Blackfolds:

a new approach to higher-dimensional black holes

Nordic String Theory Meeting

Bremen, March 27, 2009

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0902.0427 (with R. Emparan, T. Harmark, V. Niarchos)

090y.xxxx:: To appear (with R. Emparan, T. Harmark, V. Niarchos)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

0802.0519 (Springer Lectures Notes)

0701022 review CQQ (with V. Niarchos and T. Harmark)

Plan

- Introduction
- Separation of scales in higher-dimensional black holes
- Blackfold approach
- Examples of novel black hole families
- Lessons and outlook

Motivations to study higher-dimensional gravity

- Applications:
  - String/M theory
    - BH entropy, new brane solutions
  - AdS/CFT
    - new phases of thermal gauge theories, phase transitions
    - plasma balls/rings in AdS (fluid/gravity correspondence)
  - Large extra dimensions + TeV gravity
    - possible objects in universe/accelators
  - math: Lorentzian geometry

- Intrinsically interesting:

Can regard D as tunable parameter for gravity + black holes

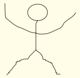
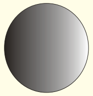
which BH properties are:

- intrinsic $\rightarrow$ Laws of BH mechanics
- D-dependent $\rightarrow$ uniqueness, topology, shape, stability

For various reviews see:

- Kol
- Harmark,Niarchos,NO
- Kleihaus,Kunz,Navarro-Larida
- Emparan,Reall
- NO

Progress in the last years

What do we know about black objects (i.e. with event horizon) in higher dimensional Einstein gravity ?

$\rightarrow$ Dynamics of BHs in D $\geq$ 5 much richer than four dimensions

In this talk: restrict (mostly) to asymptotically flat solutions of pure gravity

$$R_{\mu\nu} = 0 \qquad \mathcal{M}^D$$

but:- interesting parallels with BHs in KK spaces

- techniques are readily generalized to AdS/dS space + adding charge

- D=4: black hole uniqueness

- D=5: MP black hole (S^3), ER black ring ($S^2 \times S^1$), black Saturn, …
  - 4D inspired techniques successful

  (assuming 2 axial Killing vector fields $\longrightarrow$ integrability

  full classification of BHs in terms of "rod-structure" + asympt. charges)

- D $\geq$ 6: MP black holes (S^{D-2}) are only known exact solutions
  - full dynamics too complex to be captured by conventional approaches

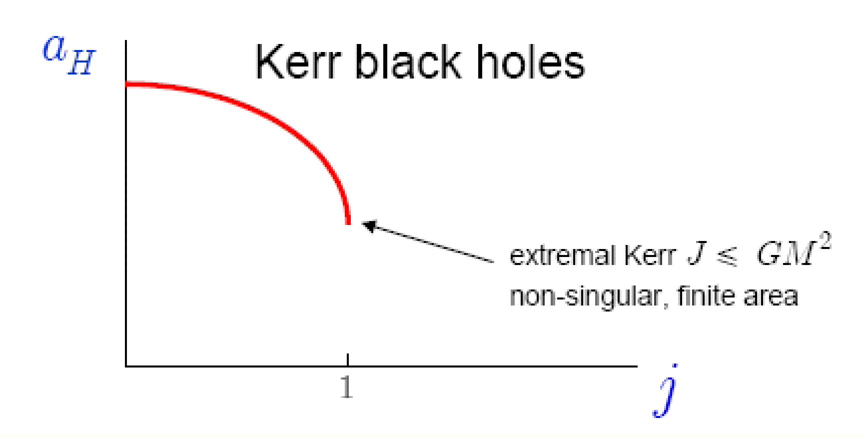
  $\Rightarrow$ but recent progress: thin black rings ($S^1 \times S^{D-3}$) in any dimension

Novel feature of higher D neutral BHs

► in some regimes horizons are characterized by (at least) two separate scales

$$r_0 \ll R$$

Cf. D=4



shape of Kerr BH is always approx. round with radius

$$r_0 \sim GM$$

D $\geq$ 5: no Kerr bound anymore

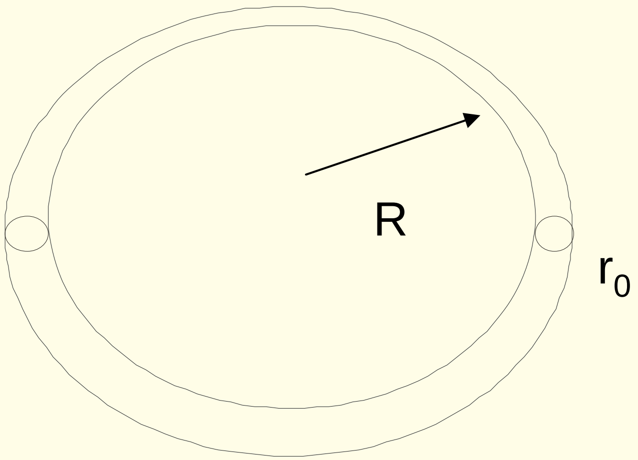
⇨ two classical length scales can be widely separated

