Inflation with a stringy minimal length (reworked)

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Acknowledgements

- Part of an ongoing collaboration with Gonzalo A. Palma.
- This work reported on in *arXiv* : 0810.5532 , to appear in JHEP.

Alternative title: 'Trans-Planckian' problem revisited

Inflation provides us with a complimentary microscope to particle accelerators. We know that the universe expanded *at least* $e^{60} \approx 10^{26}$ fold during inflation.

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- Could there be signatures of 'Trans-Planckian' physics in the CMB?
- Only an intuitive argument. Other ways of phrasing the question: why does our EFT treatment seem to work so well? Could there be a breakdown of EFT during inflation? Could observation be sensitive to higher dimensional operators? Can we infer any UV physics from the CMB?

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- Only an intuitive argument. Other ways of phrasing the question: why does our EFT treatment seem to work so well? Could there be a breakdown of EFT during inflation? Could observation be sensitive to higher dimensional operators? Can we infer any UV physics from the CMB?
- Scale of inflation could be as high as $\sim 10^{16}GeV$.

Precedent in Black Hole Physics

It seems that a similar 'Trans-Planckian' issue crops up when we compute the spectrum of a radiating Black Hole– photons which began (tunnelled out) arbitrarily close to the horizon are infinitely red shifted on their way to the asymptotic observer. Photon propagation is extrapolated to arbitrarily high frequencies.

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- Brandenberger and Martin: If we allow for non-standard dispersion relations (c.f. rotons in liquid helium), then there is a possibility of a measurable effect (although unlikely to be realized in UV complete theory).

Measurable initial state effects?

Were we to take k_c to be M_{pl} in the above, we will generically find $O(H^2/M_{pl}^2)$ corrections to the correlation functions we compute and measure in cosmology.

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- Instead of considering ad-hoc modifications in the UV, what about string theory? $M \to m_s$.

A minimal length in string theory

In string theory, we are presented with a UV complete theory of quantum gravity with the string length as its only scale. At the level of the low energy degrees of freedom (particles), this implies the Generalized Uncertainty Principle (GUP):

- $\Delta x \Delta p \geq \frac{1}{2}[1 + \beta(\Delta p)^2 + ...]$.

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- Heuristically, since $P^2_{c.o.m.} \sim N$, we lose resolution the higher energies we probe, implying a minimal length scale we can resolve if we only have strings as our probes of geometry.

Representing a stringy minimal length

We will attempt to write down a field theory that represents the GUP and study its quantization. In doing so, we will analytically compute H^2/m_s^2 corrections to observables and comment on the likelihood of measuring these corrections. In what follows, we begin in the footsteps of Kempf and collaborators.

- Try $[X^i, P_j] = i\left(f(P^2)\delta^i_j + g(P^2)P^iP_j\right)$.

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- B.C.'s for this representation: $\phi(\rho = \beta^{-1/2}) = 0$.

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- $S = -\frac{1}{2}\int dt\,\left[(\phi, \partial_t^2\phi) + (\phi, \mathbf{P}^2\phi)\right]$, where (f, g) is the scalar product $\int d^3x\; f^*(x)g(x)$.

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- Expressing fields in terms of physical co-ordinates $y^i = ax^i$, i.e. $\phi(\tau, x) \to \phi(\tau, a(\tau)x^i)$, we obtain
  $S = -\frac{1}{2}\int d\tau d^3y\, a^{-2}(\tau)\left[(A\phi)^2 - a^2(\tau)\sum_{i=1}^3(\partial_{y^i}\phi)^2\right]$, where
  $A = \partial_\tau + i\frac{a'}{a}P_i y^i - 3\frac{a'}{a}, P_i = -i\partial_{y^i}$ represents the convective derivative.

Representing a stringy minimal length

In an FRW background, the replacement of the GUP representation of P_i in $A = \partial_\tau + i\frac{a'}{a}P_i y^i - 3\frac{a'}{a}$ results in the action:

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- This action mixes different ρ modes. The change of variable $k^i = a\rho^i e^{-\beta\rho^2/2}$ decouples these modes, from which we derive the equation of motion for our mode functions:

  $\phi_k'' + \frac{\nu'}{\nu}\phi_k' + \left[\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - \frac{3a'\nu'}{a\nu}\right]\phi_k = 0$, with

  $\mu(\tau,k) := -\frac{a^2 W(-\beta k^2/a^2)}{\beta(1+W(-\beta k^2/a^2))^2} = \frac{a^2\rho^2}{(1-\beta\rho^2)^2}$ and

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- Extremely complicated to solve, numerical solutions at best. Not entirely unique prescription allows us to wonder if this is the only way to proceed.

Representing a stringy minimal length (reworked)

- Recalling that $[X^i, P_j] = i\left(f(P^2)\delta^i_j + g(P^2)P^i P_j\right)$ represents the GUP with $g = 2\beta, f(s) = 2\beta s/(\sqrt{1+\beta s} - 1)$.

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- Which yields the equations of motion for the mode functions
  $\phi'' - \frac{2}{\tau}\phi' + \frac{p^2}{(1-a^{-2}\beta p^2)^2}\phi = 0$.

Exact solutions

- We find the exact solutions for the mode functions to be
$$\phi_\pm(\tau,p) = \frac{H^2}{\sqrt{2p^3}}\frac{\sqrt{1-\beta H^2p^2\tau^2}}{\sqrt{\gamma(1+3\beta H^2)}}[1+p\tau\left(\beta H^2p\tau \mp i\gamma\right)]e^{\pm i\frac{\gamma}{\sqrt{\beta}H}\tanh^{-1}\left(\sqrt{\beta}Hp\tau\right)}$$
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- Can compute the curvature perturbations as $\langle\zeta(p)\zeta(p')\rangle = (2\pi)^3\delta^3(p+p')\frac{H_*^4}{2p^3\dot\phi_*^2}\frac{1}{\gamma_*(1+3\beta H_*^2)}$, N.B. $H^2\beta = H^2/m_s^2$ corrections.

Comparison with observation

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- If $m_s \approx 10^{16} GeV = M_{GUT}$, then we will forever be one order of magnitude away from experimental sensitivity.

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- If we find any significant 'outlier' points for the primordial spectrum, then we could perhaps deduce non-trivial effects on inflaton dynamics from UV physics.
- Effective field theory analysis of inflaton dynamics is a current and active field of research. Stay tuned for more progress and interplay with results of observations.