More line operators in $\mathcal{N} = 6$ Chern-Simons theory

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Introduction

This talk is about disorder vortex operators in $\mathcal{N} = 6$ super-Chern-Simons in three dimensions.

A vortex* is a co-dimension two singularity, so in 3d they are one-dimensional - loop operators, with some similarities to the Wilson loops presented earlier.

In 4d such a vortex is a two-dimensional object - surface operator. These were studied by Gokov and Witten in the context of the geometric Langlands program, where they wanted to classify them and understand the action of S-duality. Some physical observables were calculated last year [N.D. Gomis, Matsuura].

In this talk I present the objects in 3d, which share many properties with the surface operators in 4d and mention the calculations we have made with them.

Gauge theories have many operators. Usually one considers gauge-invariant local operators like

$$\mathrm{Tr}\, F^2\,, \qquad \mathrm{Tr}\, \Phi^2\,, \qquad \mathrm{Tr}\, \bar{\Psi}\Psi\,, \qquad \cdots$$

One can also consider "line operators" the most famous of which is the Wilson loop

$$\frac{1}{N}\mathrm{Tr}\,\mathcal{P}\exp i\int_C A_\mu dx^\mu\,.$$

Other variants will include extra couplings to other fields (they are required to make the operator BPS).

Another famous line operator in 4d is the 't Hooft loop, which is dual to the Wilson loop under strong-weak duality, but it cannot be written in a simple way in terms of the basic fields as an insertion into the path integral.

The 't Hooft loop is therefore a disorder operator and so are the vortex operators I describe here.

The ’t Hooft loop is co-dimension 3, so in 3d it is a local operator, important in the ABJM theory when constructing chiral primaries dual to D0-branes in string theory.

The vortices we discuss here are co-dimension 2, so are line operators in 3d (surface operators in 4d). One needs to consider the path integral with special prescribed singularity along a codimension-two manifold.

While the Wilson loop can be thought of as the effect induced by a probe particle, the surface operators are the result of inserting a probe anyon in $\mathcal{N} = 6$ SCS.

I will try to convince you that these objects are both beautiful and useful.

We hope that like Wilson and ‘t Hooft loops, they can also be used to characterize some subtle phases of such theories.

Outline

- Introduction
- Descriptions:
  - Semiclassical gauge theory description
  - Brane probe
  - “Bubbling geometries”
- Calculations:
  - Vacuum expectation value
  - Correlation function with local operators
- Summary

Vortex loops in $\mathcal{N}=6$ Chern Simons

The vortex is a line operator. The $1/2$ BPS ones have the geometry of a line or a circle.

The theory has $U(N)\times U(N)$ gauge symmetry that near the core of the vortex gets broken to $L=U(N_0)\times U(N_0)\times U(N_1)\times\cdots\times U(N_M)$.

The $1/2$ BPS vortex has a singularity for one scalar field

$$C^1=\frac{1}{\sqrt{z}}\begin{pmatrix}0\otimes 1_{N_0} & 0 & \cdots & 0\\ 0 & \beta^{(1)}\otimes 1_{N_1} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \beta^{(M)}\otimes 1_{N_M}\end{pmatrix}$$

where $z=x+iy$ is a complex coordinate in the transverse plane.

The diagonal gauge field has a vortex

$$A_z^+ = -\frac{i}{2kz}\begin{pmatrix} 0\otimes 1_{N_0} & 0 & \cdots & 0 \\ 0 & \alpha^{(1)}\otimes 1_{N_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha^{(M)}\otimes 1_{N_M} \end{pmatrix}$$

This solves the BPS equations

$$D_{\bar z} C^1 = 0 \qquad D_t C^1 = 0$$

And the equations of motion, with the additional gauge potential

$$A_t^+ = -4\pi C^1\, C_1^\dagger$$

The preserved supercharges are given by

$$\left\{\epsilon_{12}^+\,,\,\epsilon_{13}^+\,,\,\epsilon_{14}^+\,,\,\epsilon_{23}^-\,,\,\epsilon_{24}^-\,,\,\epsilon_{34}^-\right\}$$

Odd k

One subtlety is due to the square root, so in the presence of the vortex the field C is not single valued.

