0.003 |
| 9.807 | 0.002 |
| 9.82 | 0.13 |
| 9.852 | 0.005 |
| 9.93 | 0.04 |
| 10.022 | 0.197 |
| 9.76 | 0.04 |
| 9.88 | 0.012 |
| 9.8 | 0.3 |
| 9.81 | 0.09 |
| 9.81 | 0.009 |
| 9.78 | 0.01 |
| 9.905 | 0.005 |

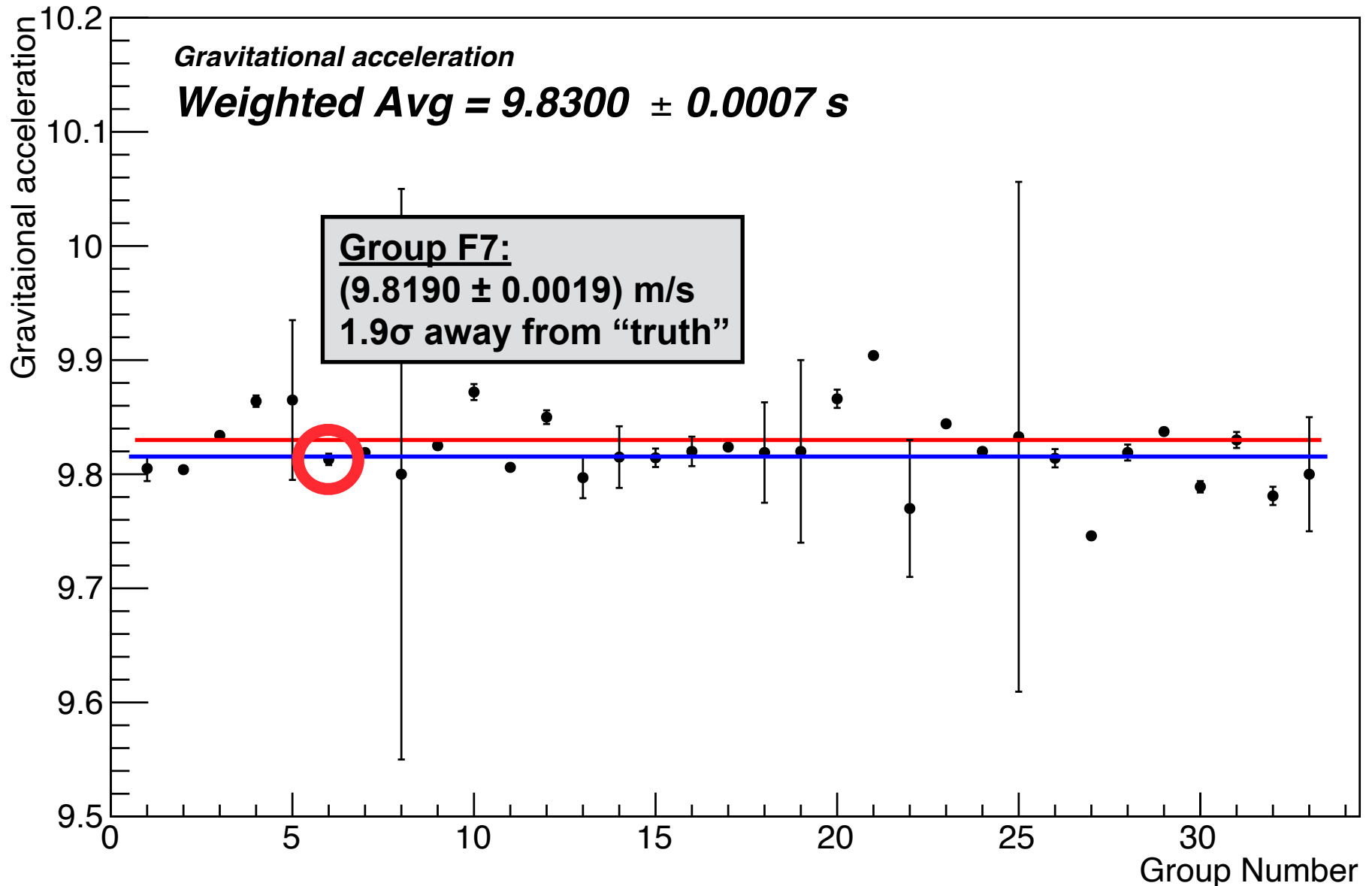
| g_Bol | sigma_g_Bol |
|------------|-------------|
| 9.2 | 0.4 |
| 9.3 | 0.3 |
| 9.85 | 0.03 |
| 9.542 | 0.036 |
| 6.8/6.6117 | 6.7/0.0019 |
| 9.439 | 0.005 |
| 9.04 | 0.01 |
| --- | --- |
| 9.51 | 0.03 |
| 9.8045 | 1.431 |
| 11.520699 | 0.8 |
| 9.42 | 0.01 |
| 9.691 | 0.049 |
| 9.518 | 0.039 |
| 8.56 | 0.04 |
| 9.88 | 0.1 |
| 9.4 | 0.02 |
| 9.27 | 0.42 |
| 9.802 | 0.051 |
| 9.25 | 0.14 |
| 10.08 | 0.2 |
| 9.4 | 0.3 |

| | |
|-------|-------|
| 9.38 | 0.06 |
| 9.57 | 0.106 |
| 9.5 | 0.08 |
| 9.6 | 0.3 |
| 9 | 1 |
| 8.9 | 0.2 |
| 9.55 | 0.04 |
| 9.588 | 0.001 |
| 9.15 | 0.11 |
| 9.314 | 0.097 |
| -- | -- |
| 9.28 | 0.12 |
| 9.83 | 0.16 |
| 8.48 | 0.11 |