$$\frac{J}{M} \quad \text{vs.} \quad (GM)^{1/(D-3)}$$

Analogue for KK black holes: size of compact manifold vs. horizon radius

Separation of scales in explicitly known solutoins

D=5



- Kerr bound for MP
but: rotating black ring can have arbitrarily large angular momentum for given mass Emparan,Reall

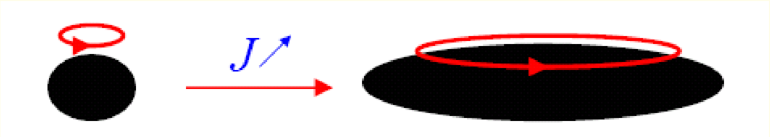
ultraspinning (small mass) limit

corresponds to: $R \gg r_0$

radius of ring $\gg$ thickness of ring

D $\geq$ 6: no Kerr bound for MP BHs:

$\longrightarrow$ ultraspinning regimes with pancaked horizons Emparan,Myers



radius of disc $\gg$ thickness of disc

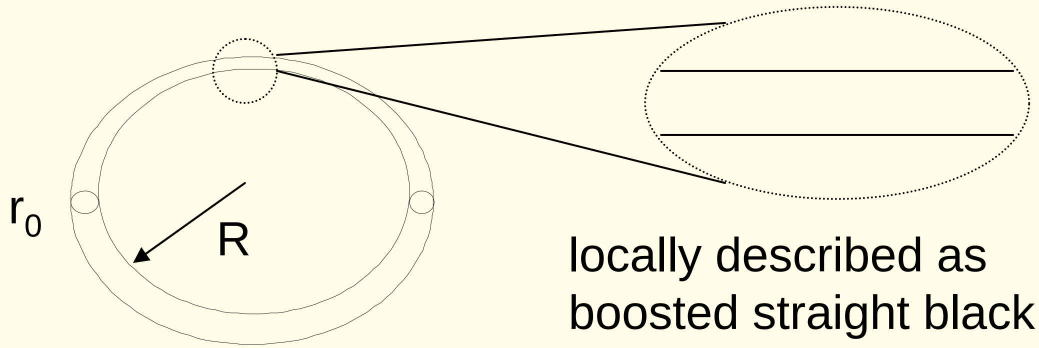
Long distance effective theory

▶ two widely separated scales ⇨ integrate out short-distance dynamics

→ long-distance effective theory

• use to construct BHs perturbatively

e.g. 5D rotating black ring



locally described as
boosted straight black string

◀ by employing method of matched asymptotic expansion (MAE)
thin black ring solution for $D \geq 6$ has been constructed Emparan,Harmark,Niarchos,NO,Rodriguez

• MAE was first developed for localized BHs in KK space in limit: $L \gg r_0$

Harmark/Kol,Gorbonos/Karsik et.al
Dias,Harmark,Myers,NO

• other technique has been developed as well: classical effective field theory (ClEFT)

Chu,Goldberger,Rothstein/Kol

Goal: use these methods to develop a leading order theory for the long-distance dynamics of higher-dimensional black holes

General idea

▸ start with general theory of gravity $S[\Psi]$ Ψ = {graviton, p-forms, scalars}

- look for BH solutions that have two characteristic scales
  + integrate out short-distance physics $\Psi = \Psi_{\mathrm{short}} + \Psi_{\mathrm{long}}$

◂ to leading order: blackfold = black-brane probe in asymptotic background

$$S_{\mathrm{full}} \to S[\Psi_{\mathrm{long}}] + S_{\mathrm{wv}}[X^\mu]$$

Aim: give general prescription for S_{wv} to leading order in r_0/R

- solve EOM of $S[\Psi_{long}]$: defines asymptotic background
- solve $S[\Psi_{long}]$ to find black brane soln with flat worldvolume:
  - asymptotic charges define the blackfold locally
    $\to$ provides short-distance input for S_{wv}
- S_{wv} describes embedding of blackfold in background

$$S_{\mathrm{wv}}[X^\mu] = \int \sqrt{-\gamma}\,\mathcal{L}[X^\mu(\sigma^a)]$$

determined by:
- consistent coupling to bulk fields Ψ (EM/charge conservation)
- use locally stress-energy/currents of the flat black brane solution
  + enforce horizon regularity

