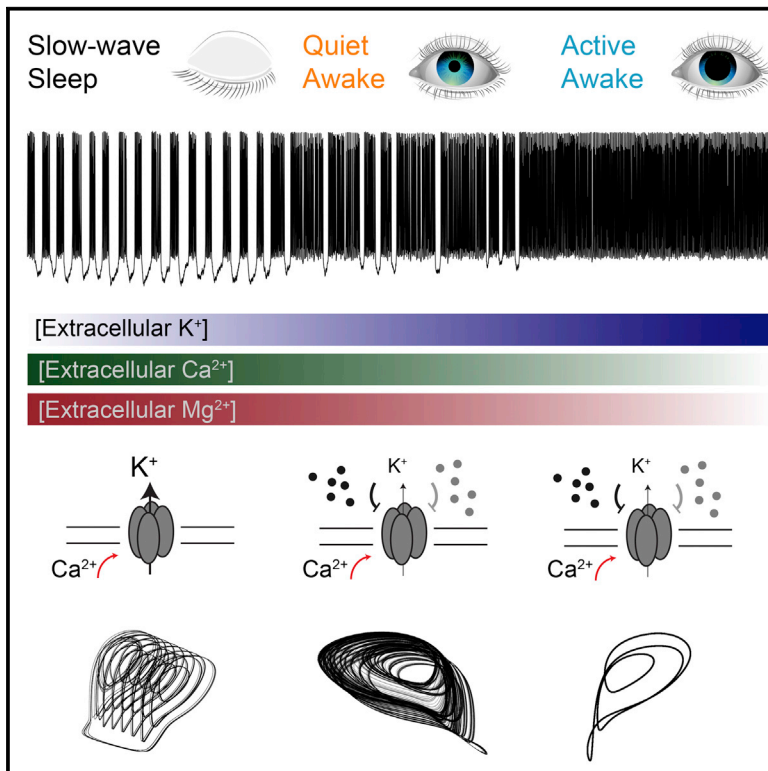


# Cell Systems

## Chaotic Dynamics Mediate Brain State Transitions, Driven by Changes in Extracellular Ion Concentrations

### Graphical Abstract



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### In Brief

By extending the Averaged-Neuron model to include changes in extracellular ion concentrations, Rasmussen et al. show that transitions between states of sleep and wakefulness are facilitated by ionic changes and governed by chaotic dynamics, which ensure smooth and robust transitions.

### Highlights

- The Averaged-Neuron model is extended to include extracellular ionic changes
- *In vivo* sleep- and wake-dependent neuronal firing patterns are recapitulated
- Extracellular ionic changes lower the threshold for transitioning to wakefulness
- Chaotic dynamics create a smooth transition between different activity states



# Chaotic Dynamics Mediate Brain State Transitions, Driven by Changes in Extracellular Ion Concentrations

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## SUMMARY

Previous studies have suggested that changes in extracellular ion concentrations initiate the transition from an activity state that characterizes sleep in cortical neurons to states that characterize wakefulness. However, because neuronal activity and extracellular ion concentrations are interdependent, isolating their unique roles during sleep-wake transitions is not possible *in vivo*. Here, we extend the Averaged-Neuron model and demonstrate that, although changes in extracellular ion concentrations occur concurrently, decreasing the conductance of calcium-dependent potassium channels initiates the transition from sleep to wakefulness. We find that sleep is governed by stable, self-sustained oscillations in neuronal firing patterns, whereas the quiet awake state and active awake state are both governed by irregular oscillations and chaotic dynamics; transitions between these separable awake states are prompted by ionic changes. Although waking is indicative of a shift from stable to chaotic neuronal firing patterns, we illustrate that the properties of chaotic dynamics ensure that the transition between states is smooth and robust to noise.

## INTRODUCTION

During transition from sleep to awake, the brain passes from a state that attenuates sensory inputs to one that often amplifies them (Lee and Dan, 2012; McGinley et al., 2015b; Polack et al., 2013). How this transition between states occurs is still incompletely understood, both at the level of the brain and at the level of the neurons that must implement the transition in a smooth and robust way.

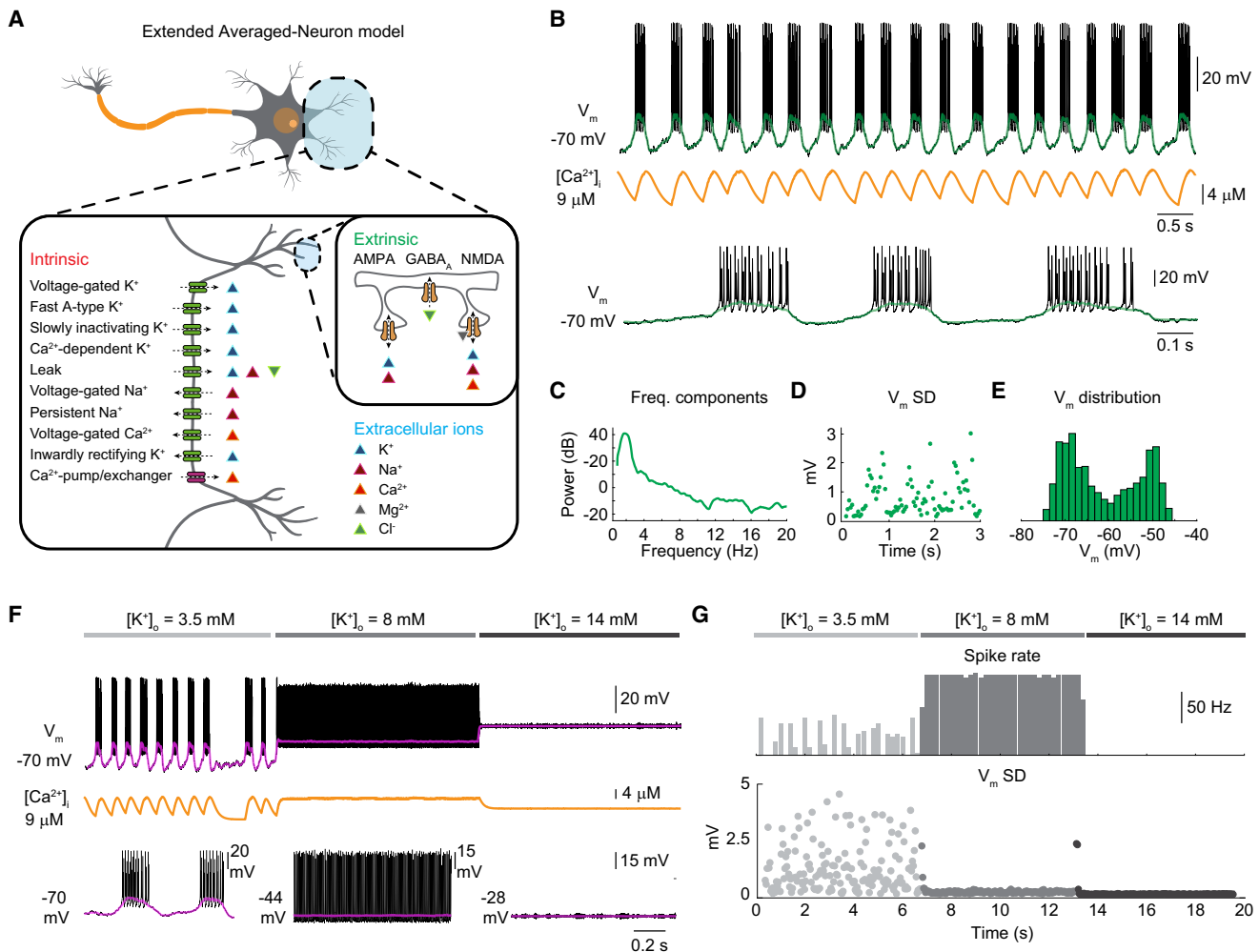
Cortical neurons are known to exhibit three distinct physiological and behavioral states that characterize the transition from sleep to active wakefulness. Specifically, slow-wave sleep is characterized by oscillating periods of synaptic barrages

(also called “upstates”) and silence (“downstates”) (Steriade et al., 2001). It is understood that this behavior reflects oscillations in membrane potential ( $V_m$ ) and therefore the neuron’s susceptibility to input. In wakefulness, these downstates are suppressed and  $V_m$  is maintained closer to threshold. Although the classic view holds that oscillations are restricted to slow-wave sleep, recent studies have demonstrated that a quiet awake state is also characterized by low-frequency  $V_m$  oscillations. Once actively awake, however, movement and arousal suppress these oscillations, and  $V_m$  is depolarized tonically, making neurons sensitive to inputs and prone to firing. Despite our understanding of the neuronal behaviors that are indicative of transitions from slow-wave sleep to quiet awake to active awake, the underlying mechanisms that initiate these transitions remain unclear.

Recently it was proposed that changes in extracellular ion concentrations ( $[K^+]_o$ ,  $[Ca^{2+}]_o$ , and  $[Mg^{2+}]_o$ ) control activity patterns during sleep and wakefulness, and that ionic changes are sufficient to shift the state from sleep to awake (Ding et al., 2016). However, given that changes in neural activity also strongly affect extracellular ion concentrations (Amzica et al., 2002; Hounsgaard and Nicholson, 1983; Lux, 1974; Lux and Neher, 1973; Nicholson et al., 1977), it is not possible to unequivocally distinguish cause and consequence *in vivo*. For example, both spiking and excitatory barrages, which originate within the neuron itself, increase  $[K^+]_o$  (Hounsgaard and Nicholson, 1983; Shih et al., 2013), making it difficult to experimentally assess causality. A remedy for solving this issue is modeling and simulations (Fröhlich et al., 2006; Krishnan et al., 2015; Tagluk and Tekin, 2014; Tatsuki et al., 2016). The ability to maintain parameters constant, thereby reducing variable space dimensionality, is a powerful tool for interrogating the impact of changes in both intracellular and extracellular ion concentrations on  $V_m$  dynamics and state transitions, as well as for formulating hypotheses and predictions for future experimental investigations.

An important step toward successfully modeling the neuronal dynamics in a simple manner, is the Averaged-Neuron model, which recently has been proposed (Tatsuki et al., 2016). This model is inspired by previously constructed neural-network models (Bazhenov et al., 2002; Chen et al., 2012; Compte et al., 2003; Hill and Tononi, 2005; Sanchez-Vives et al., 2010; Timofeev et al., 2000), but simplifies the enormous complexity of these by performing a mean-field approximation of a





### Figure 1. Implementing Ion Concentrations in the Averaged-Neuron Model

(A) Schematic diagram of the extended Averaged-Neuron model containing intrinsic ion channel conductances and  $Ca^{2+}$  pumps, extrinsic synaptic ion channel conductances, and extracellular ion concentrations.

(B) Representative slow-wave sleep membrane potential ( $V_m$ ) firing pattern and intracellular  $Ca^{2+}$  concentration ( $[Ca^{2+}]_i$ ) with the initial extracellular ion concentration parameter set ( $[K^+]_o = 3.5$  mM,  $[Ca^{2+}]_o = 1.5$  mM, and  $[Mg^{2+}]_o = 1.1$  mM). Below is an expansion of  $V_m$  dynamics.

(C)  $V_m$  frequency spectra.

(D) SD of mean  $V_m$  (spikes removed).

(E) Mean  $V_m$  distribution.

(F) Representative  $V_m$  firing pattern and  $[Ca^{2+}]_i$  with 3.5, 8, or 14 mM  $[K^+]_o$ . Below is an expansion of  $V_m$  dynamics.