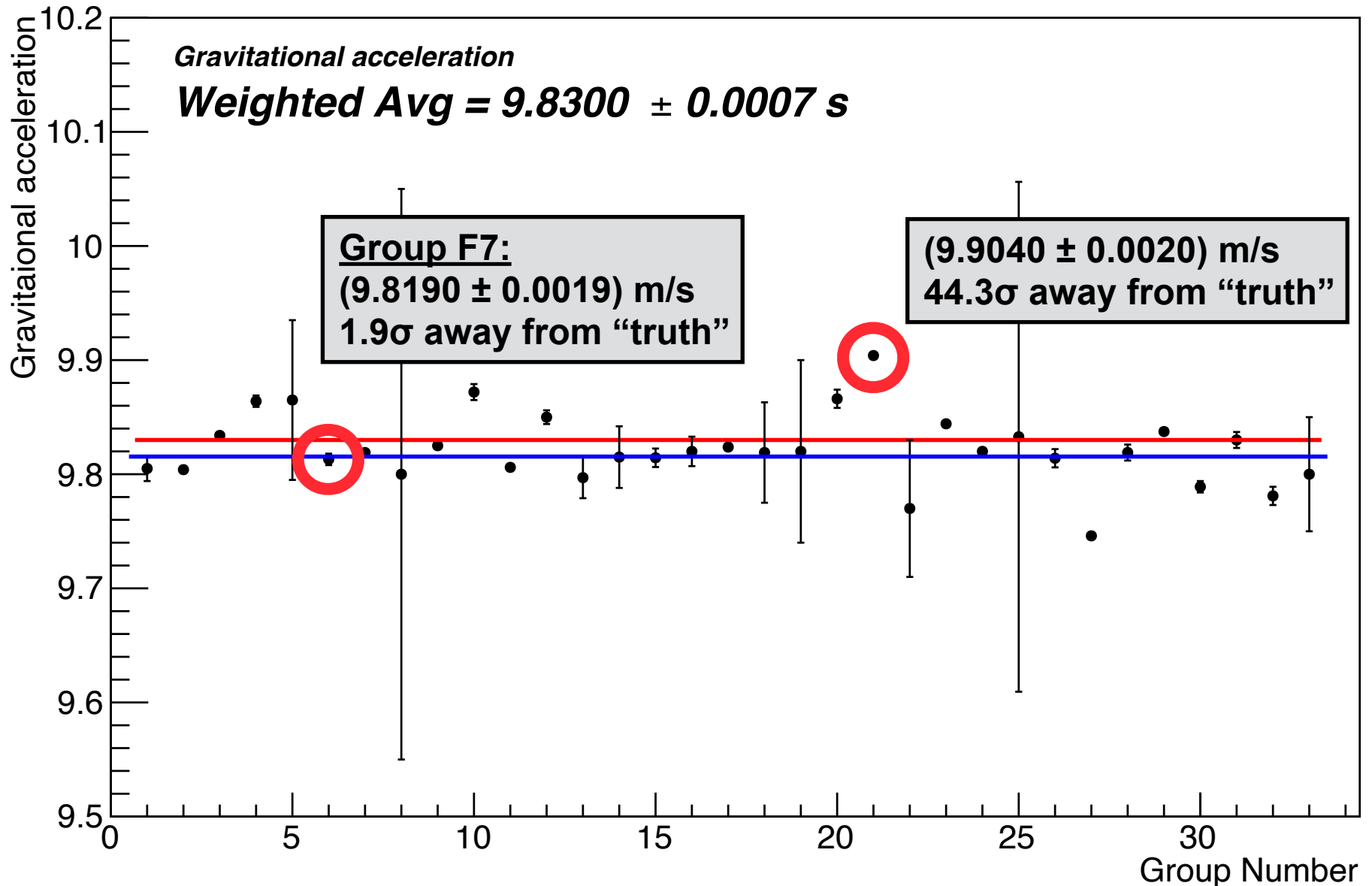
Older results - pendulum



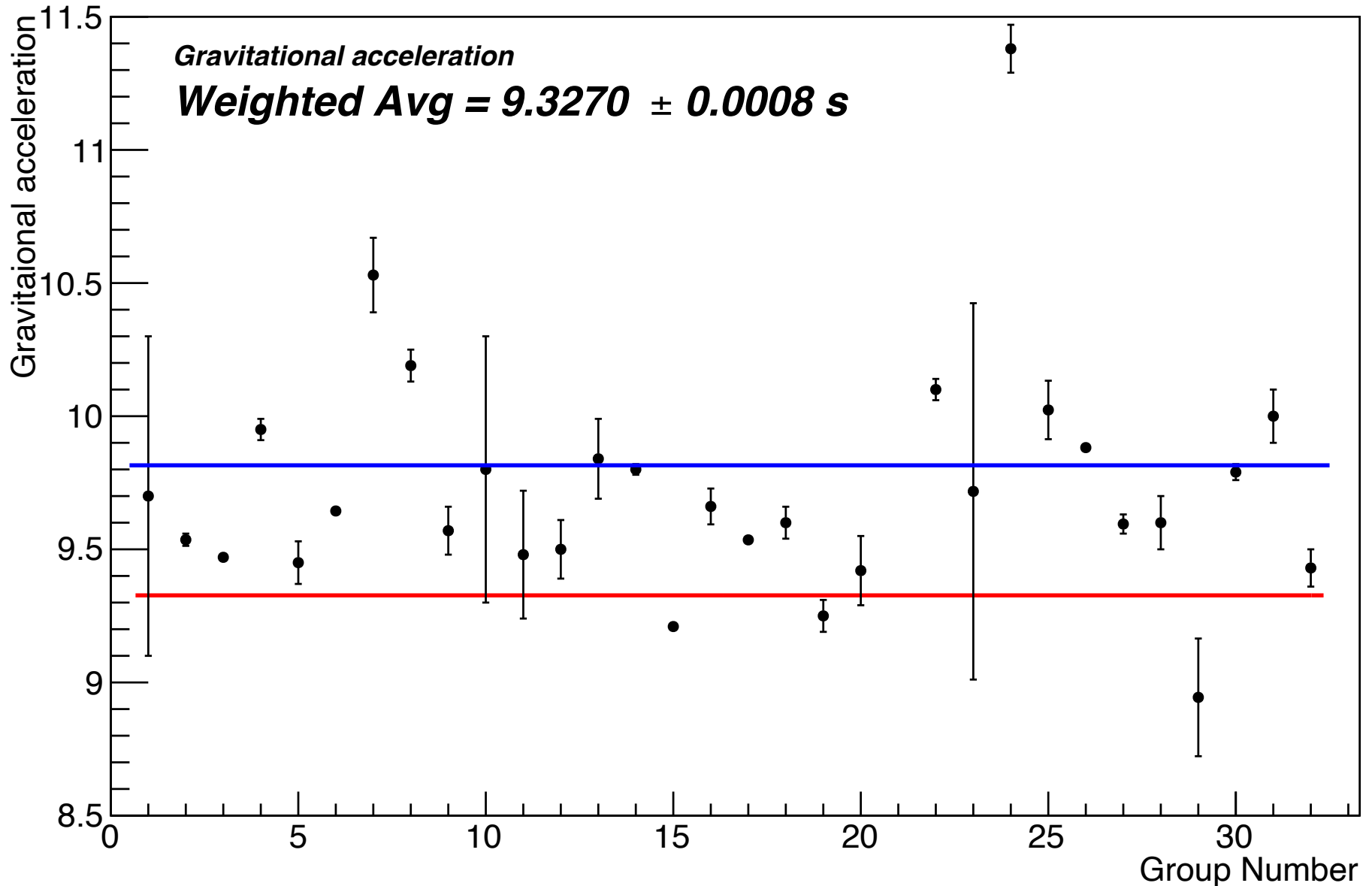
Older results - pendulum



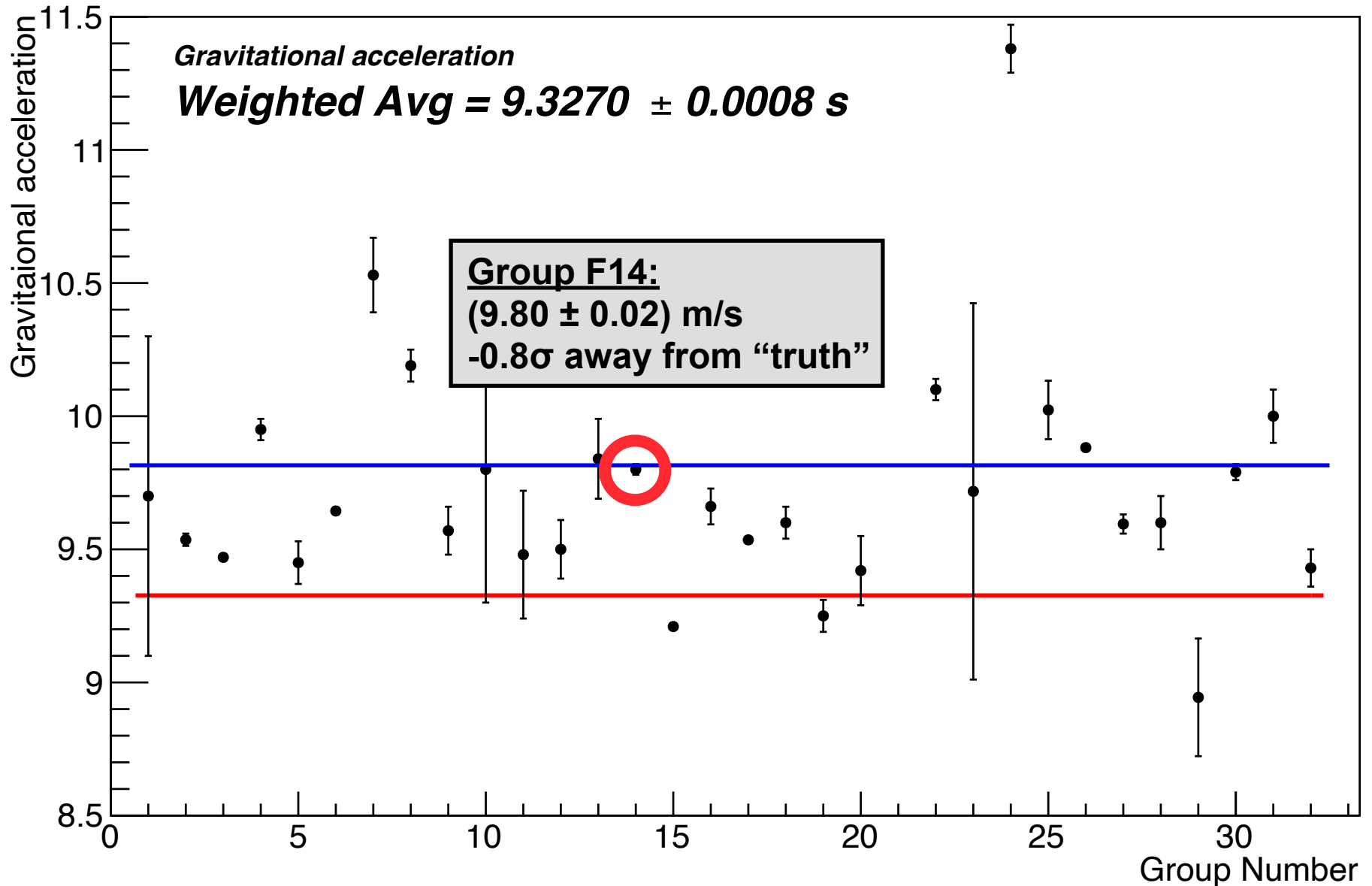
Older results - pendulum



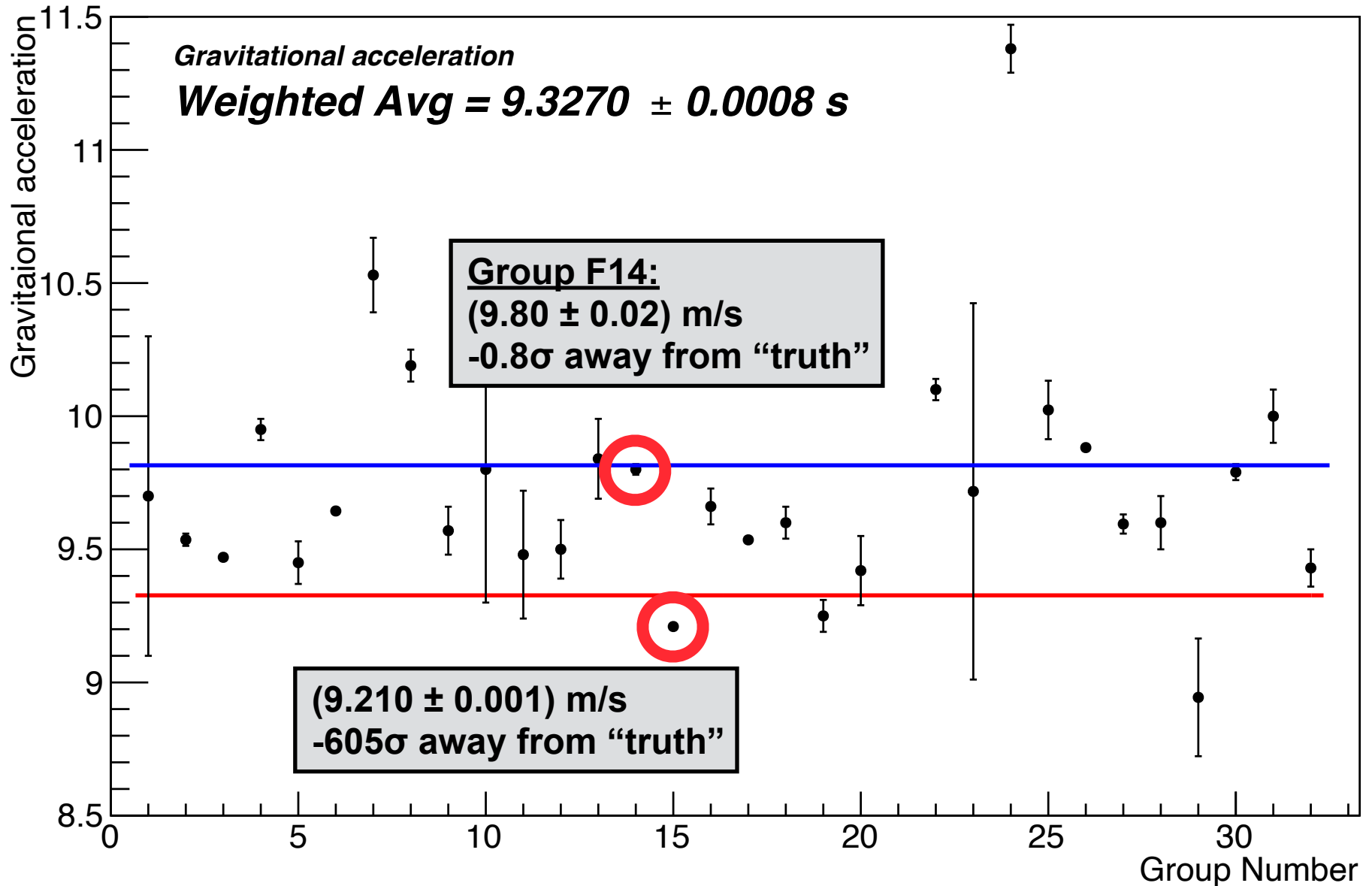
Older results - Ball on Incline



