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Hipparcos parallaxes and period–luminosity relations of high–amplitude δ Scuti stars *

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Abstract. Hipparcos parallaxes of high–amplitude δ Scuti stars are used to derive a period–luminosity relation with a scatter of about ± 0.1 mag, which is independent from photometric calibrations to absolute luminosities. Comparisons with several P–L relations from the literature show satisfactory agreement, and all deviations from the Hipparcos mean relation can be explained by uncertainties in the data available before Hipparcos. Hipparcos data for a few stars of relatively small and uncertain parallaxes indicate that they may have systematically very low luminosity. However, briefly discussing Lutz–Kelker corrections and considering the full sample of high–amplitude δ Scuti stars, it is concluded that this sample is homogeneous and has similar basic physical properties as the "normal" low–amplitude δ Scuti stars.

It is emphasized that the Hipparcos P–L relation defines a new distance scale which is independent from those of the classical Cepheids and RR Lyrae stars. Therefore, observations of high–amplitude δ Scuti stars can be used to check fundamental distance determinations to e.g. globular clusters, the Galactic bulge and the Magellanic Clouds.

Key words: methods: statistical – astrometry – stars: oscillations – stars: Cepheids: dwarf – stars: δ Sct

1. Introduction

The high–amplitude δ Scuti variables (HADS in the following) constitute a small subgroup of all δ Scuti stars ($\approx 10\%$) defined by their relatively large amplitude with conventional limit $A_V \geq 0.30$ mag. Several investigations based on photometric calibrations to absolute luminosities have shown well defined period–luminosity relations for HADS (e.g. McNamara & Powell 1990, Fernie 1992, Eggen 1994, Nemec et al. 1994 and McNamara 1995). The main purpose of the present work is to derive a new, independent P–L relation for HADS based on Hipparcos parallaxes (ESA 1997), and to compare this P–L relation to previous ones. We find satisfactory agreement. In connection with fundamental distance determination this is important because HADS used as distance indicators, provide distances in a scale that is independent from those of the primary distance indicators, classical Cepheids and RR Lyrae stars. Thus HADS may give new tests e.g. of the distance to globular clusters and the Magellanic Clouds. And since HADS have luminosities up to $M_V \approx 1$, they may be observed to almost the same distances as the RR Lyrae variables.

Because δ Sct variables often show simultaneous excitation of two or more oscillation modes, they can be used for sensitive tests of stellar models. Each observed pulsation period can in principle be utilized in a new model test. For detailed information on problems related to different δ Sct subgroups and for recent analyses and results we refer to Breger (1980, 1990), McNamara (1992), Mateo (1993), Brown & Gilliland (1994), Breger et al. (1995), Frandsen et al. (1996), Petersen & Christensen–Dalsgaard (1996), Høg & Petersen (1997) and Antonello & Mantegazza (1997).

Application of δ Sct variables both in model testing and in P–L relations requires safe identification of individual pulsation modes. For typical low–amplitude δ Sct stars ($A_V < 0.3$ mag) this problem prevents use of P–L relations for distance determinations (e.g. Antonello & Mantegazza 1997). Fortunately, the situation is much better for HADS. A considerable fraction of HADS are well understood double–mode pulsators with a period ratio identifying the excited modes as the fundamental radial mode and first overtone in most cases. And the majority

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* Based on observations made with the ESA Hipparcos satellite

Table 1. Information on three groups of high–amplitude δ Sct stars with decreasing accuracy of the parallax (see text for details). Parallax with standard error given in milliarcsec in Columns 3 and 4, and observed mean magnitude, V, in Column 6 are taken directly from the Hipparcos Catalogue. Column 5 gives the relative error in parallax, $s_\pi = \sigma_\pi/\pi$, and Columns 7 and 8 give pulsation periods [notes – (1): oscillation in fundamental mode only, (2): oscillation in first and second overtone]. For variables with $\pi > \sigma_\pi$ Columns 9, 10 and 11 give the absolute magnitude, M_V, its standard error and the weight in lsq–solutions for P–L relations, W $= \sigma_{M_V}^{-2}$