(G) Upper: Spike rate versus time for 3.5, 8, or 14 mM  $[K^+]_o$ . Lower: SD of mean  $V_m$  for 3.5, 8, and 14 mM  $[K^+]_o$ .

See also Figure S1.

population of neurons to construct an Averaged-Neuron model. The framework of the model is based on the existence of different ion channels and is mathematically described in a Hodgkin-Huxley manner; all contribute to the total change in  $V_m$ . As a first-order approximation, it is assumed that the averaged-neuron can interact with itself (directly or indirectly) through excitatory or inhibitory synaptic currents. Furthermore, intrinsic currents exist as depolarizing  $Na^+$  and  $Ca^{2+}$  currents and hyperpolarizing  $K^+$  currents. This polarization affects the  $V_m$  through nine different intrinsic channels (Figure 1A).

Here, we extend the Averaged-Neuron model to explicitly treat the changes in extracellular ion concentrations that occur during the sleep-wake cycle. We demonstrate how decreasing the

conductance of the  $Ca^{2+}$ -dependent  $K^+$  channel, in combination with physiological changes in  $[K^+]_o$ ,  $[Ca^{2+}]_o$ , and  $[Mg^{2+}]_o$ , initiates state transitions in neuronal firing patterns. We find that sleep is dominated by a stable limit cycle, resulting in self-sustained, stable firing patterns, whereas wakefulness is governed by irregular oscillations and chaotic dynamics. As the ion concentrations are changed, the firing patterns become more dominated by long high-frequency upstate periods, and we predict that a state change from quiet to active awake can be induced by subtle changes in extracellular ion concentrations. Finally, we discuss how the fundamental properties of chaotic dynamics can ensure smooth transitions between brain states in the presence of intrinsic noise.

## RESULTS

### Implementing Dynamic Ion Concentration Dependencies in the Averaged-Neuron Model

As a starting point for investigating the influence of extracellular ion concentrations on neuronal states, we first implemented and extended the Averaged-Neuron model (Tatsuki et al., 2016). In this model, a single neuron feeds back onto itself directly and indirectly, through synaptic excitatory and inhibitory conductances (AMPA, NMDA, and GABA<sub>A</sub> receptors) (Figure 1A). In addition to synaptic conductances, the Averaged-Neuron model includes the following intrinsic conductances, either depolarizing or hyperpolarizing  $V_m$ : voltage-gated (Na<sub>v</sub>) and persistent (NaP) Na<sup>+</sup> channels, voltage-gated (Ca<sub>v</sub>) Ca<sup>2+</sup> channels, voltage-gated (K<sub>v</sub>), leak (K<sub>Leak</sub>), fast A-type (K<sub>A-type</sub>), inwardly rectifying (K<sub>IR</sub>), slowly inactivating (K<sub>SI</sub>), and Ca<sup>2+</sup>-dependent (K<sub>Ca</sub>) K<sup>+</sup> channels, and finally a Ca<sup>2+</sup>-pump/exchanger expelling Ca<sup>2+</sup> ions from the intracellular compartment (Tatsuki et al., 2016) (Figure 1A). For all of these conductances, we adapted the full parameter set from the original Averaged-Neuron model published by Tatsuki et al. (2016). The original Averaged-Neuron model did not consider changes in extracellular ion concentrations as variables but as fixed constants. We extended the Averaged-Neuron model by introducing extracellular and intracellular ion concentration dependencies by implementing Nernst and Goldman-Hodgkin-Katz equations for calculating reversal potentials for all receptors and channels. We also added a [Mg<sup>2+</sup>]<sub>o</sub> dependency for the NMDA receptor that satisfies the criteria that higher [Mg<sup>2+</sup>]<sub>o</sub> leads to lower current (Mayer et al., 1984; Nowak et al., 1984) (Figure 1A). A detailed formal description of the extended Averaged-Neuron model is provided in the STAR Methods and the code is available on GitHub (<https://github.com/Neurone/IonsAndChaos>).

First, we simulated  $V_m$  in time with an initial extracellular ion concentration parameter set of [K<sup>+</sup>]<sub>o</sub> = 3.5 mM, [Ca<sup>2+</sup>]<sub>o</sub> = 1.5 mM, and [Mg<sup>2+</sup>]<sub>o</sub> = 0.8 mM. The remaining intra- and extracellular ion concentrations were maintained constant, and values were based on previous *in vivo* and *in vitro* experimental measurements (Diarra et al., 2001; Dietzel et al., 1982; Markova et al., 2008; Raimondo et al., 2013; Rose and Konnerth, 2001; Rose and Ransom, 1996). We found that the model remarkably closely recapitulated previous recordings obtained from neurons during slow-wave sleep (simply referred to as sleep from here on) (Steriade et al., 2001) (Figure 1B). The extended Averaged-Neuron model generated wave forms of slow oscillatory patterns, consisting of alternating periods of  $V_m$  depolarization and spiking (upstates) and  $V_m$  hyperpolarization and silence (downstates). This oscillatory pattern was also present in the modeled intracellular Ca<sup>2+</sup> concentration ([Ca<sup>2+</sup>]<sub>i</sub>), with upstates generating a ~8 μM rise in [Ca<sup>2+</sup>]<sub>i</sub> (Figure 1B). The preferential slow  $V_m$  oscillations were evident when analyzing frequency components for ten simulations, showing a high prevalence of 1–4 Hz delta power (Figure 1C). The moving SD of the mean  $V_m$  (spikes removed) varied between 0.1 and 3 mV (Figure 1D), similar to the seminal recordings obtained by Steriade et al. (2001). Finally, mean  $V_m$  showed a clear bimodal distribution, with peaks around –70 and –50 mV, reflecting the equal presence of up- and downstates separated by ~20 mV (Figure 1E).

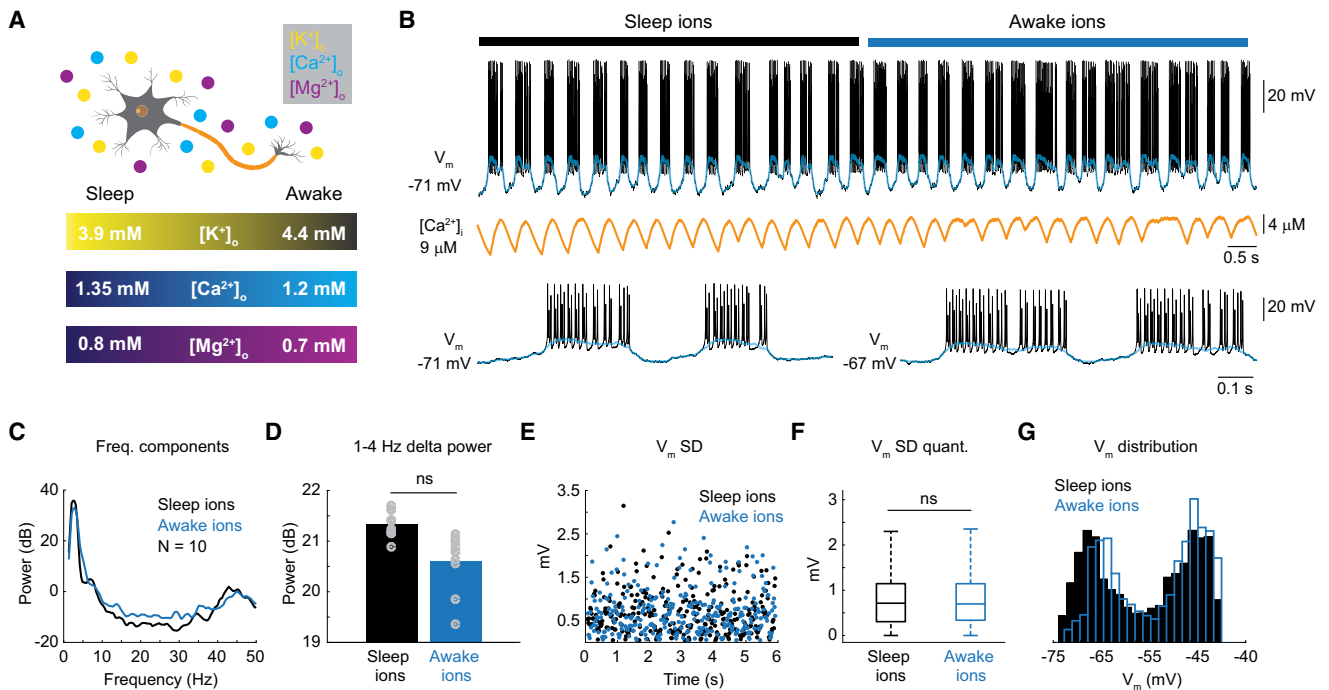
To assess the validity of our implemented ion concentration dependencies in the extended Averaged-Neuron model, we sought to test the effect of increasing [K<sup>+</sup>]<sub>o</sub> from 3.5 to 8 to 14 mM. The resting  $V_m$  of neurons is mainly determined by K<sup>+</sup>-permeable conductances, and  $V_m$  is highly sensitive to changes in [K<sup>+</sup>]<sub>o</sub> (Fröhlich et al., 2008). Thus, we expected drastic  $V_m$  changes when pushing [K<sup>+</sup>]<sub>o</sub> to the biological extremes if our model was valid. When increasing [K<sup>+</sup>]<sub>o</sub> from 3.5 to 8 mM, we observed a qualitative change in  $V_m$  dynamics. The alternating up- and downstates disappeared, and instead  $V_m$  was maintained depolarized and the neuron fired continuously at rates of ~100 Hz, giving rise to a tonic elevation in [Ca<sup>2+</sup>]<sub>i</sub> at ~9 μM (Figures 1F and 1G). This finding is in congruence with *in vitro* slice experiments, inducing epileptic seizure activity by increasing [K<sup>+</sup>]<sub>o</sub> to 7.5 or 8.5 mM in the external media (Jensen and Yaari, 1997; Traynelis and Dingledine, 1988). Further elevating [K<sup>+</sup>]<sub>o</sub> to 14 mM depolarized  $V_m$  even more but also completely abolished action potential firing, releasing the tonic [Ca<sup>2+</sup>]<sub>i</sub> elevation (Figures 1F and 1G). This effect is likely explained by depolarization-induced conductance blockage, where Na<sub>v</sub> channels are kept hostage in the inactivated configuration, resulting in the inability of the neuron to fire (De Col et al., 2008; Orkand et al., 1966). This observed effect appears similar to what is seen during *in vivo* cortical spreading depression, where [K<sup>+</sup>]<sub>o</sub> reaches levels above 12 mM (Enger et al., 2015), leading to annihilation of neural activity in the affected brain area (Grafstein, 1956; Somjen, 2001, 2002, 2004). An example of how  $V_m$  dynamics are also modulated by extreme shifts in [Ca<sup>2+</sup>]<sub>o</sub> is provided in Supplemental Information (Figure S1).

Overall, these results demonstrate that the extended Averaged-Neuron model very closely recapitulates cardinal properties of  $V_m$  dynamics during sleep and is expectedly sensitive to changes in extracellular ion concentrations.

### Shifting from Sleep to Awake Ion Concentrations Induces No Apparent State Transition

It was recently suggested that changes in extracellular ion concentrations ([K<sup>+</sup>]<sub>o</sub>, [Ca<sup>2+</sup>]<sub>o</sub>, and [Mg<sup>2+</sup>]<sub>o</sub>) control state-dependent activity patterns during sleep and wakefulness, and that ion shifts are sufficient to mediate the transition from sleep to awake (Ding et al., 2016). However, due to the intimate relation between neural activity and ionic changes, establishing causality *in vivo* is non-trivial.