Older results - Ball on Incline

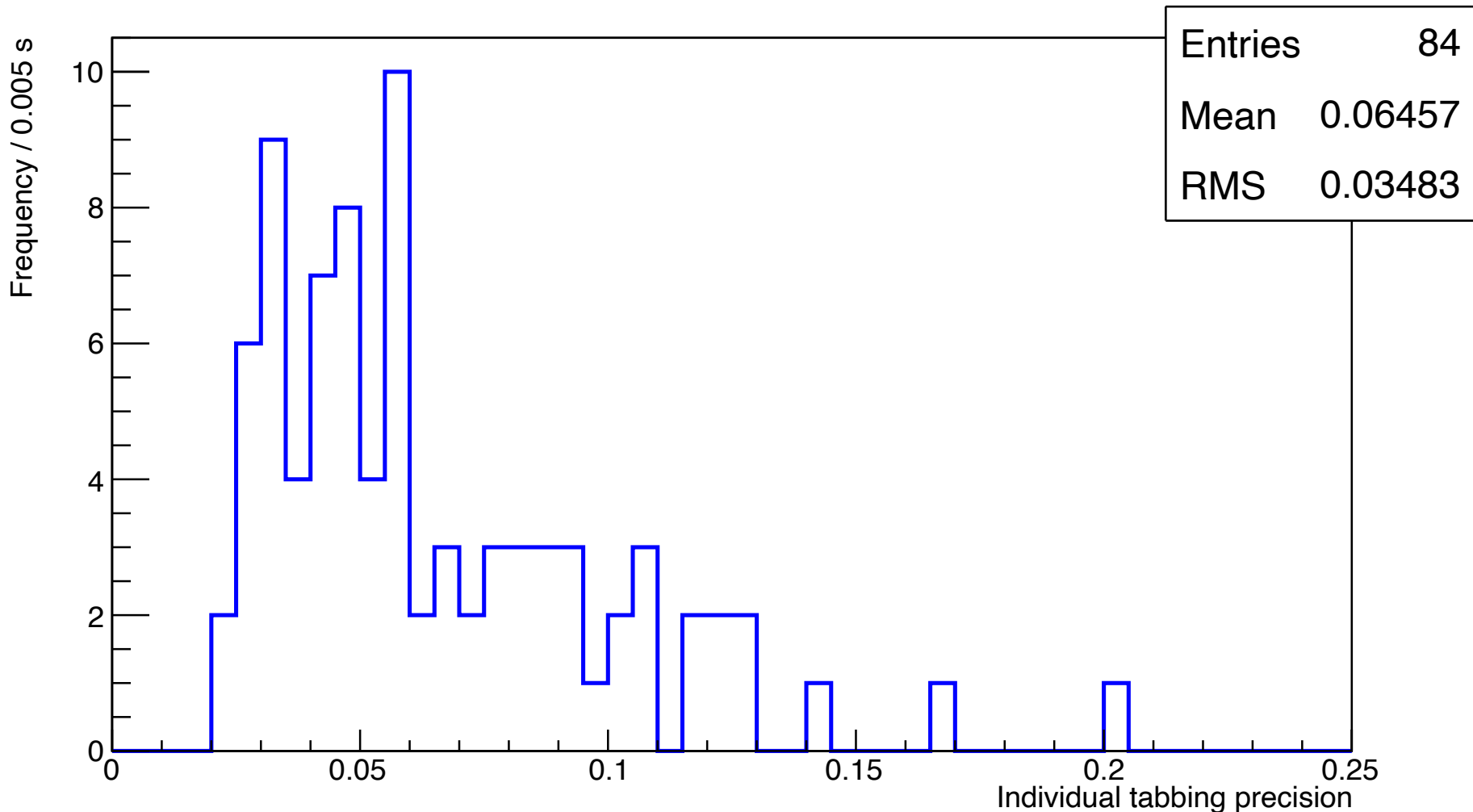


Older results - Ball on Incline



Individual tabbing precisions

The individual precision (from last year) varies quite a lot... in fact by a factor 10!



General considerations

Be careful with error estimation and propagation:

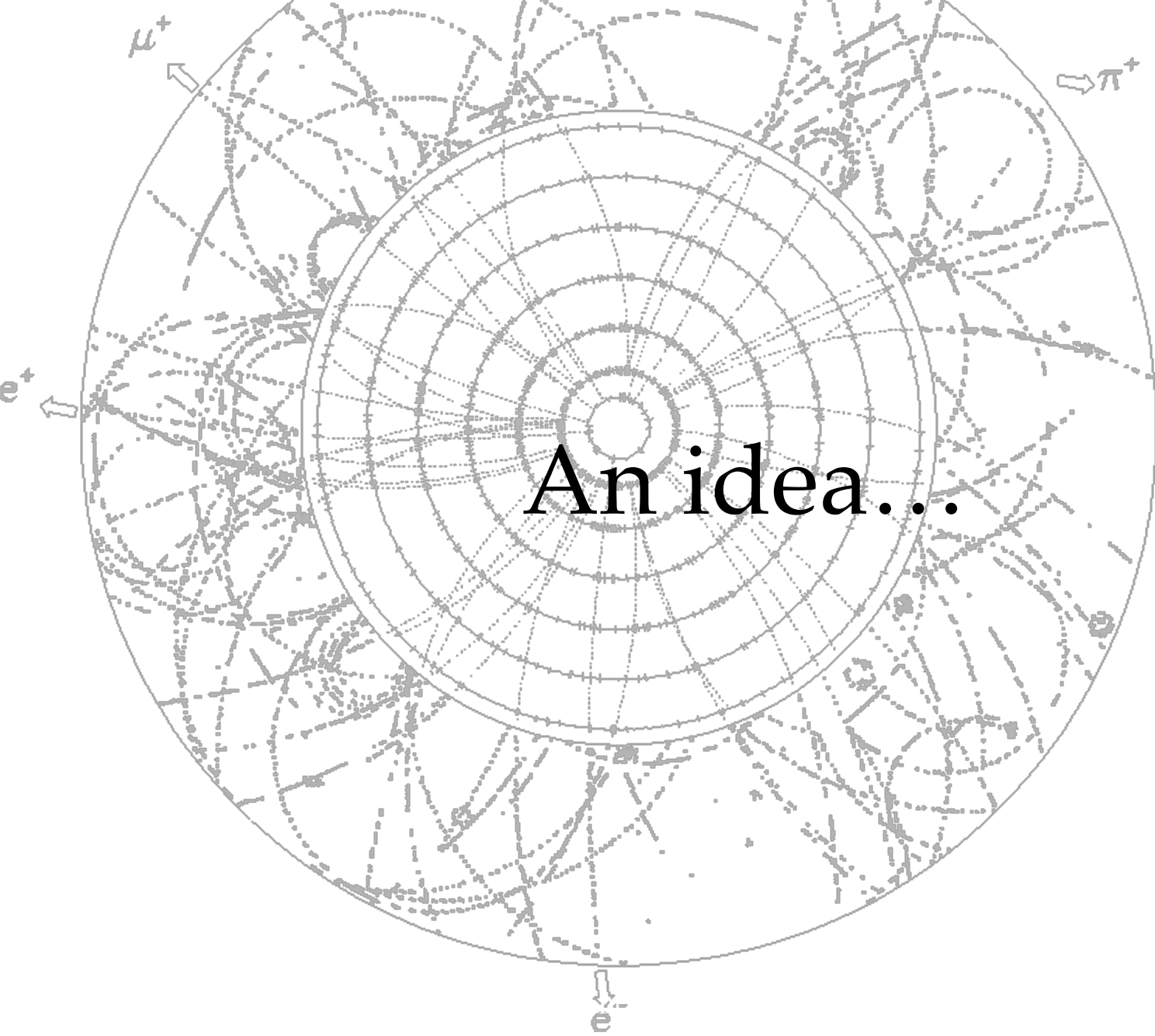
- Systematic errors do not decrease with repetition!
- **Correlated quantities** can not simply be averaged over.
- Propagation of error needs consideration - not always in quadrature.
- Be conservative, if you are uncertain about your errors.

Always show / check what you're doing:

- Make good plots.
- Give measurement tables.
- Compare the difference between results and methods.
- Combine results with uncertainties with a **weighted mean and Chi2**.

Also:

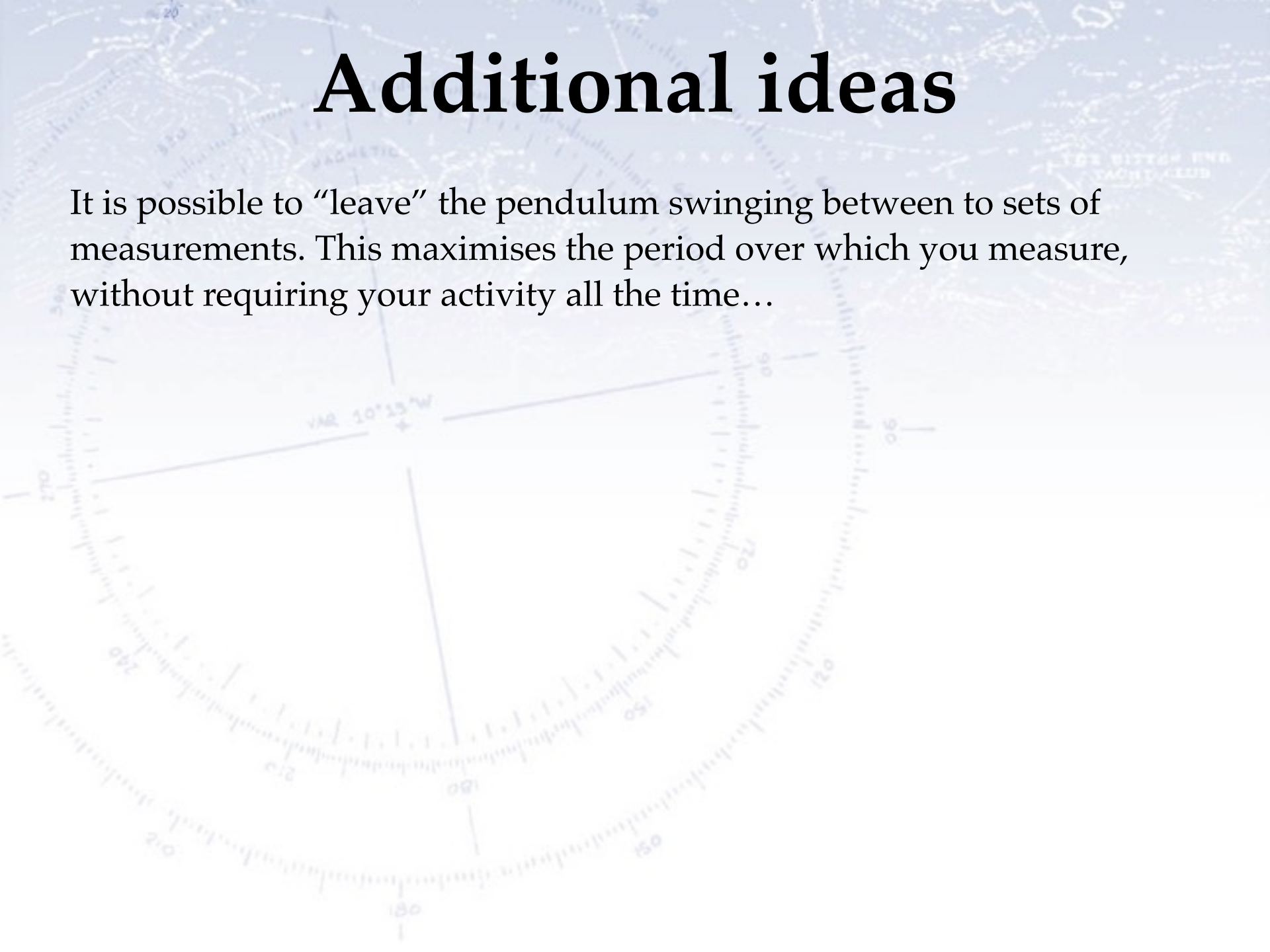
- Put the result in the abstract.
- Explain your symbols in formulae.



An idea...

Additional ideas

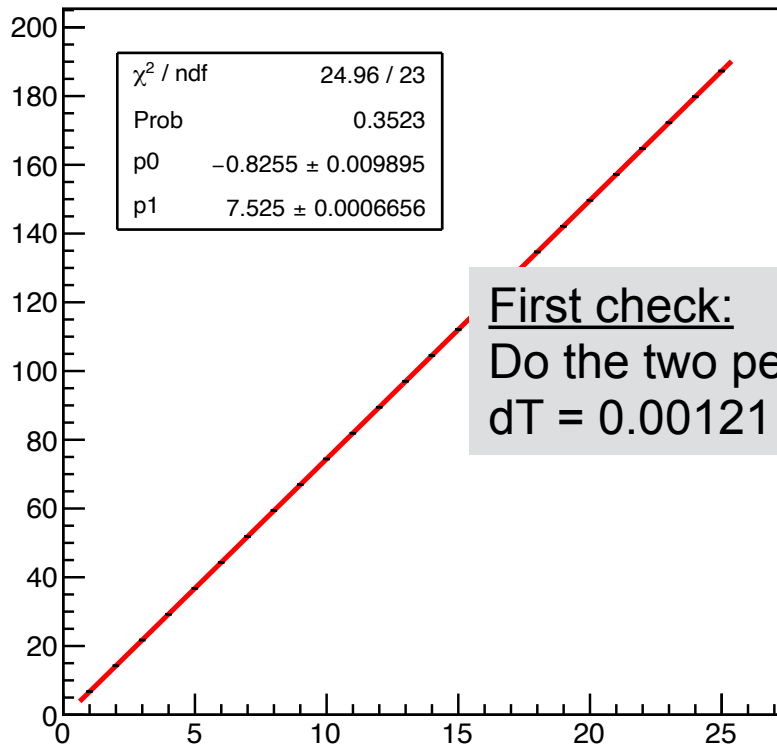
It is possible to “leave” the pendulum swinging between two sets of measurements. This maximises the period over which you measure, without requiring your activity all the time...