Star	Hipparcos Catalogue				V	Π_0	Π_1	M_V	σ_{M_V}	W
Name	#	π	σ_π	s_π	[mag]	[d]	[d]	[mag]	[mag]	[mag^{-2}]
V474 Mon	28321	10.32	0.86	0.08	6.15	0.1361	(1)	1.22	0.18	30.5
AD CMi	38473	8.40	1.73	0.21	9.31	0.1230	(1)	3.93	0.45	
AI Vel	40330	9.99	0.53	0.05	6.56	0.1117	0.0863	1.56	0.12	75.3
VZ Cnc	42594	5.43	0.99	0.18	7.73	(2)	0.1742	1.40	0.40	6.4
V703 Sco	86650	3.91	0.98	0.25	7.85	0.1500	0.1152	0.81	0.54	3.4
RS Gru	107231	4.42	1.05	0.24	8.28	0.1469	(1)	1.51	0.52	3.8
SX Phe	117254	12.91	0.78	0.06	7.33	0.0550	0.0428	2.88	0.13	58.1
GP And	4322	9.22	4.25	0.46	10.86	0.0787	(1)	5.68	1.00	
VX Hya	47904	4.28	2.66	0.62	10.68	0.2234	0.1727	3.84	1.35	
EH Lib	73315	6.71	1.97	0.29	9.85	0.0884	(1)	3.98	0.64	
YZ Boo	75373	2.53	1.53	0.60	10.63	0.1041	(1)	2.65	1.31	
DY Her	80903	6.51	1.91	0.29	10.52	0.1486	(1)	4.59	0.64	
CW Ser	77798	-2.77	3.40	-1.23	11.98	0.1892	(1)			
V974 Oph	86260	-0.38	6.24	-16.42	11.42	0.1911	(1)			
V567 Oph	87994	-1.90	2.89	-1.52	11.25	0.1495	(1)			
XX Cyg	98737	1.48	2.00	1.35	12.04	0.1349	(1)			
ZZ Mic	103684	1.45	1.51	1.04	9.48	0.0672	0.0513			
DE Lac	109420	-0.20	1.84	-9.20	10.34	0.2535	(1)			
CY Aqr	111719	0.71	2.28	3.21	10.99	0.0610	(1)			
DY Peg	114290	0.36	2.02	5.61	10.27	0.0729	(1)			
BS Aqr	117439	0.81	1.59	1.96	9.38	0.1978	(1)			

of single–mode HADS show a light curve form characteristic for fundamental mode pulsation according to standard ideas on light curve forms, although this may be a too simplified view (Bono et al. 1997).

In Sect. 2 we briefly describe the Hipparcos data on HADS, and in Sect. 3 we use the Hipparcos parallaxes to derive a new P–L relation. We assess the accuracy of this relation both by statistical estimates and by detailed comparisons between our relation and several other P–L relations from the literature. Sect. 4 contains a discussion of a small HADS subgroup, which appears to be subluminous. However, considering the full HADS sample we find that the stars of this subgroup do not deviate in physical properties. Finally, in Sect. 5 we briefly consider distance determination based on HADS P–L relations and summarize our conclusions.

2. Hipparcos data

Table 1 gives the resulting data for 21 HADS included in our proposal from 1982 for a Hipparcos project on the nature of δ Sct stars. We divide the stars into three groups with small relative error in the observed parallax, s_π, in Group 1 ($s_\pi = \sigma_\pi/\pi \leq 0.25$), somewhat larger errors in Group 2 ($0.25 < s_\pi \leq 1.0$), and very uncertain parallaxes in Group 3 ($|s_\pi| > 1.0$). In most cases the primary pulsation period has been identified as the fundamental mode of period Π_0, as indicated in Table 1. An exception is VZ Cnc oscillating in first and second overtone. We use the theoretically known period ratio $\Pi_1/\Pi_0 = 0.77$ to infer $\Pi_0 = 0.226$ d, which is used in the lsq–solutions. Cox et al. (1984) preferred an alternative mode identification of VZ Cnc, which we discuss in Sect. 3.

The absolute magnitude M_V in Column 9 of Table 1 is calculated from

$$M_V = V + 5 + 5\,\log\pi. \quad (1)$$

The error distribution of π obtained from the Hipparcos Catalogue is approximately gaussian with the standard error given in the catalogue for each star. For small values of s_π the error distribution of M_V will also be approximately gaussian with the standard error (given in Table 1)

$$\sigma_{M_V} = \frac{5}{\ln 10}\,\frac{\sigma_\pi}{\pi} = 2.171\; s_\pi. \quad (2)$$

But Eq. (1) implies that the error distribution of M_V will be very skew for large values of s_π. In Table 1 we can neglect the error of the apparent magnitude V and also the effect of interstellar absorption, because all δ Sct stars with accurate π are bright and very close.