To investigate the effect of ion concentration changes on sleep to awake transitions in the extended Averaged-Neuron model, we used the previously measured extracellular ion concentrations during sleep ([K<sup>+</sup>]<sub>o</sub> = 3.9 mM, [Ca<sup>2+</sup>]<sub>o</sub> = 1.35 mM, and [Mg<sup>2+</sup>]<sub>o</sub> = 0.8 mM) and awake ([K<sup>+</sup>]<sub>o</sub> = 4.4 mM, [Ca<sup>2+</sup>]<sub>o</sub> = 1.2 mM, and [Mg<sup>2+</sup>]<sub>o</sub> = 0.7 mM) (Ding et al., 2016) (Figure 2A). With sleep ions,  $V_m$  dynamics closely recapitulated sleep with alternating up- and downstates. When we switched to awake ions, no apparent change in  $V_m$  dynamics was obvious, except a subtle tendency for more spikes per upstate (Figure 2B). When we analyzed  $V_m$  frequency components, we found no significant difference in 1–4 Hz delta power between sleep (21.2 ± 0.24 dB) and awake ions (20.4 ± 0.62 dB) ( $p$  = 0.08, Student's  $t$  test,  $n$  = 10 simulations) (Figures 2C and 2D). Also mean  $V_m$  SD, a frequently used indicator for neuronal state (McGinley et al., 2015a; Steriade et al., 2001; Yamashita et al.,



**Figure 2. Shifting from Sleep to Awake Extracellular Ion Concentrations Does Not Induce State Change**

(A) Schematic diagram of extracellular ion concentrations used. Sleep:  $[K^+]_o = 3.9$  mM,  $[Ca^{2+}]_o = 1.35$  mM, and  $[Mg^{2+}]_o = 0.8$  mM. Awake:  $[K^+]_o = 4.4$  mM,  $[Ca^{2+}]_o = 1.2$  mM, and  $[Mg^{2+}]_o = 0.7$  mM.

(B) Representative membrane potential ( $V_m$ ) firing pattern and intracellular  $Ca^{2+}$  concentration ( $[Ca^{2+}]_i$ ) with sleep and awake ion composition. Below is an expansion of  $V_m$  dynamics.

(C)  $V_m$  frequency spectra for sleep and awake ion composition.

(D) 1–4 Hz delta power for sleep and awake ion composition (non-significant [ns]  $p = 0.08$ , Student's  $t$  test,  $n = 10$  simulations).

(E) SD of mean  $V_m$  (spikes removed) with sleep or awake ion composition.

(F) Boxplot for SD of mean  $V_m$  with sleep or awake ion composition. Center line is the median, box limits are 25<sup>th</sup> and 75<sup>th</sup> percentiles, and whiskers are maximum and minimum values (ns  $p = 0.089$ , Student's  $t$  test,  $n = 10$  simulations).

(G) Mean  $V_m$  distribution with sleep or awake ion composition.

See also Figure S2.

2013), was similar for sleep ( $0.77 \pm 0.62$  mV) and awake ions ( $0.78 \pm 0.58$  mV) ( $p = 0.089$ , Student's  $t$  test,  $n = 10$  simulations) (Figures 2E and 2F). Finally, mean  $V_m$  showed a clear bimodal distribution with both sleep and awake ion concentrations (Figure 2G).

Taken together, shifting the extracellular concentrations of  $K^+$ ,  $Ca^{2+}$ , and  $Mg^{2+}$  with the same magnitude as measured *in vivo* during sleep and awake failed to induce a cortical state transition, suggesting that ion changes of this magnitude alone are not sufficient to produce a transition from sleep to awake. To exclude that these findings were not merely a special case of one parameter set, we evaluated the effect of switching from sleep to awake ion concentrations, but with varying values of model parameters. These analyses showed the same overall result, as with the initial parameter set (Figure S2) thus substantiating our initial conclusion.

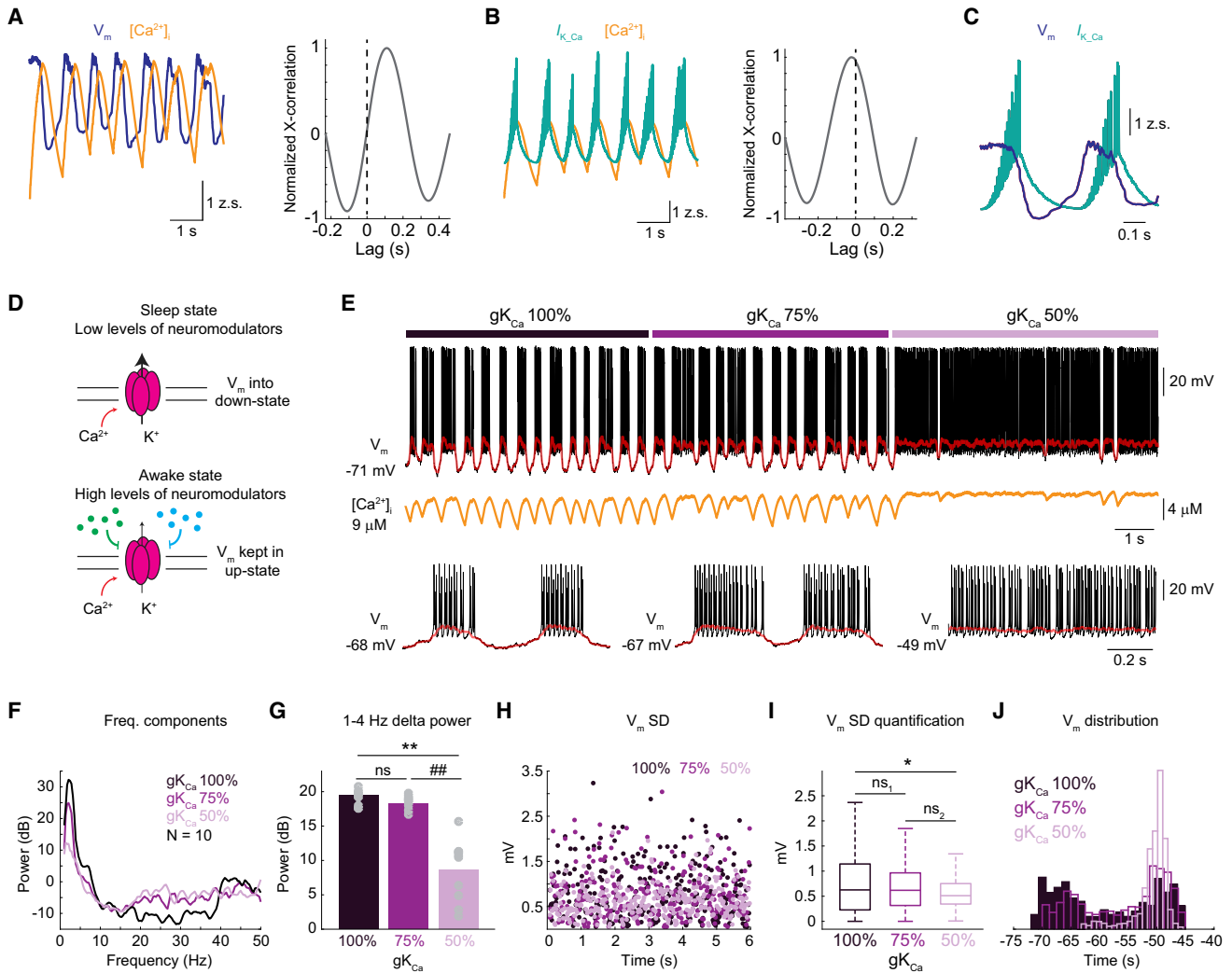
### Partially Inhibiting the $K_{Ca}$ Channel Induces Sleep to Awake State Transition

Early seminal and more recent work has implicated the  $K_{Ca}$  channel in  $V_m$  activity patterns observed during sleep and wakefulness (Steriade et al., 1993; Tatsuki et al., 2016). Following

a rise in  $[Ca^{2+}]_i$  this channel activates and generates an outward  $K^+$  current, hyperpolarizing  $V_m$ . In this way, the  $K_{Ca}$  channel was proposed as a key component for generating downstates during sleep (Steriade et al., 1993). In contrast, with elevated levels of neuromodulators (acetylcholine, norepinephrine, serotonin, and histamine) as seen during wakefulness (Lee and Dan, 2012), the  $K_{Ca}$  channel is partially inhibited (McCormick and Williamson, 1989; McCormick et al., 1993), and this could potentially induce the sleep to awake transition by preventing the occurrence of downstates (Steriade et al., 1993).

During our sleep  $V_m$  simulations, we observed a close relation between mean  $V_m$  and  $[Ca^{2+}]_i$  oscillations with a 0.12 s lag for  $[Ca^{2+}]_i$  (Figure 3A). This led us to analyze the relation between  $[Ca^{2+}]_i$  and the  $K_{Ca}$  current. Not surprisingly, we found a  $-0.013$  s lag between  $[Ca^{2+}]_i$  and  $K_{Ca}$  currents (Figure 3B). The largest  $K_{Ca}$  currents occurred when mean  $V_m$  was most depolarized, and  $V_m$  rapidly hyperpolarized following peak  $K_{Ca}$  currents (Figure 3C), suggesting that  $K_{Ca}$ -mediated outward currents could be key for generating downstates.

To assess the role of the  $K_{Ca}$  channel and to mimic the elevation in neuromodulator levels seen during wakefulness (Lee and Dan, 2012; McCormick and Williamson, 1989), we reduced the

