Spontaneous breaking of chiral symmetry

Scales in strong interactions

'Strong interactions' are about everything from π's to ^{238}U. Masses are measured in MeV:

$$m_e = 0.511\,\mathrm{MeV}, \quad m_\pi = 140\,\mathrm{MeV}, \quad m_N = 940\,\mathrm{MeV}$$

Distances and sizes are measured in $\mathrm{fm} = 10^{-15}\,\mathrm{m}$: $r_\pi \approx r_N = 1\,\mathrm{fm}$.

Quantum Chromodynamics (QCD)

The microscopic theory of strong interactions is encoded in a one-line QCD Lagrangian [Fritzsch, Gell-Mann and Leutwyler (1972)]:

$$\mathcal{L} = \sum_{f=1}^{6} \bar{q}_f \left(i\gamma_\mu \nabla_\mu - m_f\right) q_f - \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu},$$

$$\nabla_\mu = \frac{\partial}{\partial x_\mu} - iA^a_\mu t^a \qquad a = 1, 2 \ldots 8,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu + f^{abc} A^b_\mu A^c_\nu.$$

q_f are quark fields, $u, d, s, \ldots$, A^a_μ is the gluon field, $F^a_{\mu\nu}$ is the gluon field strength.

$m_u \approx 4\,\mathrm{MeV}, \quad m_d \approx 7\,\mathrm{MeV}, \quad m_s \approx 150\,\mathrm{MeV}...$

$$\left.\begin{array}{ll} \text{proton} & p \approx uud \\ \text{neutron} & n \approx udd \end{array}\right\} m_N = 940\,\mathrm{MeV}$$

How come the nucleon is 100 times heavier its constituents?

‘Transmutation of dimensions’

$\alpha_s = \frac{g^2}{4\pi}$ is the analog of $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$ but it ‘runs’:

$$\frac{2\pi}{\alpha_s(\mu)} = b_1 \ \ln\frac{\mu}{\Lambda} + \frac{b_2}{2b_1} \ln\ln\frac{\mu^2}{\Lambda^2} + O\left(\frac{1}{\ln\frac{\mu}{\Lambda}}\right).$$

$$b_1 = \frac{11}{3}N_c - \frac{2}{3}N_f, \qquad b_2 = \frac{34}{3}N_c^2 - \frac{13}{3}N_cN_f + \frac{N_f}{N_c}.$$

All dimensionfull observables come out as combinations of the UV cutoff μ and the coupling constant given at that cutoff, $g^2(\mu)$.

$$\Lambda = \mu \ \exp\left(-\frac{2\pi}{b_1\alpha_s(\mu)}\right)\left(\frac{4\pi}{b\alpha_s(\mu)}\right)^{\frac{b_2}{2b_1^2}} (1 + O(\alpha_s(\mu))) \, .$$

For example, $\mu = \frac{1}{a}$ lattice cutoff.

Chiral limit

– is an idealization: $m_u = m_d = m_s = 0$. Only 5% of the nucleon mass is due to $m \neq 0$. In this (idealized) world interaction of quarks with gluons conserves quark helicity or chirality: left-polarized u, d, s quarks remain left-polarized forever. In addition, u, d, s quarks are interchangeable.

Mathematically, the QCD lagrangian is invariant under $U(3)_L \times U(3)_R$ separate rotations of left- and right-polarized quarks,

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \rightarrow \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & A & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L ,$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_R \rightarrow \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & B & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R$$

A and B are 3×3 unitary matrices, 9+9=18 parameters!

The invariance under chiral rotations is called the chiral symmetry of strong interactions.

Since $L \leftrightarrow R$ in the mirror image,

CHIRAL SYMMETRY OF QCD means ALL STATES WITH OPPOSITE PARITY HAVE EQUAL MASSES

In reality

$$\begin{array}{llll} \rho & \text{meson} & (J^P = 1^-) & m = 770\,\text{MeV} \\ a_1 & \text{meson} & (J^P = 1^+) & m = 1250\,\text{MeV} \\ & & & \text{difference} \approx \underline{500\,\text{MeV}} \end{array}$$

For the nucleons the splitting is even larger,

$$\begin{array}{lll} N & (\frac{1}{2}^+) & m = 940\,\text{MeV} \\ N^* & (\frac{1}{2}^-) & m = 1535\,\text{MeV} \\ & & \text{difference} \approx \underline{600\,\text{MeV}} \end{array}$$

The difference is too large to be explained by nonzero current quark masses ($m_u = 4\,\mathrm{MeV},\ m_d = 7\,\mathrm{MeV}$).
$\Longrightarrow$ chiral symmetry is **spontaneously** broken

$\Longrightarrow$ pions are light [=(pseudo)-Goldstone bosons]

$\Longrightarrow$ nucleons are heavy.