Blackfold approach

Emparan,Harmark,Niarchos,NO

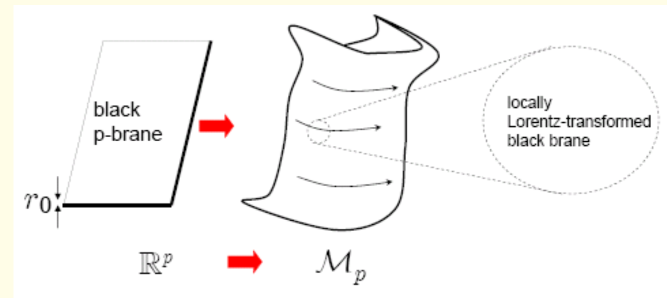
Blackfold = **Black** p-brane whose worldvolume extends along a curved submani**fold** (of embedding space)

▶ start with flat p-brane: horizon $\mathbb{R}^p \times s^{n+1}$

size: r_0

⇓

bend spatial world-volume into submanifold $\mathcal{B}_p$
characterized by length scale: R



• for asymptotically flat blackfolds in D dims:
start with a compact embedding $X^\mu(\sigma^a)$ of submanifold $\mathcal{B}_p \subset \mathbb{R}^{D-1}$

• consider regime of widely separated scales:

curvature radius of submanifold $\gg$ brane thickness

$$R \gg r_0$$

$\longrightarrow$ can approximate the blackfold locally with flat black brane

Question: which $\mathcal{B}_p$ are possible ?

Geometric Censorship

Classical brane dynamics

embedding $X^\mu(\sigma)$ of $\mathcal{B}_p$ determines induced metric: $\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

what governs dynamics of a blackfold ?
- For point particles this is Newton's 2nd law or geodesic eqn. in GR

► For infinitely thin branes we have Carter equation (brane probe approximation)

$$T^{\mu\nu} K_{\mu\nu}{}^\rho = 0 \qquad (\Leftarrow \quad \nabla_\mu T^{\mu\nu} = 0)$$

extrinsic curvature tensor (2nd fund. form)

energy momentum tensor on brane

$$T_{\mu\nu}(\sigma^\alpha) = \tau_{\mu\nu}(\sigma^\alpha)\delta^{(D-p-1)}(x - X(\sigma^\alpha))$$

• Blackfold equations are equivalent to generalized geodesic equation

$$\tau^{\alpha\beta}\left(\nabla_\alpha^{(\gamma)} \partial_\beta X^\rho + \Gamma^\rho_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu\right) = 0$$

follow from the world-volume action $I_{\mathrm{WV}}[X^\mu(\sigma^\alpha)] = \int_{\mathrm{WV}} \sqrt{-\gamma}\, \tau^{\alpha\beta}\gamma_{\alpha\beta}$

Emparan,Harmark,Niarchos,NO

Steps

▶ effective stress tensor of blackfold det'd by matching to short-distance physics
- demand that locally ($r \ll R$) blackfold is equivalent to black p-brane up to position dependent Lorentz transformation

▶ blackfold is now locally a boosted black brane
but still need to impose that it is overall black (regular horizon)

⇨ **Blackness condition:**
surface gravity and angular velocities constant on the blackfold

• determines EM tensor completely in terms of $\kappa, \Omega_i, r_i(\sigma)$

$$I_{\mathrm{WV}} = \int \sqrt{-\gamma}\, \tau^{ab} \eta_{ab} \propto -\frac{1}{\kappa^n} \int \sqrt{\gamma} [1 - \Xi(\sigma)^2]^{\frac{n}{2}}$$

- local velocity field: $\Xi(\sigma^\alpha) = \left(\sum_{i=1}^m \left(r_i(\sigma^\alpha) \Omega_{Hi} \right)^2 \right)^{1/2}$

◀ varying with respect to the embedding coordinates $r_i(\sigma)$

$\rightarrow$ Carter equation becomes a set of purely geometric equations for embedding of $\mathcal{B}$ and given temperature + angular velocities

⇨ Geometric censorship for blackfolds

Thermodynamic quantities and horizon topology

◂ can compute mass and angular momentum by integrating appropriate EM tensor components over brane worldvolume

$$M = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, \tau_{tt}\,, \qquad J_i = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, r_i \tau_{ti}$$

$$A_H = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, a_H(\sigma^\alpha)$$

- small s^{n+1}-sphere at each point of blackfold since locally we have boosted black brane