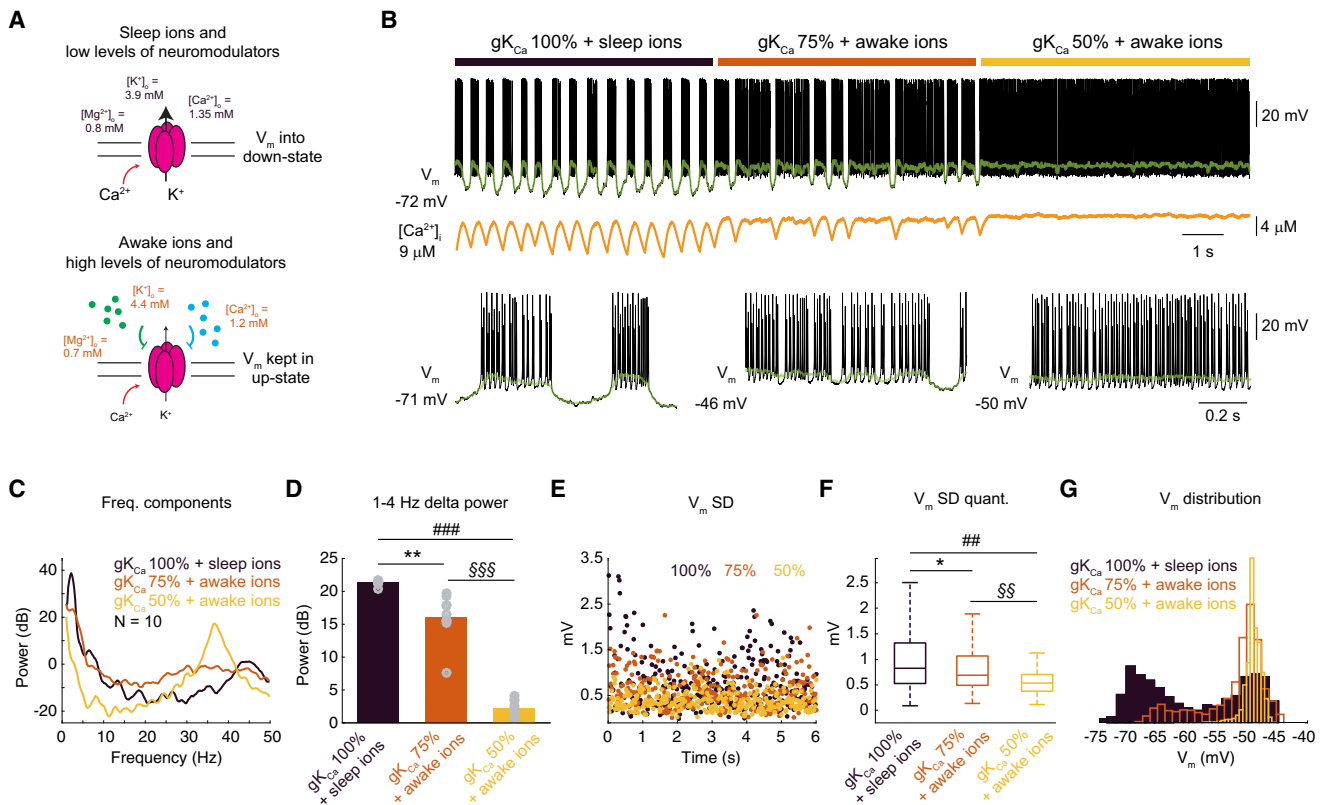


**Figure 3. Partially Inhibiting  $\text{Ca}^{2+}$ -Activated  $\text{K}^+$  Channel Can Induce Sleep to Awake State Change**

- (A) Left: Relation between membrane potential ( $V_m$ ) and intracellular  $\text{Ca}^{2+}$  concentration ( $[\text{Ca}^{2+}]_i$ ). Right: Cross-correlation between  $V_m$  and  $[\text{Ca}^{2+}]_i$ .  
 (B) Left: Relation between  $\text{Ca}^{2+}$ -activated  $\text{K}^+$  channel current ( $I_{\text{K,Ca}}$ ) and  $[\text{Ca}^{2+}]_i$ . Right: Cross-correlation between  $I_{\text{K,Ca}}$  and  $[\text{Ca}^{2+}]_i$ .  
 (C) Relation between  $V_m$  and  $I_{\text{K,Ca}}$ .  
 (D) Schematic diagram of speculated sleep and awake state and neuromodulator-mediated decrease of  $\text{Ca}^{2+}$ -activated  $\text{K}^+$  channel conductance ( $g_{\text{KCa}}$ ).  
 (E) Representative  $V_m$  firing pattern and  $[\text{Ca}^{2+}]_i$  with 100%, 75%, or 50%  $g_{\text{KCa}}$ . Below is an expansion of  $V_m$  dynamics.  
 (F)  $V_m$  frequency spectra for 100%, 75%, and 50%  $g_{\text{KCa}}$ .  
 (G) 1–4 Hz delta power for 100%, 75%, and 50%  $g_{\text{KCa}}$  (non-significant [ns],  $p = 0.383$ ,  $**p = 0.0069$ ,  $##p = 0.0083$ , one-way ANOVA,  $n = 10$  simulations).  
 (H) SD of mean  $V_m$  (spikes removed) with 100%, 75%, and 50%  $g_{\text{KCa}}$ .  
 (I) Boxplot for SD of mean  $V_m$  with 100%, 75%, and 50%  $g_{\text{KCa}}$ . Center line is the median, box limits are 25<sup>th</sup> and 75<sup>th</sup> percentiles, and whiskers are maximum and minimum values (ns<sub>1</sub>,  $p = 0.875$ ,  $*p = 0.0262$ , ns<sub>2</sub>,  $p = 0.065$ , one-way ANOVA,  $n = 10$  simulations).  
 (J) Mean  $V_m$  distribution for 100%, 75%, and 50%  $g_{\text{KCa}}$ .  
 See also [Figure S3](#).

conductance of the  $\text{K}_{\text{Ca}}$  channel ( $g_{\text{KCa}}$ ) over time (100% = 2.3  $\text{mS}/\text{cm}^2$ , 75% = 1.73  $\text{mS}/\text{cm}^2$ , and 50% = 1.15  $\text{mS}/\text{cm}^2$ ). This simulation was performed with the sleep ion concentrations being maintained throughout. When reducing  $g_{\text{KCa}}$  to 75%, we observed only a tendency toward a state change, with longer upstates and fewer downstates, whereas reducing  $g_{\text{KCa}}$  to 50% produced a clear state change, almost completely preventing  $V_m$  from entering downstates, thereby producing tonic firing ([Figure 3E](#)). This qualitative interpretation was supported when quantified. Decreasing  $g_{\text{KCa}}$  to 75% non-significantly reduced

1–4 Hz delta power ( $p = 0.383$ , one-way ANOVA,  $n = 10$  simulations), whereas 50% significantly reduced delta power ( $p = 0.0069$ , one-way ANOVA,  $n = 10$  simulations). Mean  $V_m$  SD was similar between 100% ( $0.74 \pm 0.55$  mV) and 75%  $g_{\text{KCa}}$  ( $0.67 \pm 0.46$  mV) ( $p = 0.875$ , one-way ANOVA,  $n = 10$  simulations), but significantly reduced with 50%  $g_{\text{KCa}}$  ( $0.59 \pm 0.32$  mV) ( $p = 0.0262$ , one-way ANOVA,  $n = 10$  simulations) ([Figures 3H](#) and [3I](#)). Finally, mean  $V_m$  distributions for 100% and 75%  $g_{\text{KCa}}$  appeared bimodal, whereas the 50%  $g_{\text{KCa}}$  distribution was uni-modal ([Figure 3J](#)).



**Figure 4. Awake Ion Concentrations Lowers the Threshold for Sleep to Awake State Change**

(A) Schematic diagram of speculated interaction between extracellular ion concentrations, neuromodulators, and  $Ca^{2+}$ -activated  $K^+$  channel conductance ( $gK_{Ca}$ ) during sleep and wakefulness.

(B) Representative membrane potential ( $V_m$ ) firing pattern and intracellular  $Ca^{2+}$  concentration ( $[Ca^{2+}]_i$ ) with 100%  $gK_{Ca}$  + sleep ions ( $[K^+]_o = 3.9$  mM,  $[Ca^{2+}]_o = 1.35$  mM, and  $[Mg^{2+}]_o = 0.8$  mM), 75%  $gK_{Ca}$  + awake ions ( $[K^+]_o = 4.4$  mM,  $[Ca^{2+}]_o = 1.2$  mM, and  $[Mg^{2+}]_o = 0.7$  mM) or 50%  $gK_{Ca}$  + awake ions. Below is an expansion of  $V_m$  dynamics.

(C)  $V_m$  frequency spectra for 100%  $gK_{Ca}$  + sleep ions, 75%  $gK_{Ca}$  + awake ions, or 50%  $gK_{Ca}$  + awake ions.

(D) 1–4 Hz delta power for 100%  $gK_{Ca}$  + sleep ions, 75%  $gK_{Ca}$  + awake ions, or 50%  $gK_{Ca}$  + awake ions (\*\*p = 0.026, ###p < 0.0001, §§§p < 0.0001, one-way ANOVA, n = 10 simulations).

(E) SD of mean  $V_m$  (spikes removed) for 100%  $gK_{Ca}$  + sleep ions, 75%  $gK_{Ca}$  + awake ions, or 50%  $gK_{Ca}$  + awake ions.

(F) Boxplot for SD of mean  $V_m$  for 100%  $gK_{Ca}$  + sleep ions, 75%  $gK_{Ca}$  + awake ions, or 50%  $gK_{Ca}$  + awake ions. Center line is the median, box limits are 25<sup>th</sup> and 75<sup>th</sup> percentiles, and whiskers are maximum and minimum values (\*p = 0.011, ##p = 0.0062, §§p = 0.0018, one-way ANOVA, n = 10 simulations).

(G) Mean  $V_m$  distribution for 100%  $gK_{Ca}$  + sleep ions, 75%  $gK_{Ca}$  + awake ions, or 50%  $gK_{Ca}$  + awake ions.

See also Figure S4.

To illuminate if changing  $gK_{Ca}$  is the most efficient way to induce the awake state, we performed a bifurcation analysis (i.e., gradual change in parameter values). This revealed that the parameter of  $gK_{Ca}$  was the most important parameter for creating state change in firing dynamics (Figure S3). Changing the conductance of almost all other channels had very little effect, except for the parameter  $gCa_V$ , which also had the potential to create a transition, but this was still less potent than  $gK_{Ca}$  (Figure S3).

Overall, these results demonstrate that the  $K_{Ca}$  channel most likely plays a dominant role in sleep  $V_m$  activity patterns, and inhibiting this channel, assumedly via increased levels of neuromodulators, is sufficient to induce a transition to the awake state. However, decreasing the conductance of this channel by 25% was not sufficient to invoke a full state transition, suggesting a threshold level for when the state is shifted from sleep to wakefulness.

### Awake Ion Concentrations Lower the Threshold for Sleep to Awake State Transition

Although changing extracellular ions from sleep to awake concentrations was not in itself sufficient to induce a state transition (Figures 2 and S2), we speculate if awake ions could lower the threshold for invoking the  $K_{Ca}$  channel-dependent state change and in this way be permissive and modulatory on state changes (Figure 4A).

To investigate this, we changed the extracellular ions from sleep to awake concentrations while reducing  $gK_{Ca}$  to 75% and 50% as in the previous section (Figure 3). In contrast to Figure 3, when we reduced  $gK_{Ca}$  to 75% in the presence of awake ions, we observed a state transition with  $V_m$  kept in the upstate for long periods and with few downstates (Figure 4B). As before, when reducing  $gK_{Ca}$  to 50%, the state change was pronounced, with  $V_m$  tonically depolarized and the neuron continuously firing (Figure 4B). We now found a significant reduction in 1–4 Hz delta

power with the combination of 75%  $g_{K_{Ca}}$  and awake ions ( $p = 0.026$ , one-way ANOVA,  $n = 10$  simulations), as well as with 50%  $g_{K_{Ca}}$  and awake ions ( $p < 0.0001$ , one-way ANOVA,  $n = 10$  stimulations) (Figures 4C and 4D). The prevalence of delta power was significantly smaller with 50%  $g_{K_{Ca}}$  compared with 75%  $g_{K_{Ca}}$  ( $p < 0.0001$ , one-way ANOVA,  $n = 10$  simulation) (Figure 4D), likely reflecting that wakefulness comprises two states (i.e., quiet awake and active awake) (Crochet and Petersen, 2006; McGinley et al., 2015b). Further supporting a permissive effect of awake ions on the sleep to awake transition was the significant decrease in mean  $V_m$  SD from 100%  $g_{K_{Ca}}$  and sleep ions ( $0.81 \pm 0.15$  mV) to 75%  $K_{Ca}$  and awake ions ( $0.64 \pm 0.13$  mV) ( $p = 0.011$ , one-way ANOVA,  $n = 10$  simulations). Reducing  $g_{K_{Ca}}$  to 50% with awake ions caused a significant decrease in mean  $V_m$  SD to ( $0.51 \pm 0.07$  mV) ( $p = 0.0062$ , one-way ANOVA,  $n = 10$  simulations), significantly smaller than with 75%  $g_{K_{Ca}}$  ( $p = 0.0018$ , one-way ANOVA,  $n = 10$  simulations), again pointing to multiple awake states (Figure 4F). Finally, the mean  $V_m$  distribution was shifted toward uni-modality with 75%  $g_{K_{Ca}}$  and awake ions, and this was even more apparent with 50%  $g_{K_{Ca}}$  and awake ions (Figure 4G).