$\Longrightarrow$ nuclei exist

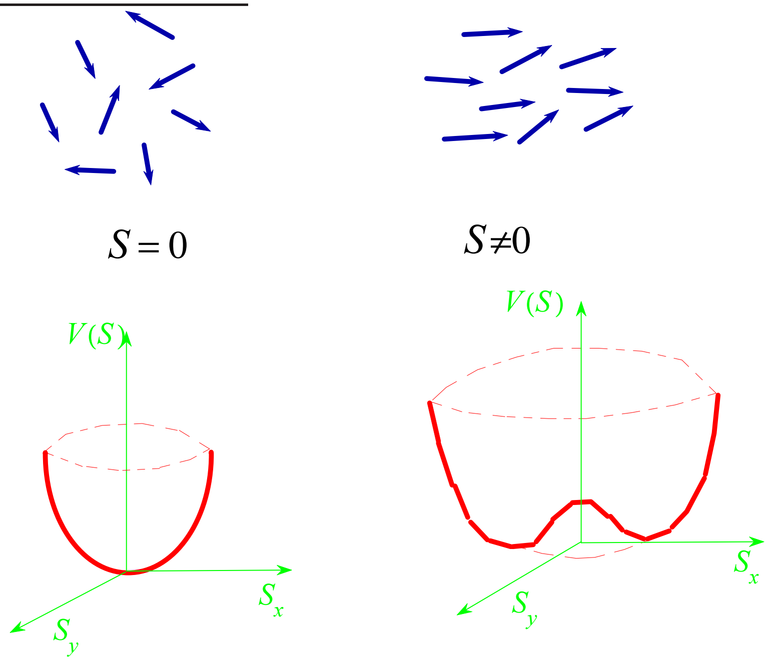
$\Longrightarrow$ ... and we exist.

Were chiral symmetry unbroken, it would be the other way: pions would be heavy while nucleons would be light.

95% of the (visible) mass around us is due
to the spontaneous chiral symmetry breaking.
It is a basic event in QCD.

Spontaneous breaking of any continuous symmetry is marked by a nonzero order parameter, in this case called quark condensate $<\bar{u}u>=<\bar{d}d>=<\bar{s}s>\neq 0$.

Analogy: Spontaneous magnetization



Order parameter $<S_x>\neq 0$.

Symmetry with respect to rotation,

$$\begin{array}{rcl} S_x & \longrightarrow & S_x \cos\phi + S_y \sin\phi, \\ S_y & \longrightarrow & -S_x \sin\phi + S_y \cos\phi \end{array}$$

is broken! As a result, there is one Goldstone or massless or gapless excitation in the system. The energy is $\mathcal{E} = \frac{1}{2}(\nabla\phi)^2$.

If magnetic field $\mathbf{B}$ is added, rotational symmetry is broken from the start. The "Mexican hat" is tilted, and the energy gets a term

$$\begin{array}{rcl} \mathcal{E}_{\rm pot} & = & -\mu(\mathbf{B}\cdot\mathbf{S}) = -\mu|B| < S > \cos\phi \\ & = & \mu|B| < S > (-1 + \frac{1}{2}\phi^2) \end{array}$$

such that the Goldstone excitation gets a mass

$$m_G^2 = \mu|B| < S >$$

Notice that m_G is **square root** of $|B|$.

With chiral symmetry, out of 18 symmetries 9 are spontaneously broken. It means that there must be nine (pseudo) massless Goldstone bosons. These are the pseudoscalar mesons:

$$\pi^+ \sim \bar{d}u, \quad \pi^- \sim \bar{u}d, \quad \pi^0 \sim \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$$

$$K^+ \sim \bar{s}u, \quad K^0 \sim \bar{s}d, \quad K^-, \quad \bar{K}^0$$

$$\eta \sim \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}}$$

$$\eta' \sim \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{3}}$$

Recall nonzero $m_{u,d,s}$ – analogs of magnetic field $\overrightarrow{B}$ which breaks symmetry explicitly. It lifts the degeneracy of the bottom of the valley, and makes Goldstone bosons massive, although still light (Gell-Mann–Oakes–Renner formula):

$$m_\pi^2 = \frac{1}{F^2}(m_u + m_d) < \bar{q}q >= (140\,\mathrm{MeV})^2$$

$$m_K^2 = \frac{1}{F^2}(m_s + m_{u(d)}) < \bar{q}q >= (495\,\mathrm{MeV})^2$$

$$\Longrightarrow \quad \frac{m_s}{m_u + m_d} \approx 12.5$$

$$\Longrightarrow \quad m_s \simeq 140\,\mathrm{MeV}, \quad m_d \simeq 7\,\mathrm{MeV}, \quad m_u \simeq 4\,\mathrm{MeV}$$

A check:

$$\eta \sim \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}} \Rightarrow$$

$$m_\eta^2 \simeq \frac{4}{3}\frac{1}{F^2} m_s < \bar{q}q >\simeq \frac{4}{3} m_K^2$$

$$547\,\mathrm{MeV} \quad \mathrm{vs.} \quad 570\,\mathrm{MeV}, \qquad \text{works well!}$$

$$m_{\eta}'^{2} \quad \simeq \quad \frac{2}{3} m_K^2$$

$$958\,\mathrm{MeV} \quad \mathrm{vs.} \quad 404\,\mathrm{MeV}, \qquad \text{completely wrong!}$$

This paradox was called "the $U(1)$ problem" [Weinberg (1974)!]

The mechanism of chiral symmetry breaking must be such that η' is NOT A GOLDSTONE BOSON.