$\rightarrow$ horizon is fibration of s^{n+1} over $\mathcal{B}_p$

• if fiber is regular, horizon topology: $(\text{topology of } \mathcal{B}_p) \times S^{n+1}$

• but $r_o(\sigma)$ can go go to zero at codimension-1 locus on $\mathcal{B}$ (where local boost is light-like)

- e.g. if $\mathcal{B}_p$ is p -ball with s^{n+1} shrinking at boundary: $S^{p+n+1} = S^{D-2}$

1st law of thermodynamics

► consider Gibbs free energy funtional: $I_G[X^\mu(\sigma)] = M - \Omega_i J_i - 4\pi\kappa A_H$

by explicit computation on finds that this is proportional to the worldvolume brane action: $(D-2) I_G = - I_{\rm WV}$

varying I_G $\Rightarrow$ 1st law of thermodynamics

⇨ 1st law of thermo $\Leftrightarrow$ geometric blackfold equations

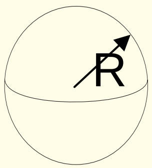
• same as integrated version of local Smarr for black p-brane ! Harmark,NO/Kastor,Traschen

asymptotically flat solutions should obey this Smarr with zero tension:

$$\mathcal{T} = -\int \sqrt{-\gamma} \sum_{i=1}^{p} \tau_{ii} = 0$$

→ total tension vanishes for blackfold (explicitly checked in examples)

New solutions: odd-spheres (and products)



► black brane wrapped on $\mathcal{B}_{2k+1} = S^{2k+1}$

- assume R = const. and take all Ω_i equal (simple solution ansatz)

► action is: $I_{\text{WV}} \propto \int R^p (1 - \Omega^2 R^2)^{\frac{n}{2}}$

EOM solved by $$R = \sqrt{\frac{p}{n+p}\frac{1}{\Omega}}\ , \quad p = 2k+1$$

Novel family of blackfolds with horizon topology: $S^{2k+1} \times S^{n+1}$

- includes black rings for $k = 0$
- for $k \geq 1$: boosts depend on location on the S^{2k+1}
- uniform thickness
- generalizes easily to any product of odd-spheres

Ultraspinning MP BHs as even-ball blackfolds

► blackfold eqs. do not admit even-sphere solutions for $\mathcal{B}_p$

- tension at fixed points of rotation group cannot be counterbalanced by centrifugal forces

⟶ instead solutions with $\mathcal{B}_p$ = ellipsoidal even-ball

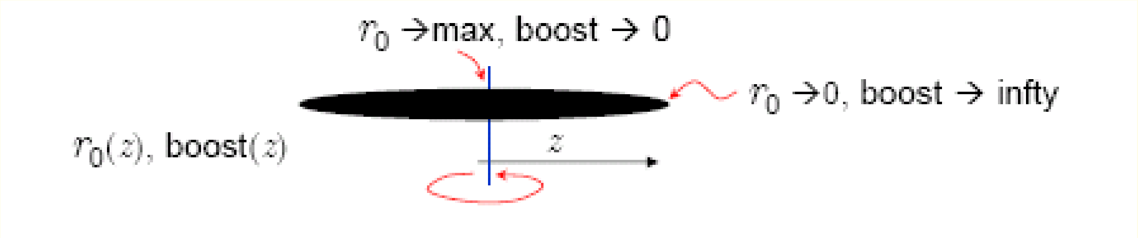
thickness r_0 shrinks to zero at boundary of ball so including the s^{n+1} fibers, horizon topology is S^{D-2}

• reproduce precisely all physical quantities of MP BH with $p/2$ ultra-spins

- highly non-trivial check on approach (rotation has fixed points at center of ball, $r_0(\sigma)$ varying)

◄ simplest example: black disc: $D_2 \subset \mathbb{R}^2$

boost depends on radius:



• corresponds to MP BH with one angular momentum in ultraspinning limit

Blackfold Bestiary

► blackfold construction shows existence of new types of asymptotically flat stationary black holes in higher dimensions