3. P–L relations

The standard form of the P–L relation is

$$M_V = A \log \Pi_0 + B, \tag{3}$$

where the coefficients A and B are determined from a set of observed $(\Pi_0,\ M_V)$ for a sample of stars. We always use periods in unit of [d]. Previous HADS P–L relations have been based on M_V–values taken from photometric calibrations. Now for the first time the Hipparcos data can be used to derive a P–L relation directly based on trigonometric parallaxes. In Høg & Petersen (1997) we briefly mentioned the period–luminosity relation derived from the accurate Hipparcos data for Group 1 of Table 1:

$$M_V = -3.73 \log \Pi_0 - 1.90. \tag{4}$$

This lsq–solution is calculated with weights given by $W = \sigma_{M_V}^{-2}$, and the standard errors of the coefficients are $\sigma_A = 0.57$ and $\sigma_B = 0.59$. However, Eq. (4) is based on only six stars because we had to exclude AD CMi (see Høg & Petersen for details). In the P–L diagram in Fig. 1 we compare Hipparcos data for the Group 1 stars with Eq. (4) and several P–L relations from the literature. The standard deviation of the six points (excluding AD CMi) in Fig. 1 around the Hipparcos mean line is ±0.46 mag, using equal weight for all points, and ±0.09 mag, using the weights just mentioned.

The resulting scatter of ±0.09 mag consists of two parts: one contribution from the error of M_V–values, which we can assume is due to the error in parallaxes only, and another contribution due to intrinsic differences in basic physical properties between the stars of the sample. It seems from the figure that the scatter may be entirely due to the observational error of the parallaxes, implying that the scatter of the Hipparcos P–L relation could be even smaller than ±0.1 mag. However, we do not want to draw this conclusion, considering that HADS are known to be an inhomogeneous group of variables e.g. in metal content.The deviation of AD CMi from Eq. (4) is 2.44 mag, which confirms the exceptional position of this star according to the Hipparcos parallax.

HADS period–luminosity relations have been studied by e.g. McNamara & Powell (1990), Fernie (1992) and Eggen (1994). In Table 2 we give the coefficients of all P–L relations included in our analysis. The P–L relation given by McNamara & Powell (1990) is in remarkably good agreement with Eq. (4) as seen in Fig. 1. Perhaps the most remarkable P–L relation of all those considered here is the relation of Fernie (1992), which can be used from a period of $\Pi_0 \approx 0.05$ d to ≈ 50 d, i.e. over an interval of 3 dex. Fernie discussed δ Sct stars as small Cepheids, extending the Cepheid period–luminosity law to δ Sct periods. Fernie remarked that the pop. II star SX Phe also fits his relation very well. However, Fernie used for SX Phe the photometrically determined $M_V = 2.5$, which is

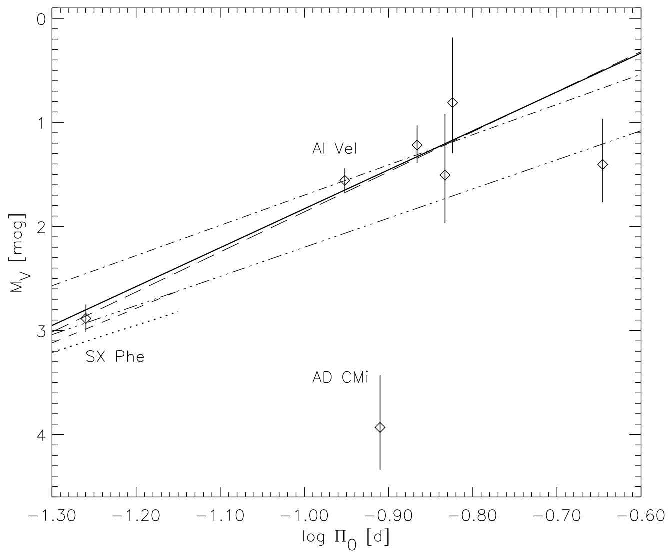

Fig. 1. Period–luminosity diagram for the seven nearest field high–amplitude δ Sct stars. Hipparcos data with 1σ error bars are marked by diamonds. Lines give the P–L relations discussed in the text (full line: Hipparcos mean relation Eq. (4) with formal scatter ±0.09 mag, long dashes: McNamara & Powell 1990, dash–dots: Fernie 1992, dash–triple dots: Eggen 1994, short dashes: McNamara 1995, dots: Nemec et al. 1994). The large differences between Eq. (4) and two of the other P–L relations are due to uncertainties in the data available before Hipparcos (see text)