Taken together, these data suggest that the awake ion concentrations reduce the threshold for invoking a sleep to awake state transition, and thereby are permissive on this state change. We found that  $[K^+]_o$  is the most potent mediator of this ion-concentration-mediated effect, but the combined concurrent shift in all three ion species triggers the greatest sleep to awake state change (Figure S4). What is more, the results point to and support current *in vivo* evidence that the awake state comprises multiple sub-states, with differences in delta oscillations and mean  $V_m$  dynamics.

### Subtle Change in Extracellular Ion Concentrations Shifts the State from Quiet Awake to Active Awake

After demonstrating that the awake ion concentrations are permissive for shifting  $V_m$  dynamics from sleep to awake (Figures 4 and S4), we next investigated the two apparent awake states in greater detail, and in particular the role of extracellular ion concentrations on the transition from the quiet awake to active awake state. For this, we speculate that an arousal-related “hyper-awake” ion composition (in addition to the sleep and awake ion compositions) might exist and could be involved in shifting  $V_m$  into the active awake state (Bennett et al., 2013; Crochet and Petersen, 2006; Gentet et al., 2010; McGinley et al., 2015a; Polack et al., 2013; Reimer et al., 2014; Zagha et al., 2013). As an estimate, we changed extracellular ions with the same magnitude as observed between sleep and awake for formulating the hypothesized hyper-awake ion concentrations ( $[K^+]_o = 4.9$  mM,  $[Ca^{2+}]_o = 1.05$  mM, and  $[Mg^{2+}]_o = 0.6$  mM) (Figure 4A).

We initially simulated the quiet awake state (75%  $g_{K_{Ca}}$  and awake ions) and then selectively shifted the extracellular ion concentrations to hyper-awake ions while maintaining  $g_{K_{Ca}}$  at 75% (Figures 5A and 5B). We observed that with awake ions,  $V_m$  alternated between long periods of firing (upstate) and occasional silent downstates (Figure 5B), similar to what is described in quiet awake rodents (Crochet and Petersen, 2006; McGinley et al., 2015a; Yamashita et al., 2013). When we shifted to hyper-awake ion concentrations, we observed an annihilation

of downstates, and  $V_m$  was maintained in the depolarized upstate with continuous firing as a result (Figure 5B). We next analyzed these two  $V_m$  states further. We found that with hyper-awake ions, 1–4 Hz delta power significantly decreased ( $p = 0.002$ , Student’s *t* test,  $n = 10$  simulations) and 25–45 Hz gamma power significantly increased ( $p = 0.0023$ , Student’s *t* test,  $n = 10$  simulations) (Figures 5C–5E). This result is similar to what has been observed when rodents transition from the quiet to the active awake state (Niell and Stryker, 2010; Reimer et al., 2014; Vinck et al., 2015). With hyper-awake ions, mean  $V_m$  was significantly more depolarized than with awake ions ( $-49.38 \pm 0.11$  mV and  $-52.62 \pm 0.23$  mV;  $p = 0.002$ , Student’s *t* test,  $n = 10$  simulations) (Figure 5F), keeping  $V_m$  closer to the firing threshold. Finally, mean  $V_m$  distribution was shifted toward the depolarized upstate with hyper-awake ions, representing the lack of downstates (Figure 5G). All of these results are in congruence with *in vivo* recordings obtained in awake rodents, where the active awake state is characterized by a depolarized  $V_m$  and a lack of downstates (Crochet and Petersen, 2006; McGinley et al., 2015a; Yamashita et al., 2013).

Taken together, these results demonstrate that subtle changes in extracellular ion concentrations are sufficient for shifting  $V_m$  dynamics from the quiet awake state to the active awake state. Shifting to the hypothesized hyper-awake ion concentrations recapitulated key properties of the active awake state  $V_m$  dynamics, including increased gamma oscillations and depolarized  $V_m$ . Further analysis revealed that shifting  $[K^+]_o$  alone was sufficient for invoking this state transition, but the combined effect of shifting all three ion species was greater (Figure S5), suggesting a biological importance of concerted extracellular ion changes. We advocate that the existence of such hyper-awake ion concentrations should be investigated in future *in vivo* experiments, as well as its ability to causally invoke the proposed awake state changes.

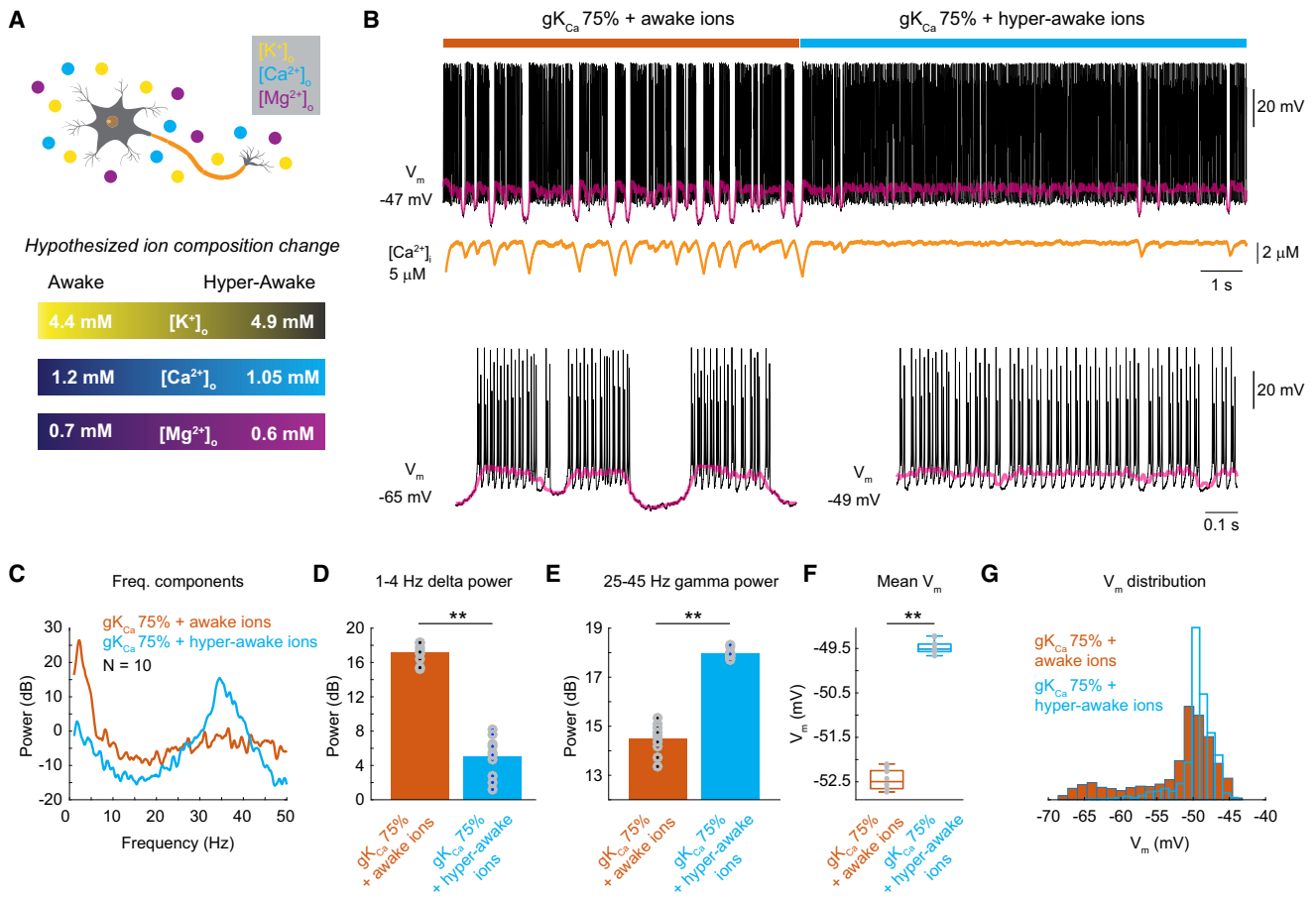
### Transitions between Brain States Are Governed by Chaotic Dynamics

After finding that awake ion concentrations are permissive for shifting the state from sleep awake (Figure 4), and that hyper-awake ion concentrations can invoke the transition from quiet to active awake (Figure 5), we next investigated the state-dependent properties of  $V_m$  dynamics in more analytical and physical terms, as we altered extracellular ion concentrations.

First, we performed a detailed simulation of the sleep state, where in the deterministic system we found spikes and  $V_m$  oscillations to be stable and periodic in the sense that the pattern was self-repeating, and small perturbations in the initial conditions did not make the trajectories diverge (Figure 6A). This therefore had to be a closed trajectory with several small loops that occur when the neuron fires, which is best visualized in a three-dimensional phase space spanned by three variables ( $V_m$ ,  $Ca^{2+}$ , and  $n_K$ ) (Figure 6B). This means that  $V_m$  dynamics in the sleep state is well defined by regular and stable oscillations, exactly as what one would expect from a synchronous system.

Next, we considered which ion currents are most important for changing the state from sleep to awake. From what was observed in the awake state (Figures 3 and 4), we characterize the transition in terms of (1) the rate at which a silent downstate period is initiated, (2) the mean time duration of a spiking upstate period, and





**Figure 5. Transition between Two Awake States Can Be Controlled by Extracellular Ion Concentrations**

(A) Schematic diagram of hypothesized shift in extracellular ion concentration from awake to hyper-awake. Awake:  $[K^+]_o = 4.4$  mM,  $[Ca^{2+}]_o = 1.2$  mM, and  $[Mg^{2+}]_o = 0.7$  mM. Hyper-awake:  $[K^+]_o = 4.9$  mM,  $[Ca^{2+}]_o = 1.05$  mM, and  $[Mg^{2+}]_o = 0.6$  mM.

(B) Representative membrane potential ( $V_m$ ), firing pattern, and intracellular  $Ca^{2+}$  concentration ( $[Ca^{2+}]_i$ ) with 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions. Below is an expansion of  $V_m$  dynamics.

(C)  $V_m$  frequency spectra for 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions.

(D) 1–4 Hz delta power for 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions (\*\* $p = 0.002$ , Student's  $t$  test,  $n = 10$  simulations).

(E) 25–45 Hz gamma power for 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions (\*\* $p = 0.0023$ , Student's  $t$  test,  $n = 10$  simulations).

(F) Boxplot for mean  $V_m$  for 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions. Center line is the median, box limits are 25<sup>th</sup> and 75<sup>th</sup> percentiles, and whiskers are maximum and minimum values (\*\* $p = 0.002$ , Student's  $t$  test,  $n = 10$  simulations).