$D=4$	$D=5$	$D=6$	$D=7$	$D=8$	$D=9$	
S^2	S^3	S^4	S^5	S^6	S^7	Kerr, MP BH
		$\mathcal{B}_2 \otimes s^2$	$\mathcal{B}_2 \otimes s^3$	$\mathcal{B}_2 \otimes s^4$ $\mathcal{B}_4 \otimes s^2$	$\mathcal{B}_2 \otimes s^5$ $\mathcal{B}_4 \otimes s^3$	ultraspinning MP BH
	$S^1 \times s^2$	$S^1 \times s^3$	$S^1 \times s^4$	$S^1 \times s^5$	$S^1 \times s^6$	black ring
		$\mathbb{T}^2 \times s^2$	$\mathbb{T}^2 \times s^3$	$\mathbb{T}^2 \times s^4$	$\mathbb{T}^2 \times s^5$	black torus
			$S^3 \times s^2$ $\mathbb{T}^3 \times s^2$	$S^3 \times s^3$ $\mathbb{T}^3 \times s^3$	$S^3 \times s^4$ $\mathbb{T}^3 \times s^4$	
				$S^1 \times S^3 \times s^2$	$S^1 \times S^3 \times s^3$ $\mathbb{T}^4 \times s^3$	

◄ for product odd-sphere and even-ball blackfolds with equal sizes and angular momenta (at fixed mass):

$$A(J) \sim J^{-p/n}$$

tori dominate entropically

Caveats

• regularity of black brane horizon after bending ?
- shown for black 1-folds (i.e. black strings)
- extension to p-folds (to appear)
(use matched asymptotic expansion)

• backreaction of blackfold on background geometry is neglected
(to leading order in r_0/R)
- could make it impossible for leading-order solution to remain stationary
(must be analyzed case-by-case)

• blackfolds may be (classically) unstable
- can use blackfold equations to analyze stability under long wavelength perturbations ($\lambda \gg r_0$)
- there are short wavelength ($\lambda \sim r_0$) instabilities (GL-type) outside approach

Lessons from blackfold approach

► dynamics of higher-dimensional black holes naturally organized in relative value of scales

$0 \leq J \lesssim M(GM)^{\frac{1}{D-3}}$ • single length scale: Kerr BH behavior

$J \gtrsim M(GM)^{\frac{1}{D-3}}$ • regime of mergers and connections between phase
when two horizon scales meet $r_0 \sim R$
- not accesible to effective methods;
requires extrapolation or numerics

$J \gg M(GM)^{\frac{1}{D-3}}$ • blackfolds
- extreme rich physics in this regime;
study dynamics rather than exact solutions for all possible BHs

◄ blackfold horizon topologies $\mathcal{B}_p \times S^{n+1}$

($\mathcal{B}_p$) supported by mechanical equilibrium

(S^{n+1}) supported by internal structure of the BH

- purely topological analysis cannot distinguish between these two factors

Other cases

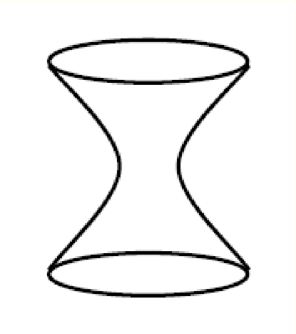
► static minimal blackfolds $\tau_{ij} = -P\eta_{ij}$

(no boost)

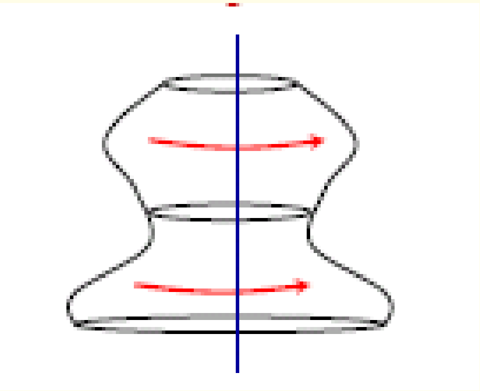
$\longrightarrow \; K^\rho = 0$ (mean curvature vector)

minimal submanifold

e,g. hyperboloid (static non-compact blackfold)



► axisymmetric blackfolds



use numerics or further perturbative approach ?

Outlook

- new blackfolds in 5D: helical rings and strings — Emparan,Harmark,Niarchos,NO (in progress)
  - -have only single spatial U(1) isometry: first evidence of such a solution in 5D ! (as admited by rigidity theorem) — Hollands, Ishibashi, Wald
- charged blackfolds
  - - dipole rings in any dimension
- method can also be applied to blackfolds in other backgrounds (AdS, dS)
  - - black rings in (A)dS — Caldarelli,Empran,Rodriguez
- SUSY blackfolds ?
  - - extremal black holes and black rings — Figueras,Kunduri,Lucetti,Rangamani
  - cf. 5D supersymmetric black ring — Elvang,Emparan,Mateos,Reall
- stability analysis
- relation with DBI
- higher-order analysis (via MAE/ClEFT) (in progress: horizons stay regular)
- blackfold motion + relation to fluid/gravity correspondence
- duality of higher D black holes to plasma balls + rings in AdS (cf. Lahiri,Minwalla) – many similar features

The end