0.4 mag too bright according to the Hipparcos result. This explains the deviation between Fernie's mean line and Eq. (4) for short periods. Eggen's (1994) relation agrees with Eq. (4) for short periods, but deviates by more than 0.5 mag for longer periods. Eggen's Table 3 gives for SX Phe the photometric $M_V = 3.03$ in good agreement with the Hipparcos value, while his M_V values for AI Vel and V474 Mon are more than 0.4 mag too faint according to Hipparcos data, which for AI Vel corresponds to more than 3σ. Since AI Vel and V474 Mon fix Eq. (4) for long periods, this difference explains the deviation between Eggen's P–L relation and our Eq. (4) for periods above 0.1 d.

Nemec et al. (1994) investigate period–luminosity–metal relations for population II variable stars. They give for SX Phe variables oscillating in the fundamental mode: $M_V(SX) = 0.36 - 2.56(\pm0.54) \log \Pi_0 + 0.32\,[\mathrm{Fe/H}]$. In Fig. 1 we show this relation for the representative metal content $[\mathrm{Fe/H}] = -1.5$ together with the relation: $M_V = -3.29 \log \Pi_0 - 1.16$ preferred by McNamara (1995). It is seen that these relations agree with the Hipparcos result for SX Phe itself within estimated uncertainties. We conclude that all deviations between the Hipparcos Eq. (4) and other P–L relations larger than about 0.1 mag can be explained by uncertainties in the data available before the Hipparcos Catalogue.

Table 2. Coefficients $A = a, B$ and b (given in mag) defined by Eq. (3) and Eq. (5) with Π_r = 0.1 d for all P–L relations considered here. Standard errors are given when available

P–L relation	$A = a$	B	b
Present Eqs. (4) & (6)	-3.73	-1.90	1.83
	±0.57	±0.59	±0.10
Present Eq. (8)	-4.10	-2.34	1.76
	±0.49	±0.51	±0.09
van Leeuwen (1998)	-3.98	-2.18	1.80
	±0.52	±0.52	±0.09
McNamara & Powell (1990)	-3.85	-1.99	1.86
Fernie (1992)	-2.90	-1.20	1.70
Eggen (1994)	-2.80	-0.60	2.20
Nemec et al. (1994)	-2.56	-0.12	2.44
McNamara (1995)	-3.29	-1.16	2.13

3.1. Error estimates

Table 2 contains all coefficients in the P–L relations used here as well as their standard errors when available. The large errors given for the coefficients A and B in Eq. (4) do not directly represent the accuracy of an M_V–value predicted from an observed Π_0 due to strong correlation between A and B. For a more detailed discussion of errors in the Hipparcos P–L relation it is useful to rewrite the P–L relation

$$M_V = a \log(\Pi_0/\Pi_r) + b, \qquad (5)$$

where Π_r is a conveniently chosen constant reference period and a and b again are parameters to be determined by the lsq–solution for a known sample. In the following we choose Π_r = 0.1 d, near the middle of the abscissa range of our data, with the advantage that covariances may be neglected and only standard errors of the parameters a, b need to be considered, for the degree of approximation we require.

The new form of the Hipparcos P–L relation [based on precisely the same data as Eq. (4)] becomes

$$M_V = (-3.73 \pm 0.57) \log(\Pi_0/0.1) + (1.83 \pm 0.10). \qquad (6)$$

The advantage of using this form rather than Eq. (3) clearly is that the zero–point error σ_b = 0.10 now becomes much smaller than the corresponding σ_B = 0.59. And differences in b in Table 2 directly correspond to differences in M_V between the P–L relations in Fig. 1 near the middle of the period interval.

The formal standard error of M_V predicted from Eq. (5) is given by

$$\sigma^2_{M_V} = [\log(\Pi_0/\Pi_r)]^2\, \sigma^2_a + \sigma^2_b. \qquad (7)$$

Thus for the period $\Pi_0 = \Pi_r$ = 0.1 d, $\sigma_{M_V} = \sigma_b$ = 0.10 mag. This is very similar to the scatter of ±0.09 mag around the mean line mentioned above. Eq. (7) shows that σ_{M_V} increases with changes in the ratio Π_0/Π_r from 1, e.g. a factor of 2 gives σ_{M_V} = 0.20 mag.