Gauge invariant local operator can be of the form $\mathrm{Tr}\, C\bar{C}\cdots\bar{C}$, which are single valued also in the presence of a vortex.

The other type of local operators, dual to D0-branes include C^k and an 't Hooft vertex. For odd k they are not well defined.

In this case, it is still possible to construct vortices, but one has to take all N_m positive $m = 1, \cdots M$.

Each block in the matrix of C^1 will be split in two with the two possible roots $\beta^{(m)}$ and $-\beta^{(m)}$. The connection A_z will include a "twist" that exchanges the two eigenvalues as one goes around the vortex (by multiplying with a σ_1 Pauli matrix).

Then also the expectation value of C^k is single valued.

1/3 BPS vortex

Another interesting thing to do is to turn on an *anti*-holomorphic vortex for a second scalar

$$C^2 = \frac{1}{\sqrt{\bar{z}}}\begin{pmatrix} 0 \otimes 1_{N_0} & 0 & \cdots & 0 \\ 0 & \beta_2^{(1)} \otimes 1_{N_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_2^{(M)} \otimes 1_{N_M} \end{pmatrix}$$

This guy will preserve

$$\left\{\epsilon_{12}^-\,,\epsilon_{23}^-\,,\epsilon_{24}^-\,,\epsilon_{13}^+\,,\epsilon_{14}^+\,,\epsilon_{34}^+\right\}$$

Comparing to those of C^1: $\left\{\epsilon_{12}^+\,,\epsilon_{13}^+\,,\epsilon_{14}^+\,,\epsilon_{23}^-\,,\epsilon_{24}^-\,,\epsilon_{34}^-\right\}$

They share four Poincaré supercharges, so together are 1/3 BPS.

They also preserve the same number of conformal superchanges.

Enhanced SUSY

For $k = 1, 2$ the theory is expected to have $\mathcal{N} = 8$ supersymmetry [Bagger, Lambert, Gustavsson].

$U(4)$ R-symmetry is enlarged to $SO(8)$.

$\bar{C}_2$ has a holomorphic vortex and with the enlarged $SO(8)$, it can be rotated into C^1.

So the $1/3$ BPS configuration is enlarged to $1/2$.

The enhancement of the $1/2$ BPS vortex is $24/2 = 12 \to 16 = 32/2$.

The enhancement of the $1/3$ BPS vortex is $24/3 = 8 \to 16 = 32/2$.

For $k = 1, 2$ both are $1/2$ BPS with either four or eight extra superchages getting broken for larger k.

Symmetry

Apart for the supersymmetry and superconformal symmetry the $1/2$ BPS vortex preserves also an $SL(2,\mathbb{R}) \times U(1) \times SU(3)$.

$SL(2,\mathbb{R})$ is the conformal symmetry of the line or circle.

$SU(3)$ is the unbroken R-symmetry ($SU(2)$ in the $1/3$ BPS case).

$U(1)$ is a diagonal subgroup of rotation + R-symmetry.

A nice way to write the vortices is to make a conformal transformation of $\mathbb{R}^3$ and take the gauge theory on $AdS_2 \times S^1$:

The scalar field depends only on the S^1 direction $C^1 \sim e^{i\psi/2}$.

The $SL(2,\mathbb{R})$ symmetry is the AdS_2 isometry.

M2-brane probe

The gravity dual of the ABJM theory is M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ or IIA string theory on $AdS_4 \times \mathbb{CP}^3$.

The duals of the vortices are M2-branes. In all cases they wrap an $AdS_2 \times S^1 \subset AdS_4 \times S^1$

$$ds^2 = du^2 + \cosh^2 u(d\rho^2 + \sinh^2 u\, d\varphi^2) + \sinh^2 u\, d\psi^2 + d\phi^2$$

given by $u = \text{const}$ and $\phi = \psi + \phi_0$.

Some features we want to reproduce:

- Symmetry and supersymmetry.
- The distinction between the $1/2$ and $1/3$ BPS vortices and the symmetry enhancement for $k = 1, 2$.
- The subtleties that arise at odd k.