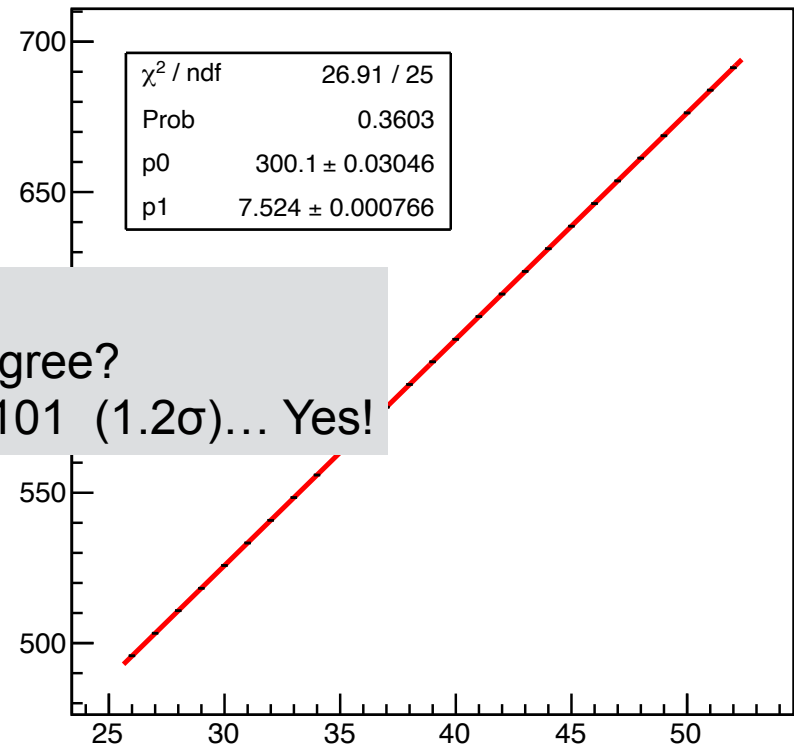
Additional ideas

It is possible to “leave” the pendulum swinging between two sets of measurements. This maximises the period over which you measure, without requiring your activity all the time...

Graph



Graph



First check:

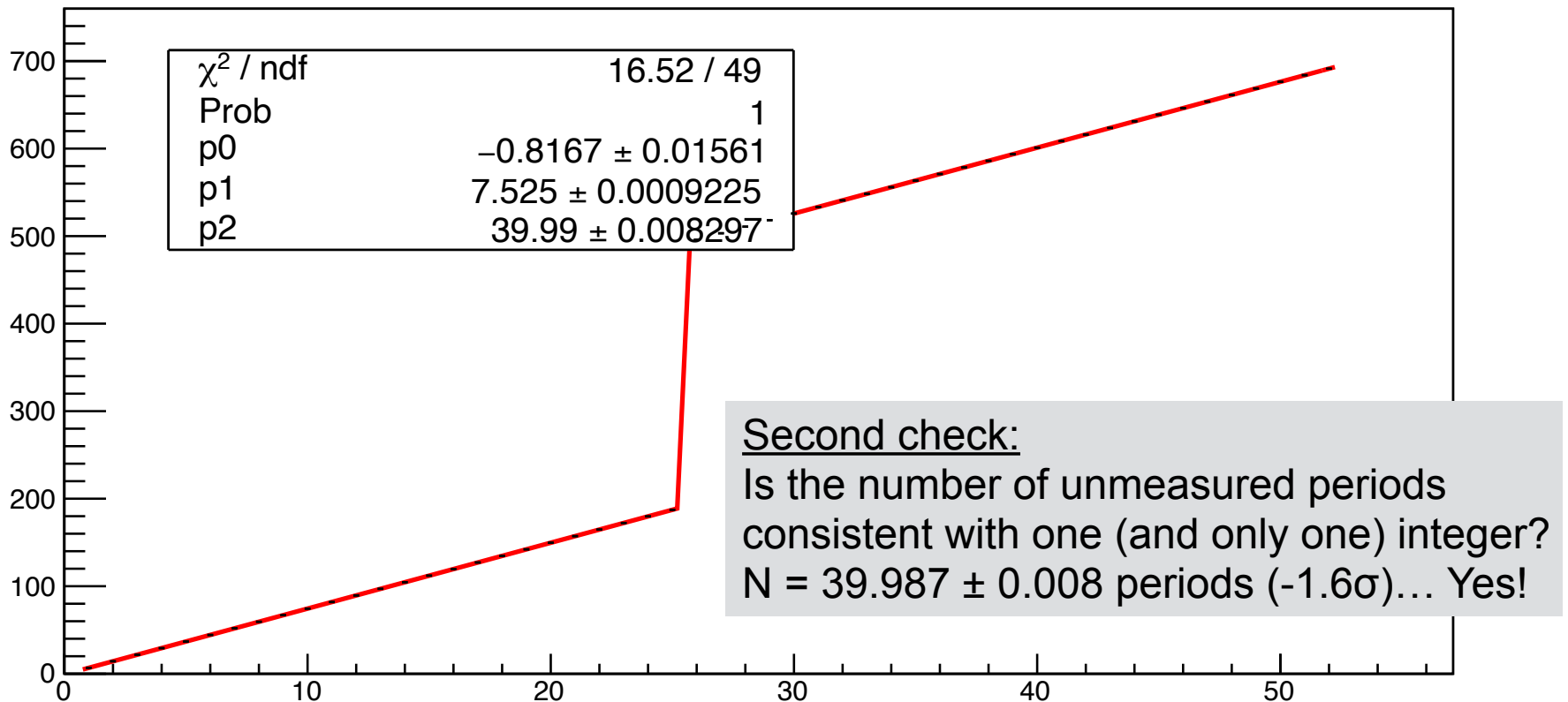
Do the two periods agree?

$dT = 0.00121 \pm 0.00101$ (1.2σ)... Yes!

Additional ideas

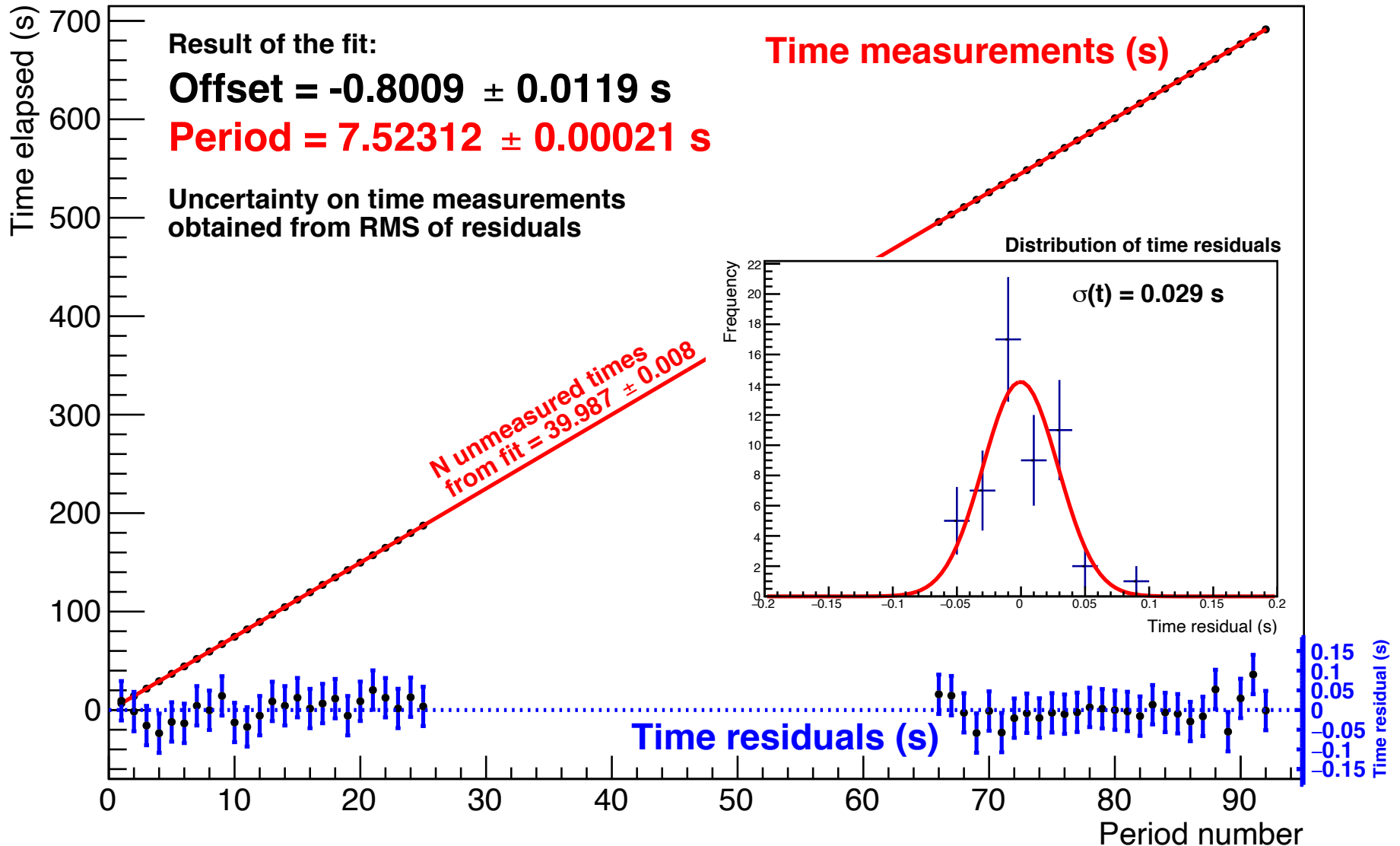
It is possible to “leave” the pendulum swinging between two sets of measurements. This maximises the period over which you measure, without requiring your activity all the time...