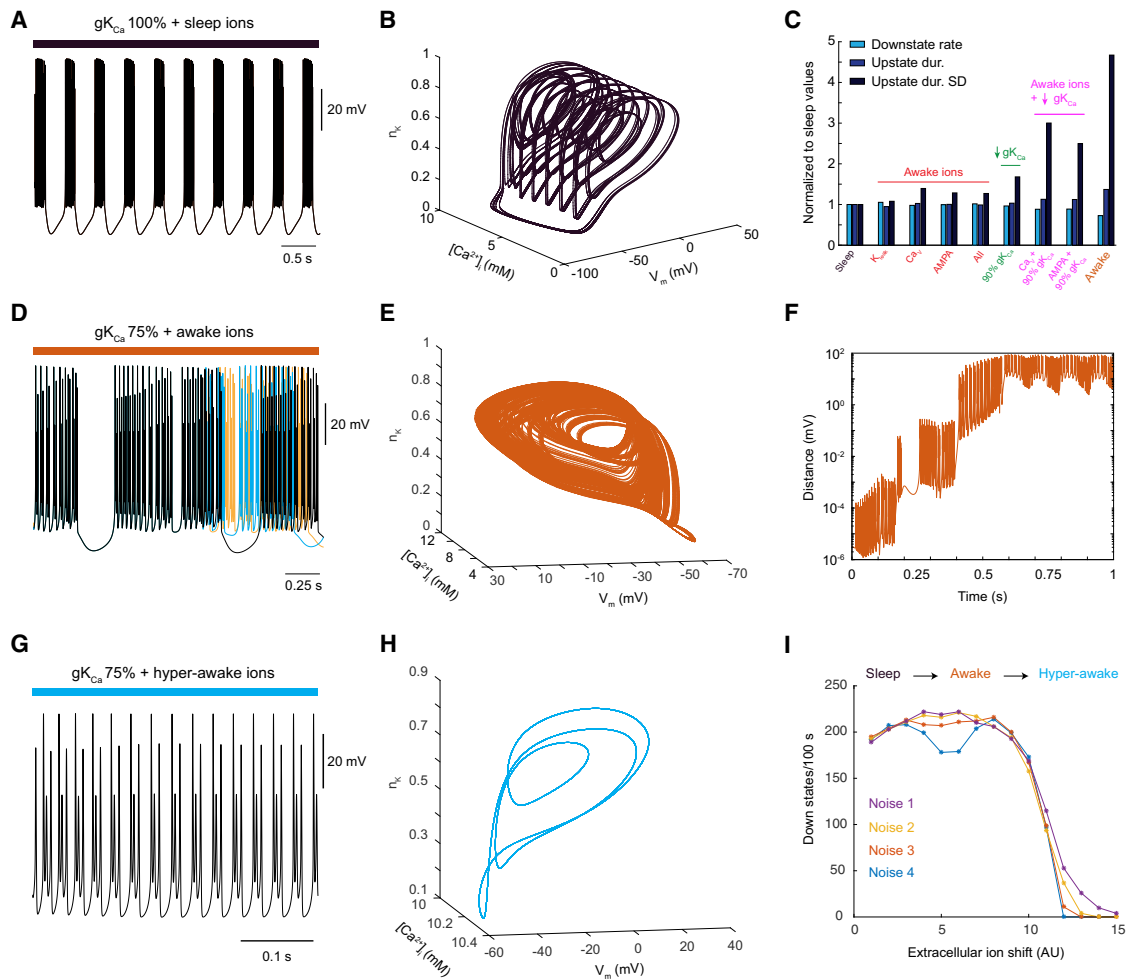
(G) Mean  $V_m$  distribution for 75%  $gK_{Ca}$  + awake ions and 75%  $gK_{Ca}$  + hyper-awake ions.

See also Figure S5.

(3) the SD of this measure. These numbers combined describe much of the change in  $V_m$  dynamics that happens between the sleep and awake state. By affecting only one of the currents at a time by the change in extracellular ion concentrations from sleep to awake, we found the largest ion-concentration-mediated effect was produced on the  $Ca_v$ -mediated current (Figure 6C). This was even better observed if we combined the sleep to awake ion-mediated changes in  $Ca_v$  current with a small perturbation in  $gK_{Ca}$  (Figure 6C). Thus, here we found that affecting only the  $Ca_v$  channel by changes in sleep to awake ion concentration can account for much of the irregularities of the awake state, and that a similar ion-mediated effect can occur for the extrinsic AMPA receptor, and this induces a big step toward the awake state (Figure 6C). We also show that decreasing  $gK_{Ca}$  is the most important factor in moving  $V_m$  dynamics toward the awake

state, but that changes in extracellular ion concentrations amplify this effect markedly (Figure 6C).

As observed in the sections above, transition from the sleep to the awake state could be invoked by a 25% reduction in  $gK_{Ca}$  and a concomitant change to awake ion concentrations (Figure 4). With the combination of 75%  $gK_{Ca}$  and awake ions,  $V_m$  dynamics are more irregular and significantly long periods of upstates and spiking can occur before a silent downstate is recovered (Figure 4B). This suggests that  $V_m$  dynamics in the awake state is chaotic, meaning that the phase space has a strange attractor. Deterministic chaos is defined by the fact that two initial conditions, being infinitesimally perturbed, will have diverging trajectories as time evolves, and that one cycle will never repeat itself and therefore no closed cycles exist on a strange attractor (Lorenz, 1963) (Box 1). We tested this by



**Figure 6. Transition from Sleep to Wakefulness Is Governed by Chaotic Dynamics**

(A) Representative membrane potential ( $V_m$ ) firing pattern in the sleep state with 100%  $g_{K_{Ca}}$  and sleep ion composition ( $[K^+]_o = 3.9$  mM,  $[Ca^{2+}]_o = 1.35$  mM, and  $[Mg^{2+}]_o = 0.8$  mM).

(B) Phase space plot showing the trajectory in the sleep state.

(C) Measures for the effect of different ion channel-mediated currents on  $V_m$  dynamics. Light blue is the average rate of initiating a silent downstate periods. Blue is the average duration of the spiky upstate periods. Dark blue is the SD in the duration of upstate periods. All measures are normalized to the sleep state.

(D) Representative  $V_m$  firing patterns with 75%  $g_{K_{Ca}}$ , awake ion composition ( $[K^+]_o = 4.4$  mM,  $[Ca^{2+}]_o = 1.2$  mM, and  $[Mg^{2+}]_o = 0.7$  mM) and 1 mV perturbations showing the development of different trajectories.

(E) Phase space plot showing the trajectory in the awake state with 75%  $g_{K_{Ca}}$  and awake ion composition.

(F) Difference between two  $V_m$  trajectories in the awake state with 75%  $g_{K_{Ca}}$  and awake ion composition.

(G) Representative  $V_m$  firing pattern in the awake state with 75%  $g_{K_{Ca}}$  and hyper-awake ion composition ( $[K^+]_o = 4.9$  mM,  $[Ca^{2+}]_o = 1.05$  mM, and  $[Mg^{2+}]_o = 0.6$  mM).

(H) Phase space plot showing the trajectory in the awake state with 75%  $g_{K_{Ca}}$  and hyper-awake ion composition.

(I) Number of silent periods for a period of 100 s as the values of  $[K^+]_o$ ,  $[Ca^{2+}]_o$ , and  $[Mg^{2+}]_o$  are perturbed linearly from values for the sleep state to values of the active awake state. On the x axis, 1 corresponds to the sleep state and 9 corresponds to the awake state.

perturbing the initial concentration of  $[Ca^{2+}]_i$  by only 1 pM ( $10^{-12}$ ), and even though  $V_m$  trajectories are very similar in the beginning, after some seconds they are completely different (Figure 6D). Another way of visualizing the chaotic nature of the awake state  $V_m$  dynamics can be seen in Figure 6E, where the trajectory is shown in three dimensions. Here, it seems that the trajectory never repeats and that the dynamics are irregular compared with the regular oscillations in the sleep state. To test if this had the signature of chaos, we calculated the difference in all dimensions between two trajectories and found that they diverge

in a complex manner that seems however to be on top of a general exponential diverging, which is what we would expect from a chaotic system (Lorenz, 1963) (Figure 6F).

As shown in previous sections, the sleep to awake state transition that we have now characterized is likely not the end of the story, and an additional active awake state exists (Crochet and Petersen, 2006; McGinley et al., 2015b) (Figures 4 and 5). Furthermore, we hypothesize that a parameter set of hyper-awake ion concentrations exist (Figure 5A). As in Figure 5, we observed that with hyper-awake ions,  $V_m$  dynamics is a

### Box 1. What Is Chaos?

Chaos is a well-defined mathematical concept that occurs in non-linear dynamical systems with three or more dimensions. It is defined by the property that two initial conditions infinitesimally apart exhibit exponentially diverging trajectories as time evolves. Such systems are deterministic in the sense that if one knows the initial state of the system exactly, then the trajectory will be the same every time it is initiated in that state. Chaos is thus a mathematical definition that should not be confused with the use of chaos in normal language. Furthermore, it is important to distinguish chaos from randomness or stochasticity in general. Randomness normally refers to non-deterministic responses due to stochastic noise, which in principle appears in any system of nature. The effects of stochasticity can sometimes be negligible but may, in some cases, give rise to a different dynamics than seen in a deterministic system. Examples of this can be sustained oscillations or multi-stability. Since systems always exhibit some uncertainty, the initial conditions in a system are never completely known and therefore chaotic systems, from a practical point of view, might appear random since it is impossible to predict the dynamical outcome.

continuously spiking process with no occurrence of silent downstates (Figure 6G), which likely correspond to an aroused and alert active awake state (Crochet and Petersen, 2006; Polack et al., 2013). We find that this state is not chaotic, but rather a well-defined, high-frequency three cycle (Figure 6H). This means that the initial transition from the sleep state can be regarded as a mediator between two robust states (sleep and active awake) defined by closed cycles. We consider it of great importance that the dynamics of  $V_m$  changes from sleep to the awake state and even further into an active awake state, using effects of chaotic dynamics, and how these changes are sensitive to subtle changes in extracellular ion concentrations.

Next, we performed the  $V_m$  simulation while applying white noise, which in the awake state, with awake ions, gives rise to  $V_m$  dynamics as shown in Figure 6D. Since stochasticity is present in every system and intrinsic biological noise is assumed to appear around the ion channels, noise is a very important element in this region where multi-stable solutions seem to arise. Since the most striking difference between the sleep and awake states is the occurrence/lack of silent downstates, we used the number of silent periods as a measure to characterize the gradual transition from sleep to the active awake state. We investigated this for four different noise levels while we linearly perturbed the values of  $[K^+]_o$ ,  $[Ca^{2+}]_o$ , and  $[Mg^{2+}]_o$  from the sleep to the hyper-awake concentrations. As can be observed from Figure 6I, the complete state transition in this measure is a continuous process for the noise levels tested, and shifting extracellular ion concentrations is sufficient to fully transition from the quiet awake to the active awake state.

In summary, we found that the transition from sleep to full wakefulness is modulated by changes in extracellular ion concentrations. Moreover, we show that if extracellular ions change, for instance if  $[K^+]_o$  is either below or above 3.9 and 4.5 mM, respectively, the dynamics of the system is determined by a stable limit cycle with several smaller loops included, whereas for  $[K^+]_o$  within the range of 3.9 and 4.5 mM, the system is chaotic, which can ensure a smooth and robust transition between the sleep state and the active awake state.