For $k = 1, 2$ this solution preserves 16 supercharges.

For $k > 2$ there are two cases:

- If the ϕ circle is aligned with the orbifolded circle, the solution preserves 12 supercharges and is $1/2$ BPS.
- If the ϕ circle is not aligned with the orbifolded circle, the solution preserves 8 supercharges and is $1/3$ BPS.

The two circles, ϕ and the orbifold direction, fit inside an $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$.

In the string theory picture the M2 is a D2 still with geometry $AdS_2 \times S^1$.

The $S^3/\mathbb{Z}_k$ becomes an S^2 at large k and the circle - a latitude. When the two circles coincide the latituded degenerates to a point.

By wrapping more branes on different circles one gets more general $1/3$ and $1/6$ BPS vortices.

Odd k

The solution has $\phi = \psi$, and the ϕ circle has period $4\pi/k$.

For even k we can wrap the ψ circle an arbitrary number of times, N_1 corresponding to a vortex with a block of size N_1.

For odd k, we need to take N_1 to be even, otherwise the solution is not closed.

This is the realization in M-theory of the non-single-valudness of the solution for odd k in the gauge theory.

This can be solved if all the blocks are of even size, or all the branes wrapped an even number of times.

Bubbling geometry

The probe description is appropriate for a small number of branes.

With N branes, they back-react leading to “bubbling geometries”.

In our case there exist a class of such solutions found by Lunin.

They are the Wick-rotation of giant gravitons in $AdS_4 \times S^7$.

One needs to further orbifold his geometries.

I will not discuss this any further.

Calculations

Vacuum expectation value

The first question we might want to ask is what is the VEV of such an object.

For the line we expect unity, which is indeed the answer.

For the circle it is simplest to calculate in $AdS_2 \times S^1$ where the relevant part of the action is

$$\mathcal{L} = k \, \mathrm{Tr} \left(D_\mu C_I^\dagger D^\mu C^I + \frac{R^{(3)}}{8} C_I^\dagger C^I \right) ,$$

and the scalar curvature is $R^{(3)} = -2$

Since $C^1 \sim e^{i\psi/2i}$, the two terms exactly cancel each-other and we find

$$\langle V_C \rangle = 1$$

Same can be found also from an M-theory calculation.

Correlator with local operator

For surface operators in 4d we found very good agreement between correlators with local operators at weak and at strong coupling.

The string theory calculation reproduced correctly the weak coupling result and gives a finite series of quantum corrections to it.

So we tried to see if we can reproduce that agreement here too.

We calculated the correlation functions with some local operators. For example:

$$\mathcal{O}_{1,0} = \frac{2\pi}{\sqrt{3}\lambda} \mathrm{Tr}\left[C^I C_I^\dagger - 4C^1 C_1^\dagger\right],$$

To calculate the correlation function, just plug in the classical value

$$\frac{\langle \mathcal{O}_{1,0} \cdot V_C \rangle}{\langle V_C \rangle} = -\frac{\pi\sqrt{6}}{\lambda} \frac{|\beta|^2}{|z|}$$

The calculation in M-theory is more complicated.

One considers the emission of a supergravity field dual to $\mathcal{O}_{1,0}$ from the brane probe and its propagation to the boundary.

The answer is

$$\frac{\langle \mathcal{O}_{1,0} \cdot V_C \rangle}{\langle V_C \rangle} = \frac{1}{(2\pi^2\lambda)^{1/4}} \frac{3\sqrt{2}}{4} \frac{\cosh u}{|z|}$$

Unlike the surface operator, there is no agreement. ABJM theory is more complicated....

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works	doesn't work

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- Subtleties for odd k: All N_m even.
- $1/2$ and $1/3$ BPS become the same for $k=1,2$.
- Full map of parameters.

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doesn't work

- Calculated the correlator with local operators, but cannot find a reasonable interpolating function.
- Similar to the factor of $h(\lambda)$ in the dispersion relation.
- May help in understanding weak to strong coupling interpolation in ABJM.

The end