Graph



Additional ideas

After checks, fit the entire time span to get “insanely great” precision.



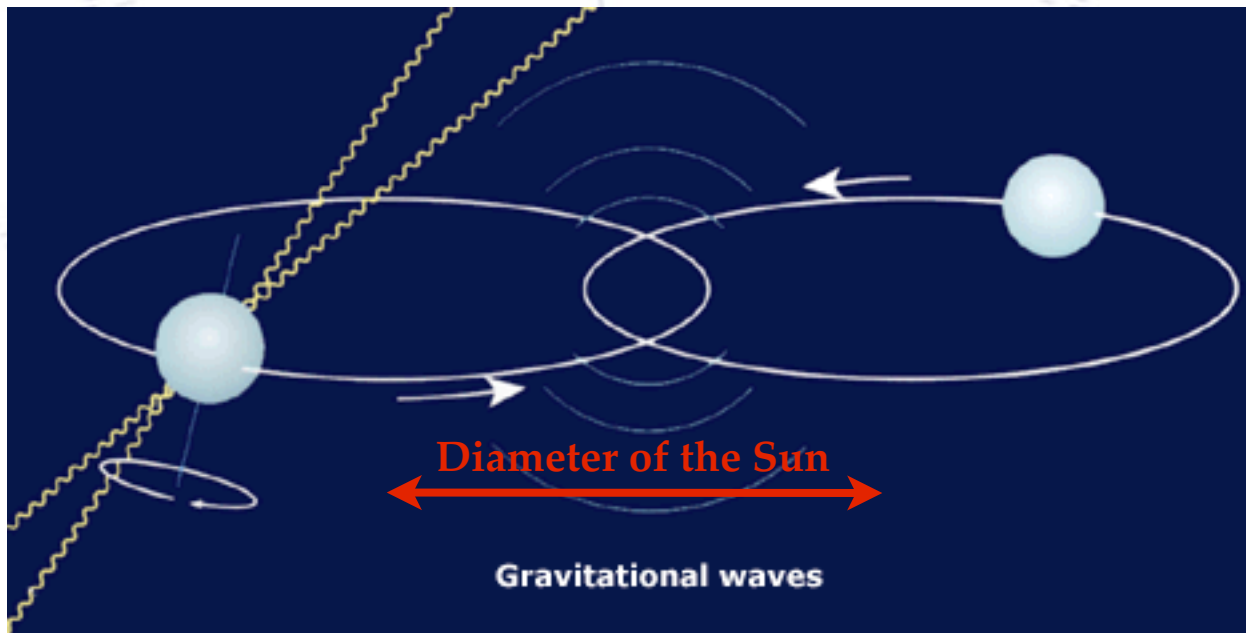
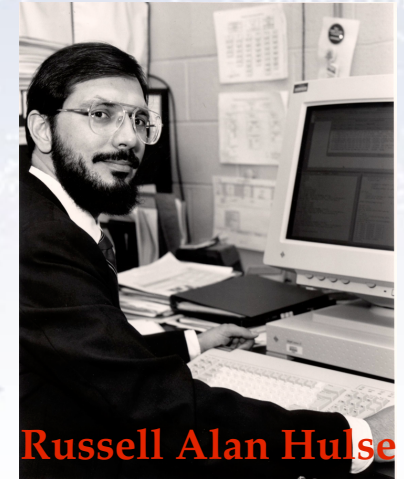
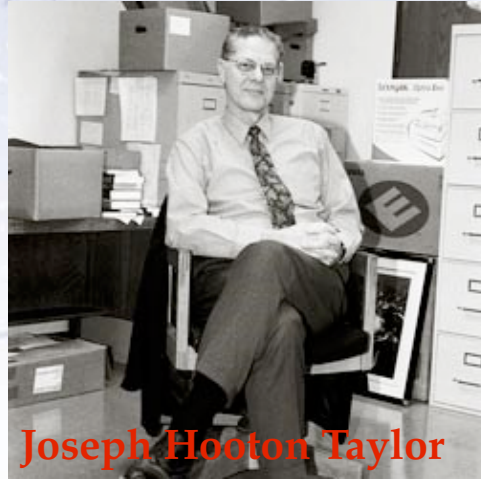
A small intermezzo...

...the Hulse-Taylor-pulsar



Hulse-Taylor-pulsaren

In 1973 Hulse & Taylor discovered a very special pulsar... the period for its signal was NOT constant, but had a variation with a period of 8 hours! As it turns out, this was to become a “jewel” in the test of Einstein’s theory of relativity.

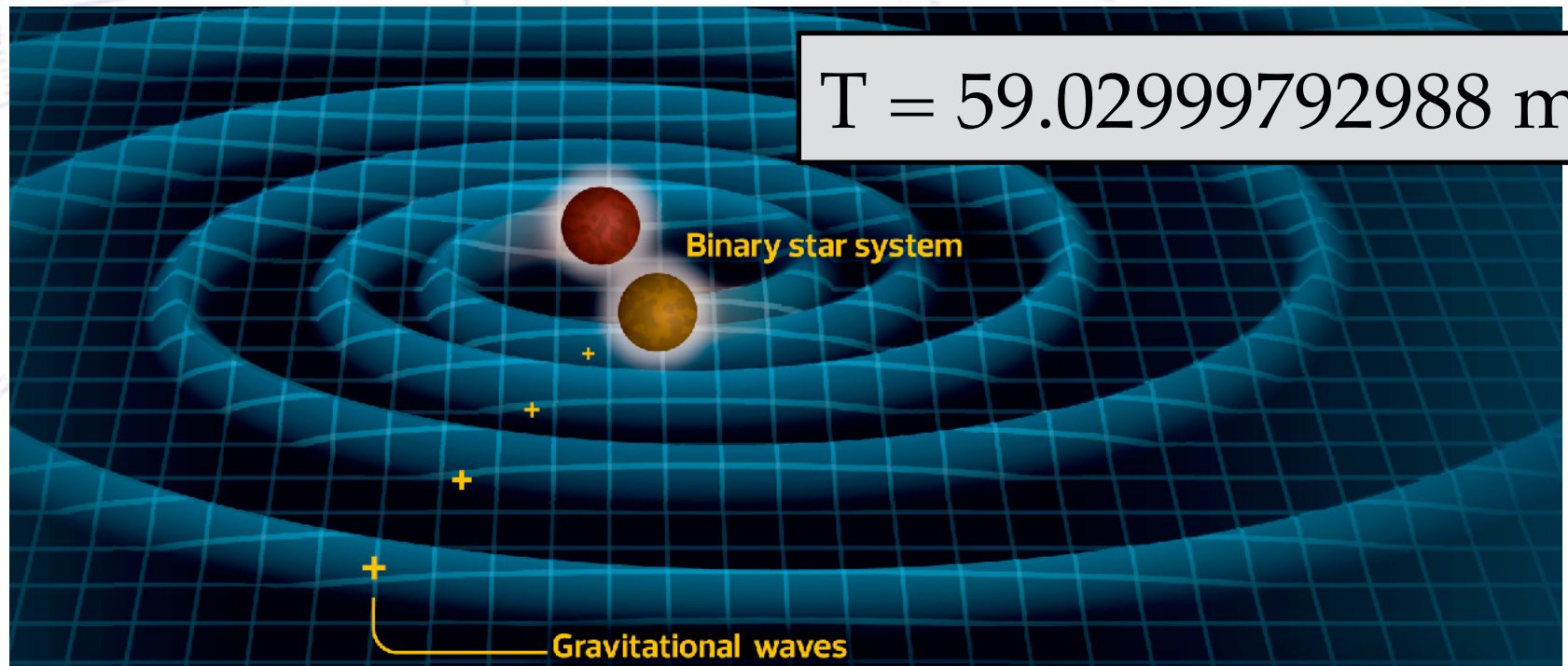


Præcisionsmålinger

In the following years, they measured the pulsar parameters with great precision:

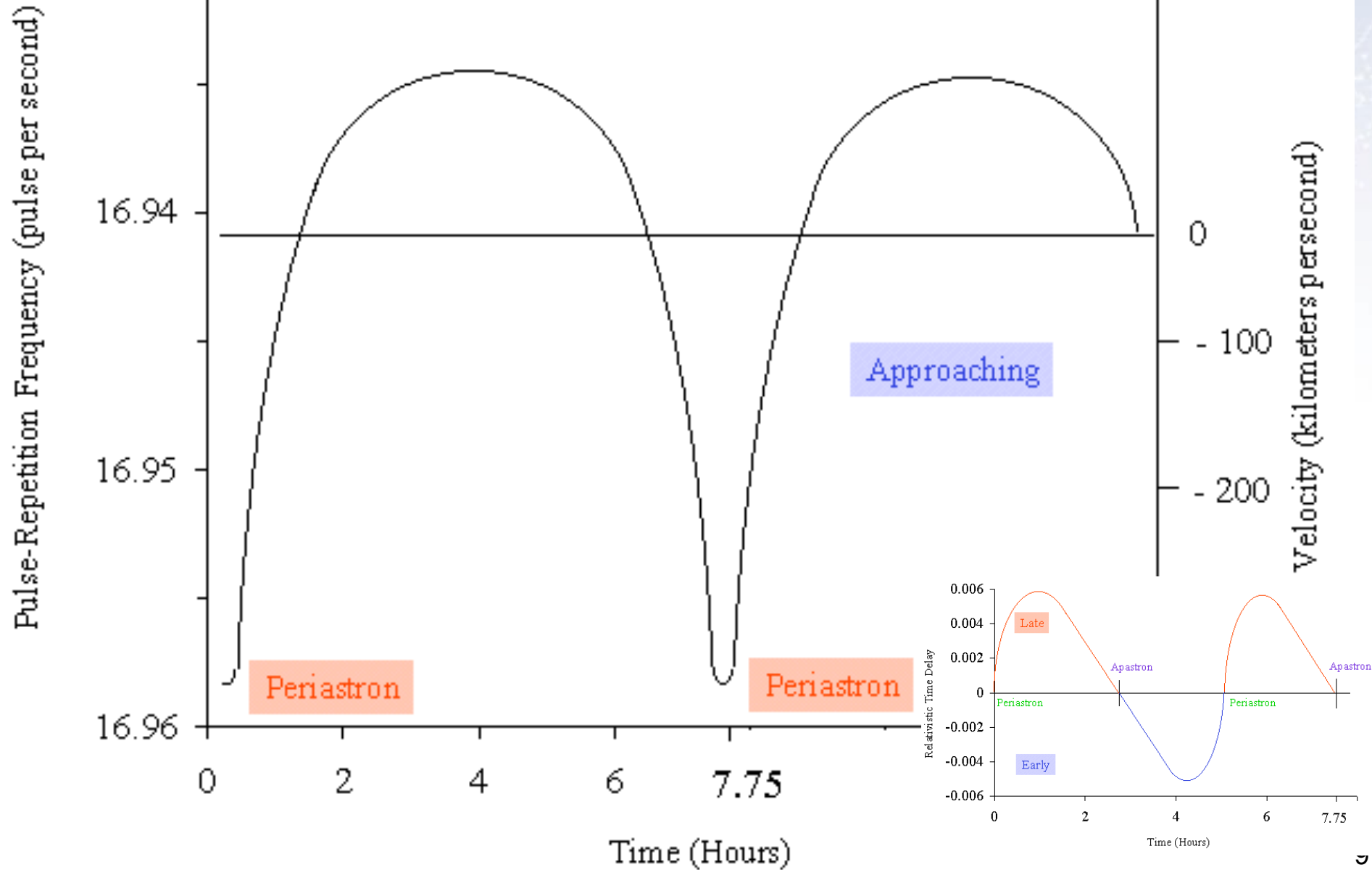
- Mass of companion: 1.387 MSun
- Total mass of the system: 2.828378(7) MSun
- Orbital period: 7.751938773864 hr
- Eccentricity: 0.6171334
- Semimajor axis: 1,950,100 km
- Periastron separation: 746,600 km
- Apastron separation: 3,153,600 km
- Orbital velocity of stars at periastron (relative to center of mass): 450 km/s
- Orbital velocity of stars at apastron (relative to center of mass): 110 km/s

The measurements were possible, partly because of the large relativistic effects. What takes **a century** for Mercury, takes **a day** for the Hulse-Taylor-pulsar!



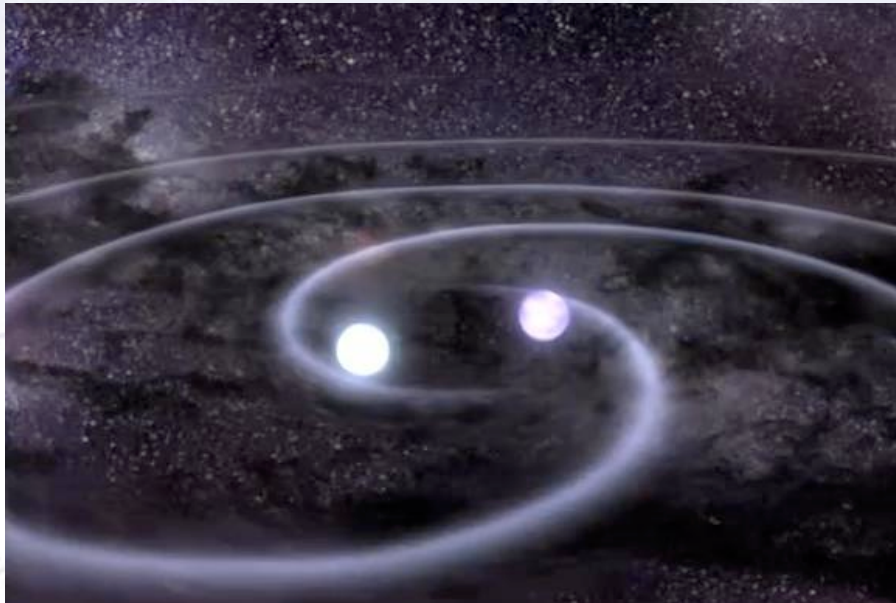
$$T = 59.02999792988 \text{ ms}$$

Original plot of measurements

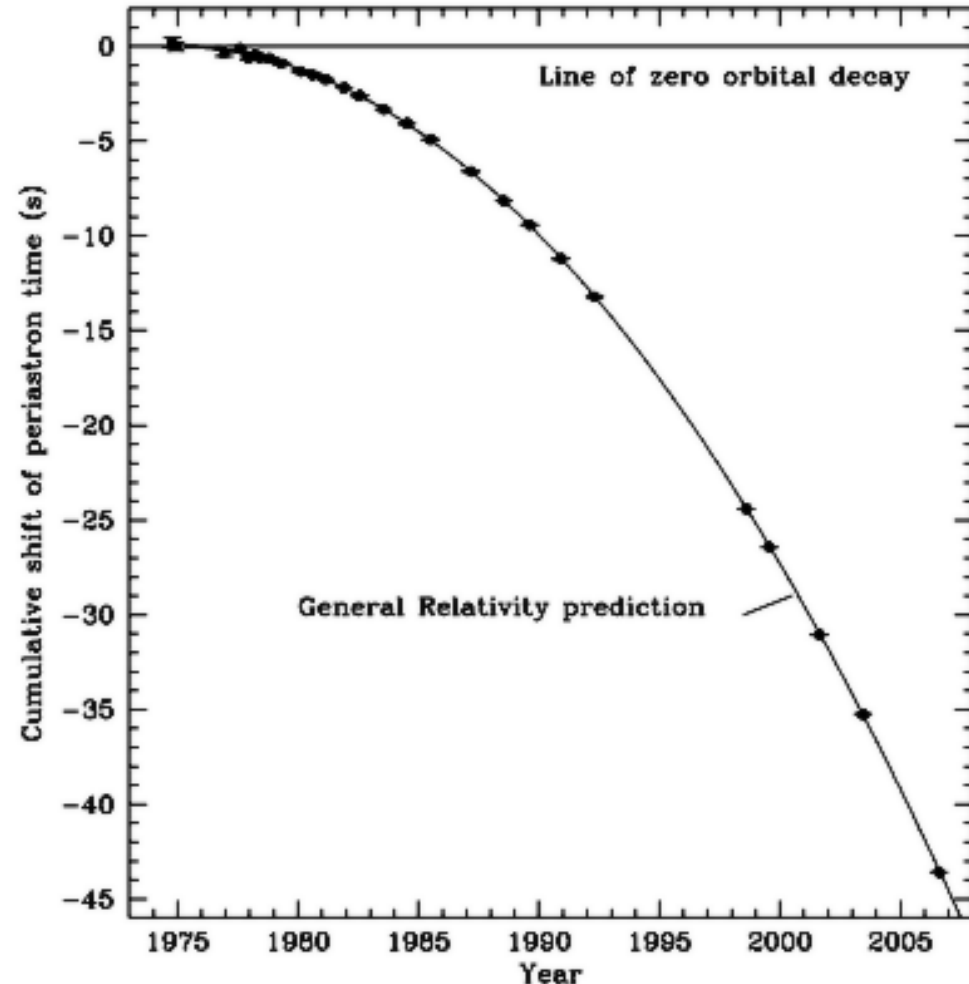


The discovery

After years of observation it became clear, that the pulsars spiral towards each other.
Conclusion: They loose energy (fast). Immediate question: How?!?