## DISCUSSION

Neuronal states powerfully influence sensory processing and perceptions (Harris and Thiele, 2011; Lee and Dan, 2012; Lee et al., 2014; McGinley et al., 2015a; Vinck et al., 2015). Here, we have shown that the concentrations of extracellular ions

influence  $K_{Ca}$  channel-mediated sleep to awake transitions and can themselves drive the switch from quiet awake to active awake. We demonstrate that the state change from sleep to awake is characterized by the transition from stable to chaotic dynamics, and argue that chaotic dynamics mediates a smooth transition between the quiet awake and active awake state, remarkably robust to intrinsic noise.

Previous studies have causally implicated ion concentrations in sleep to awake transitions (Ding et al., 2016). However, due to the intimate relation between neural activity and ionic changes, causality is difficult to establish *in vivo*. In our model, exclusively shifting ion concentrations by the same magnitude as measured *in vivo* was insufficient to cause a complete sleep to awake state transition (Figures 2, 6C, and S2), suggesting that other parameters need to change in time in order to transition the neuron into the awake state, and that ion concentration changes are not causal for this state transition. This might seem as an advantageous built-in gate mechanism, since extracellular ion concentrations are affected by even slight changes in neuronal firing (Hounsgaard and Nicholson, 1983; Lux, 1974; Lux and Neher, 1973; Nicholson et al., 1977), and global brain states should be robust to such small ion fluctuations, not causing the system to constantly cycle between sleep and awake.

We determined that decreasing the conductance of the  $K_{Ca}$  channel is a powerful way to induce the sleep to awake state change (Figures 3, 6C, and S3), in congruence with previous work implicating this channel as a prime mechanism for  $V_m$  downstates (McCormick and Williamson, 1989; McCormick et al., 1993; Steriade et al., 1993; Tatsuki et al., 2016). Shifting to the awake ion concentrations was permissive to decreasing the conductance of the  $K_{Ca}$  channel (Figures 4, 6C, and S4), advocating that subtle changes in extracellular ion concentrations can lower the threshold for how much ion channel modulation is required to shift the state. Changing the global environment surrounding neurons, in addition to modulating selective intrinsic ion channels, might be a simple, yet powerful, way to shift the state of entire brain-wide neuronal networks and ultimately for changing the behavioral state.

In more abstract terms, transitions between states in living organisms are fundamental for the functions and complexity of that organism. However, it is often speculated that very rapid state changes might be harmful for cells, since they might not be able to adapt properly and fast enough to the new dynamics. In this regard, the state transition described here for neuronal  $V_m$  dynamics has the beneficial property that it represents a gradual

and smooth change between the two stereotypical behavioral states, sleep and wakefulness. Furthermore, the chaotic nature of the phase space even guarantees the smoothness for a range of different intrinsic noise levels, which ensures that neurons can transition between states in a controlled and noise-robust manner when this is needed and appropriate.

It is increasingly appreciated that the awake state is comprised of at least two sub-states, namely quiet awake and active awake, related to the level of arousal and motor behavior (Bennett et al., 2013; Crochet and Petersen, 2006; Gentet et al., 2010; McGinley et al., 2015a; Polack et al., 2013; Reimer et al., 2014; Zagha et al., 2013). Rodent studies have suggested that the ongoing awake state shapes how the brain processes incoming sensory inputs. For example, the gain of visual-evoked responses in the visual cortex is increased when neurons are in the active awake state, whereas auditory responses in the auditory cortex are suppressed (Bennett et al., 2013; McGinley et al., 2015a; Polack et al., 2013; Schneider et al., 2014). This indicates that the brain state powerfully and differentially scales what sensory modalities are up- and downregulated at a given moment in time for optimizing sensory processing and neural computations, making it important to understand at the mechanistic level what determines brain states and the transition between them.

Here, we hypothesized that in addition to sleep and awake extracellular ion concentrations (Ding et al., 2016), a hyper-awake ion composition might exist (Figures 5, 6, and S5). When  $K_{Ca}$  was kept partially inhibited (75%  $g_{K_{Ca}}$ ), shifting from the awake to the hyper-awake ion concentrations produced a state change similar to what is observed when rodents transition from quiet to active (Crochet and Petersen, 2006; Yamashita et al., 2013) (Figures 5, 6, and S5). This ion-mediated state change was characterized by a decrease in delta power, an increase in gamma power,  $V_m$  depolarization, and a uni-modal  $V_m$  distribution (Figure 5); similar to what has been observed *in vivo* with intracellular recordings from cortical neurons in behaving rodents. We predict that subtle concerted changes in extracellular ion concentrations could be a key mechanism for the rapid and often-occurring state shifts in the awake brain, allowing sensory processing and neural computations to be rapidly modified based on the different situational demands. Whether our hypothesized shifts in extracellular ion concentrations do occur in the awake brain during active behavior and states of elevated arousal, as well as whether this ionic shift is primary or secondary to neural activity, will be interesting and important to resolve in future experiments.

We consider the frequency and length of the silent downstate periods a key signature defining the different dynamics in the two awake states (Figure 6I). It can be argued that in the active awake state,  $V_m$  is sustained depolarized and only high-frequency spikes occur, and one might entertain the speculative hypothesis that this is required for rapid responses to incoming barrages of sensory stimuli and for decision making. In this case, one would ideally avoid the hyperpolarized silent downstates and in general slow oscillations in the  $V_m$ . From a dynamical systems point of view, this means that the trajectory should not visit a large part of the phase space. Since this is the case for a chaotic system, the fact that the chaotic dynamics disappears

in the active awake state can be regarded as a beneficial mechanism that ensures the occurrence of only high-frequency spikes and maintains  $V_m$  close to the firing threshold, keeping the neuron in a state for optimal responsiveness to sensory inputs from the periphery. We note that from a theoretical point of view, it is possible to imagine a stable limit cycle that can guarantee a smooth transition. In this picture, the temporal period between downstates gets shorter and shorter as the parameters change in the quiet awake state, for it to completely disappear in the end in the hyper-awake state. However, we believe there are several intriguing elements in a chaotic transition to the awake state in this neuronal state transition. Firstly, since not only one, but several, parameters are changed in the sleep to awake state transition, chaotic dynamics is a clever way for the system to make a path-independent transition between the two states. Due to the never-repeating trajectories, the system does not change significantly if, for example, the changes in  $[Ca^{2+}]_o$  occur before  $[K^+]_o$  or vice versa. This would normally not be the case for a stable limit cycle, and thus the chaotic transition makes the transition to the awake state change more robust. Secondly, we believe that chaotic dynamics is an important property of neurons in the awake state since one would expect that neurons in this state need the capacity to create a variety of different combinatorial outputs on a population level. If the neuronal dynamics in the awake state were purely stable limit cycles differing in period, small differences in incoming inputs would lead to very small differences in output firing, and thus information encoding. However, since they are governed by chaotic dynamics, the sensitivity to the always existing small perturbations that accompany external stimulation, can lead to a much more complex variety of outcomes that are likely highly important and necessary for higher-order neuronal computations. Thirdly, it is intriguing to imagine that since all neurons are governed by the dynamics of a stable limit cycle in the sleep state, they have the potential to synchronize their firing outputs through their common oscillations. Now, perturbing the system could change the period of oscillations but would not necessarily destroy this activity synchrony between neurons. However, as the awake state is chaotic, this population synchrony would naturally disappear. In this way, the awake state would make each neuron more independent, since it is no longer “bound” by the dynamics of the state, and the specific feature-selective inputs to each neuron can have a great effect on the firing output for that particular neuron in the end.

This work uncovers an important function of extracellular ion concentrations and chaos dynamics in neuronal state transitions that, to the best of our knowledge, has not been previously reported. The transition between global brain states needs to be strictly controlled and well regulated to avoid risky diseases such as narcolepsy or insomnia. As we show, the concentration of extracellular ions is a key parameter for neuronal state changes, and these state transitions are governed by chaotic dynamics. Concerted regulation and shifts in ion concentrations therefore expand the toolbox available to the brain for controlling state-dependent activity, and thus need to be considered as an integrated mechanism in future investigations of what determines ongoing activity and state transitions in the brain.

## STAR★METHODS

Detailed methods are provided in the online version of this paper and include the following:

- KEY RESOURCES TABLE
- CONTACT FOR REAGENT AND RESOURCE SHARING
- EXPERIMENTAL MODEL AND SUBJECT DETAILS
  - Description of the Extended Averaged-Neuron Model
- METHOD DETAILS
  - Membrane Potential ( $V_m$ ) Analysis
- DATA AND SOFTWARE AVAILABILITY
  - Software
  - Algorithms
- QUANTIFICATION AND STATISTICAL ANALYSIS
- ADDITIONAL RESOURCES

## SUPPLEMENTAL INFORMATION

Supplemental Information includes five figures and can be found with this article online at <https://doi.org/10.1016/j.cels.2017.11.011>.

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## AUTHOR CONTRIBUTIONS

Conceptualization, R.R. and M.L.H.; Methodology, R.R., M.H.J., and M.L.H.; Investigation, R.R., M.H.J., and M.L.H.; Writing, R.R. and M.L.H.; Visualization, R.R. and M.L.H.

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## STAR★METHODS

### KEY RESOURCES TABLE

REAGENT or RESOURCE	SOURCE	IDENTIFIER
Software and Algorithms		
Simulations made in c++	This Paper	
MATLAB 6.1	The MathWorks Inc. 2010	<a href="https://se.mathworks.com/products/matlab/">https://se.mathworks.com/products/matlab/</a>
Chronux 2.0 Toolbox	Chronux	<a href="https://chronux.org/">https://chronux.org/</a>
fcn, d02cjk and d02cjl library packages in Fortran	The NAG Library, The Numerical Algorithms Group (NAG), Oxford, United Kingdom	<a href="http://www.nag.com">www.nag.com</a>

### CONTACT FOR REAGENT AND RESOURCE SHARING

Further information and requests for resources should be directed to and will be fulfilled by the Lead Contact, Mathias L. Heltberg ([heltberg@nbi.ku.dk](mailto:heltberg@nbi.ku.dk)).

### EXPERIMENTAL MODEL AND SUBJECT DETAILS

#### Description of the Extended Averaged-Neuron Model

We consider the membrane potential ( $V_m$ ) given by:

$$\frac{dV}{dt} = -\frac{1}{C} \left( \sum I_{Ext} \right) - \frac{1}{AC} \left( \left( \sum I_{Int} \right) \right)$$

intrinsic  $\in [Leak, Na_V, K_V, K_{A-type}, K_{Sl}, Ca_V, K_{Ca}, NaP, K_{IR}]$

extrinsic  $\in [NMDA, AMPA, GABA_A]$

The units on the left side is mV/ms, but the extrinsic currents are in units of  $\mu A$  whereas the intrinsic currents are in nA, and therefore the first should be multiplied by 1000. This reduces to 10, since we multiply with A, that should be in  $cm^2$  and should thus be divided by 100.

#### Intrinsic Channel Conductances

For the leak channel we have:

$$I_{Leak} = g_{Leak} (V - V_{Leak})$$

$$g_{Leak} = 0.03573 \text{ mS/cm}^2 \quad V_{Leak} = \frac{RT}{zF} \ln \left( \frac{\rho K [K]_o + pNa [Na]_o + pCl [Cl]_i}{\rho K [K]_i + pNa [Na]_i + pCl [Cl]_o} \right) \quad (\text{Equation 1})$$

$$R = 8.314472 \text{ J/K/mol} \quad T = 310 \text{ kelvins} \quad z = \text{valence}_{ion} \quad F = 9.64853399 \times 10^4 \text{ C/mol}$$

For the voltage-gated sodium channel we have:

$$I_{Na_V} = g_{Na_V} m_{Na_V}^3 h_{Na_V} (V - V_{Na})$$

$$m_{Na_V} = \frac{\alpha_m}{\alpha_m + \beta_m} \quad \begin{cases} \alpha_m = 0.1 \frac{V + 33}{1 - e^{-(V + 33)/10}} \\ \beta_m = 4e^{-(V + 53.7)/12} \end{cases} \quad (\text{Equation 2})$$

$$\dot{h}_{Na_V} = 4(\alpha_h(1 - h_{Na_V}) - \beta_h h_{Na_V}) \quad \begin{cases} \alpha_h = 0.07e^{-(V + 50)/10} \\ \beta_h = \frac{1}{1 + e^{-(V + 20)/10}} \end{cases}$$

$$g_{Na_V} = 12.2438 \text{ mS/cm}^2 \quad V_{Na} = \frac{RT}{zF} \ln \left( \frac{[Na]_o}{[Na]_i} \right)$$

For the voltage-gated potassium channel we have:

$$\begin{aligned}
 I_{K_V} &= g_{K_V} n_K^4 (V - V_K) \\
 \dot{n}_K &= 4(\alpha_n(1 - h_{K_V}) - \beta_n h_{K_V}) \begin{cases} \alpha_n = 0.01 \frac{V + 34}{1 - e^{-(V+34)/10}} \\ \beta_n = 0.125e^{-(V+44)/25} \end{cases} \\
 g_{K_V} &= 2.61868 \text{ mS/cm}^2 \quad V_K = \frac{RT}{zF} \ln\left(\frac{[K]_o}{[K]_i}\right)
 \end{aligned} \tag{Equation 3}$$

For the fast A-type potassium channel we have:

$$\begin{aligned}
 I_{A\text{-type}} &= g_{A\text{-type}} m_{A\text{-type}}^3 h_{A\text{-type}} (V - V_K) \\
 m_{A\text{-type}} &= \frac{1}{1 + e^{-(V+50)/20}} \\
 \dot{h}_{A\text{-type}} &= \frac{h_{A\text{-type}\infty} - h_{A\text{-type}}}{\tau_{hA\text{-type}}} \\
 h_{A\text{-type}\infty} &= \frac{1}{1 + e^{(V+80)/6}} \\
 g_{A\text{-type}} &= 1.79259 \text{ mS/cm}^2 \quad \tau_{hA\text{-type}} = 15\text{ms} \quad V_K = \frac{RT}{zF} \ln\left(\frac{[K]_o}{[K]_i}\right)
 \end{aligned} \tag{Equation 4}$$

For the slowly inactivating potassium channel we have:

$$\begin{aligned}
 I_{K_{Sl}} &= g_{K_{Sl}} m_{K_{Sl}} (V - V_K) \\
 m_{K_{Sl}} &= \frac{h_{m_{K_{Sl}\infty}} - m_{K_{Sl}}}{\tau_{m_{K_{Sl}}}} \\
 m_{K_{Sl}\infty} &= \frac{1}{1 + e^{-(V+34)/6.6}} \\
 \tau_{m_{K_{Sl}}} &= \frac{8}{e^{-(V+55)/30} + e^{(V+55)/30}} \\
 g_{K_{Sl}} &= 0.0350135 \text{ mS/cm}^2 \quad V_K = \frac{RT}{zF} \ln\left(\frac{[K]_o}{[K]_i}\right)
 \end{aligned} \tag{Equation 5}$$

For the voltage-gated calcium channel we have:

$$\begin{aligned}
 I_{Ca_V} &= g_{Ca_V} m_{Ca_V\infty}^2 (V - V_{Ca}) \\
 m_{Ca_V\infty} &= \frac{1}{1 + e^{(V+20)/9}} \\
 g_{Ca_V} &= 0.0256867 \text{ mS/cm}^2 \quad V_{Ca} = \frac{RT}{zF} \ln\left(\frac{[Ca]_o}{[Ca]_i}\right)
 \end{aligned} \tag{Equation 6}$$

For the calcium-dependent potassium channel we have:

$$\begin{aligned}
 I_{K_{Ca}} &= g_{K_{Ca}} m_{K_{Ca}\infty} (V - V_K) \\
 m_{K_{Ca}\infty} &= \frac{1}{1 + \frac{K_D^{3.5}}{[Ca]_i}} \\
 \dot{[Ca]_i} &= -\alpha_{Ca}(10 \cdot AI_{Ca} + I_{NMDA}) - \frac{[Ca]_i}{\tau_{Ca}} \\
 g_{K_{Ca}} &= 2.34906 \text{ mS/cm}^2 \quad K_D = 30\mu\text{M} \quad \tau_{Ca} = 121.403\text{ms} \quad V_K = \frac{RT}{zF} \ln\left(\frac{[K]_o}{[K]_i}\right)
 \end{aligned} \tag{Equation 7}$$



For the persistent sodium channel we have:

$$\begin{aligned}
 I_{NaP} &= g_{NaP} m_{NaP\infty} (V - V_{Na}) \\
 m_{NaP\infty} &= \frac{1}{1 + e^{-(V+55.7)/7.7}} \\
 g_{NaP} &= 0.0717984 \text{ mS/cm}^2 \quad V_{Na} = \frac{RT}{zF} \ln \left( \frac{[Na]_o}{[Na]_i} \right)
 \end{aligned}
 \tag{Equation 8}$$

For the inwardly rectifying potassium channel we have:

$$\begin{aligned}
 I_{K_{IR}} &= g_{K_{IR}} h_{K_{IR}\infty} (V - V_K) \\
 h_{K_{IR}\infty} &= \frac{1}{1 + e^{(V+75)/4}} \\
 g_{K_{IR}} &= 0.0166454 \text{ mS/cm}^2 \quad V_K = \frac{RT}{zF} \ln \left( \frac{[K]_o}{[K]_i} \right)
 \end{aligned}
 \tag{Equation 9}$$

### Extrinsic Channel Conductances

We start by defining the saturating function

$$f(V) = \frac{1}{1 + e^{-(V-20)/2}}$$

We now consider the AMPA receptor:

$$\begin{aligned}
 I_{AMPA} &= g_{AMPA} s_{AMPA} (V - V_{AMPA}) \\
 s_{AMPA} &= 3.48f(V) - \frac{s_{AMPA}}{\tau_{AMPA}} \\
 g_{AMPA} &= 0.513425 \text{ } \mu\text{S/cm}^2 \quad V_{AMPA} = \frac{RT}{zF} \ln \left( \frac{p_K [K]_o + p_{Na} [Na]_o}{p_K [K]_i + p_{Na} [Na]_i} \right)
 \end{aligned}
 \tag{Equation 10}$$

For the NMDA receptor we have:

$$\begin{aligned}
 I_{NMDA} &= \frac{1.1}{1.0 + [Mg]_o / 8.0 \text{ mM}} g_{NMDA} s_{NMDA} (V - V_{NMDA}) \\
 s_{NMDA} &= 0.5 x_{NMDA} (1 - s_{NMDA}) - \frac{s_{NMDA}}{\tau_{sNMDA}} \\
 x_{NMDA} &= 3.48f(V) - \frac{x_{NMDA}}{\tau_{xNMDA}} \\
 g_{NMDA} &= 0.00434132 \text{ } \mu\text{S/cm}^2 \quad V_{NMDA} = \frac{RT}{zF} \ln \left( \frac{p_K [K]_o + p_{Na} [Na]_o + p_{Ca} [Ca]_o}{p_K [K]_i + p_{Na} [Na]_i + p_{Ca} [Ca]_i} \right)
 \end{aligned}
 \tag{Equation 11}$$

For the GABA<sub>A</sub> receptor we have:

$$\begin{aligned}
 I_{GABA_A} &= g_{GABA_A} s_{GABA_A} (V - V_{GABA}) \\
 s_{GABA_A} &= f(V) - \frac{s_{GABA_A}}{\tau_{sGABA_A}} \\
 g_{GABA_A} &= 0.00252916 \text{ } \mu\text{S/cm}^2 \quad V_{GABA} = \frac{RT}{zF} \ln \left( \frac{[Cl]_i}{[Cl]_o} \right)
 \end{aligned}
 \tag{Equation 12}$$

### Ion Concentrations

We used the following intra- and extracellular ion concentrations:

$$\begin{aligned}
 [Na]_o &= 140 \text{ mM} \quad [Na]_i = 7 \text{ mM} \\
 [K]_o &= [3.5; 3.9; 4.4; 4.9; 8; 14] \text{ mM} \quad [K]_i = 7 \text{ mM} \\
 [Ca]_o &= [1.05; 1.2; 1.35] \text{ mM} \quad [Ca]_i = -\alpha_{Ca} (10 \cdot A_{Ca} + I_{NMDA}) - \frac{[Ca]_i}{\tau_{Ca}} \text{ } \mu\text{M} \\
 [Cl]_o &= 140 \text{ mM} \quad [Cl]_i = 10 \text{ mM} \\
 [Mg]_o &= [0.6; 0.7; 0.8] \text{ mM}
 \end{aligned}$$

## METHOD DETAILS

### Membrane Potential ( $V_m$ ) Analysis

$V_m$  frequency component analysis was performed using procedures in MATLAB similar to previously described (Rasmussen et al., 2016). For exploring frequency-domain dynamics we used the *mtspectrumc* function, a multi-taper method implemented in the Chronux 2.0 toolbox, an open-source, MATLAB-based toolbox available at <http://chronux.org/> (Mitra and Bokel, 2008). For this analysis we used a padding factor of 2, time-bandwidth product of 3 and 5 tapers. For determining power content we used the *bandpower* function and afterwards the *pow2db* functions in MATLAB. All power measurements are reported in units of decibel unless otherwise stated. For removing spikes from the simulated  $V_m$ , yielding the mean  $V_m$ , we median filtered the raw  $V_m$  (window size: 80 ms).  $V_m$  standard deviation was determined over a 200 ms moving window.

## DATA AND SOFTWARE AVAILABILITY

### Software

All simulations were performed using scripts written in Fortran, C++ and MATLAB. All data-analysis were performed from scripts written in MATLAB and python using the ROOT software. Figures were composed in Adobe Illustrator. All scripts used for simulation and data analysis from the model, will be available upon reasonable request to Mathias L. Heltberg ([heltberg@nbi.ku.dk](mailto:heltberg@nbi.ku.dk)).

### Algorithms

All deterministic simulations were performed using the *fcn*, *d02cix* and *d02cixw* library packages in Fortran and tested by a similar script written in c++. All stochastic simulations were performed by scripts written in c++, and using the Mersenne Twister to draw random numbers.

## QUANTIFICATION AND STATISTICAL ANALYSIS

Statistical testing was carried out in MATLAB. All group comparisons were performed using two-sided parametric paired Student's t test. For comparisons of multiple groups the parametric repeated measures One-way ANOVA with a Geisser-Greenhouse correction and a Dunnett's multiple comparisons correction was used. Lower-case 'n' refers to the number of simulations and is noted in the manuscript text, figure legends and on figure panels whenever appropriate. Statistical significance was considered with P-values less than 0.05. When a statistical test was used, the precise P-value is noted in the manuscript text, and depicted in figures with asterisks: \*  $P < 0.05$ , \*\*  $P < 0.01$ , \*\*\*  $P < 0.001$ .

## ADDITIONAL RESOURCES

The custom code for the extended Averaged-Neuron model is publicly available online at <https://github.com/Neurone/IonsAndChaos>.