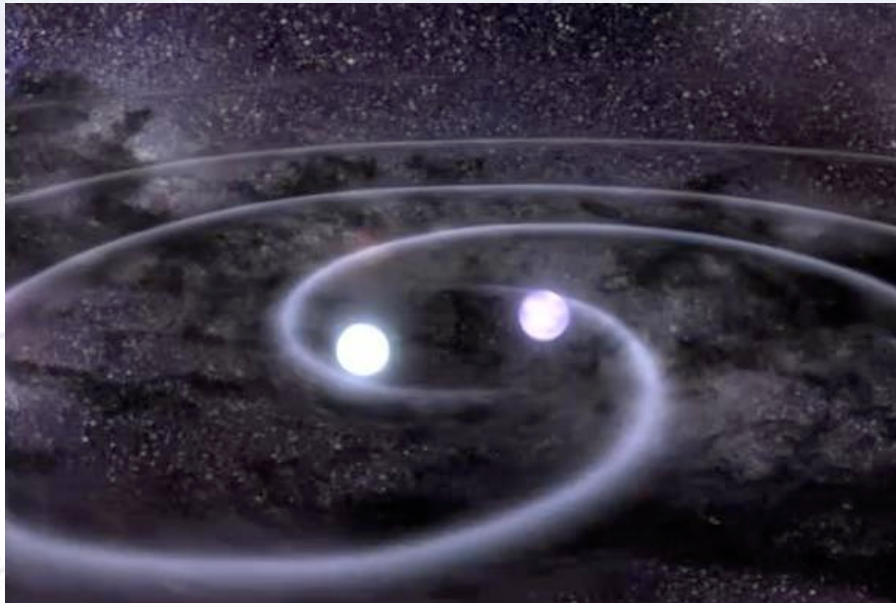


The answer is gravitational waves.
What is normally a tiny effect is here so large (100x suns output!), that we (with a natural high precision clock) can see it.
(Sun+Earth: 200 W, 10^{-15} m radius “lost” pr. day)

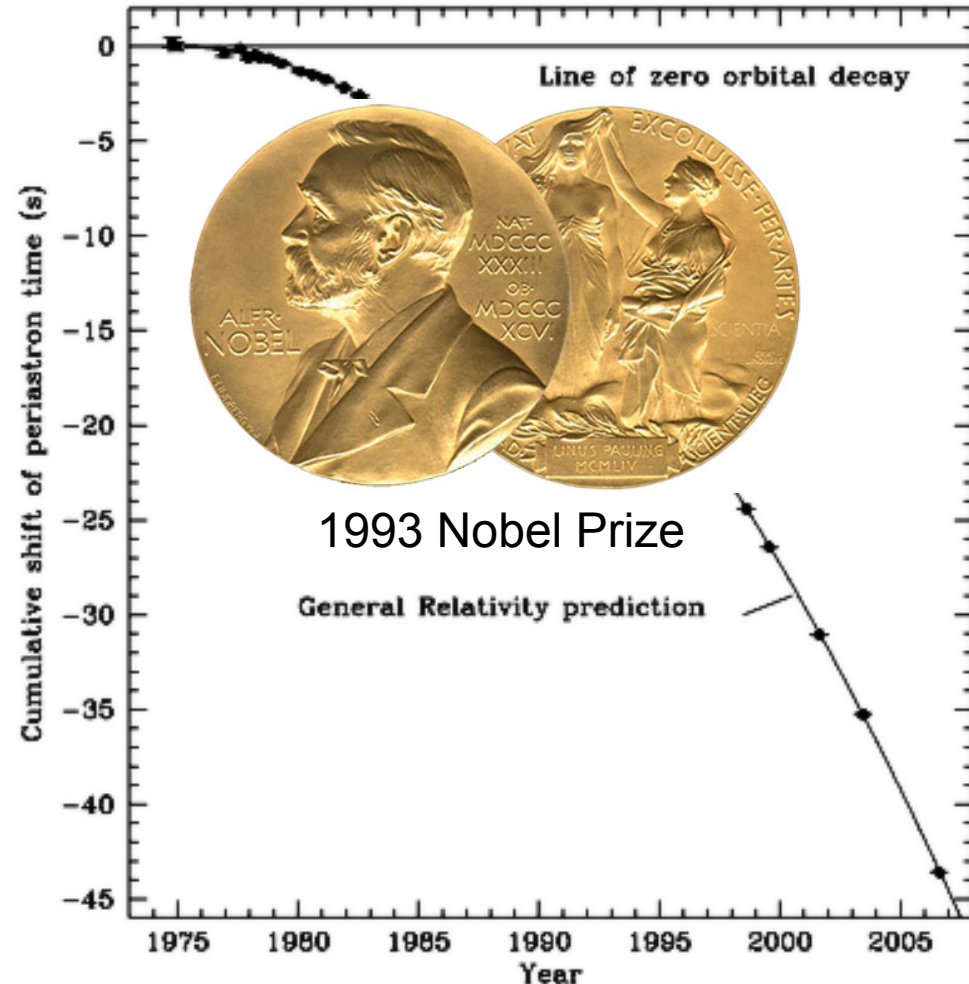


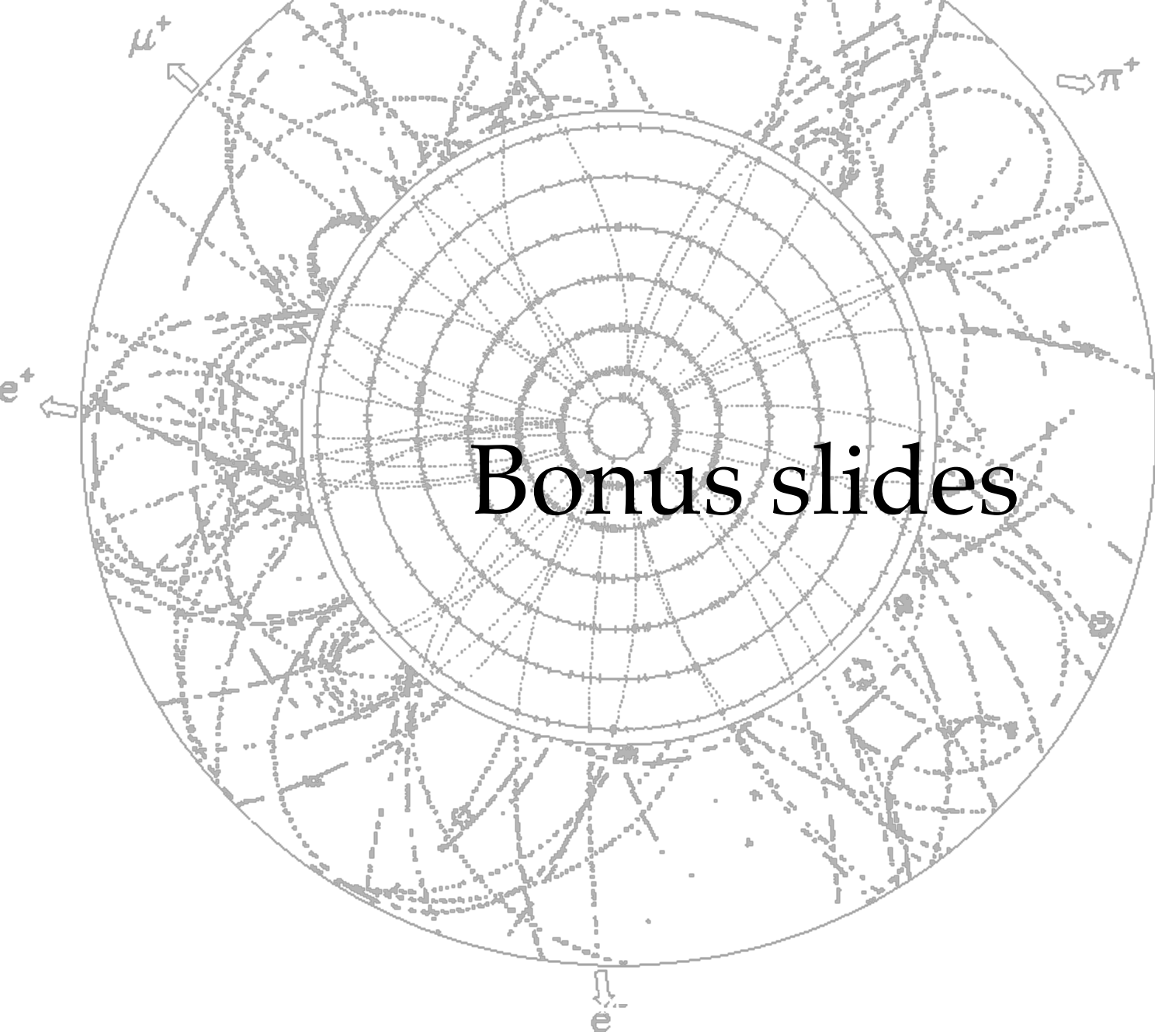
The discovery

After years of observation it became clear, that the pulsars spiral towards each other.
Conclusion: They loose energy (fast). Immediate question: How?!?



The answer is gravitational waves.
What is normally a tiny effect is here so large (100x suns output!), that we (with a natural high precision clock) can see it.
(Sun+Earth: 200 W, 10^{-15} m radius “lost” pr. day)





Bonus slides