3.2. Mode identification of VZ Cnc

Cox et al. (1984) argue that the oscillations observed in VZ Cnc are fundamental mode and first overtone with Π_1/Π_0 = 0.8006, explaining the difference from the standard value $\Pi_1/\Pi_0 \approx 0.77$ by helium diffusion. We prefer as mode criterion for all double–mode variables including VZ Cnc to require agreement between the observed period ratio and the ratios calculated from standard stellar evolution models (e.g. Petersen & Dalsgaard 1996). We have three arguments for our mode identification based on Π_2/Π_1 = 0.8006: (i) It is in perfect agreement with the standard pop. I models of Petersen & Dalsgaard. (ii) For SX Phe and AI Vel we believe that no significant effect on Π_1/Π_0 from diffusion (and also from possible rotation — which could also give a considerable effect) is present (Høg & Petersen 1997). Rather, the observed interval in Π_1/Π_0 for pop. I HADS (excluding VZ Cnc) is surprisingly narrow: 0.771 ± 0.002 (cf. Petersen & Dalsgaard 1996). If these effects were present much more scattered values should be expected. (iii) Kurtz (1988) gave an argument against diffusion in high-amplitude δ Sct stars as VZ Cnc: Diffusion removes He from the driving zone, so it is strange that such stars have large amplitudes in contrast to the Am and Ap stars where helium–settling is supposed to result in very low amplitudes.

We have also calculated lsq–solutions assuming the Cox et al. mode identification, i.e. Π_0 = 0.1742 d. The resulting P–L relation corresponding to Eq. (4): $M_V = (-4.00 \pm 0.39) \log \Pi_0 - 2.19 \pm 0.41$ has somewhat smaller formal errors than Eq. (4). This fact can be taken as an argument for the Cox et al. interpretation. However, we assess that our arguments (i) – (iii) for Π_1 = 0.1742 d are more convincing. We note that the difference between the two alternative P–L relations is only marginal and changes nothing in the astrophysical discussion.

4. Existence of an AD CMi group?

Figure 2 shows the P–L diagram for the stars of Groups 1 and 2 in Table 1. It is seen that seven stars agree nicely with the Hipparcos mean relation, while five stars seem to be similar to AD CMi. They are situated ≈ 3 mag below the standard P–L relation. This corresponds to several σ_{M_V} according to Table 1, e.g. for AD CMi to 5.4 σ. Do we have a separate deviating group of HADS here?

In the literature there is no indication that these five AD CMi variables are different from "normal" HADS in any physical property. Therefore, it seems difficult to understand that they could be about 3 mag weaker than other HADS. Before seriously considering the existence of a deviating HADS group, we must discuss possible effects of both statistical and systematic errors in the parallaxes. Selecting twelve variables with $|s_\pi| < 0.65$ from a larger sample, we may have neglected several variables with very small observed π, which should be taken into account in

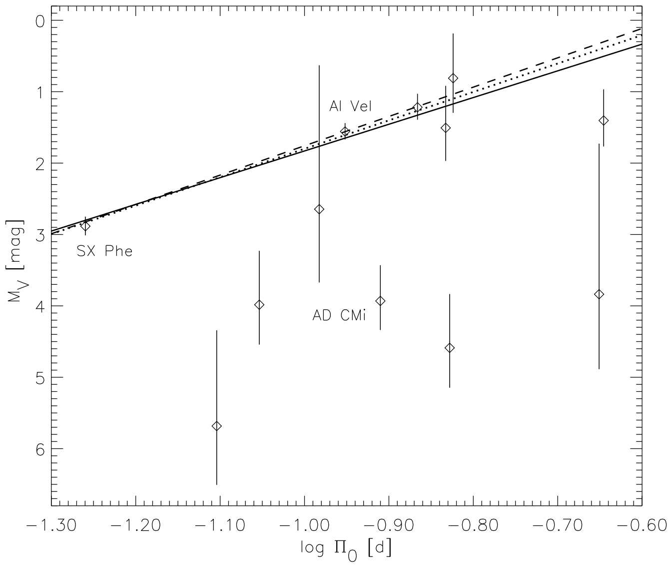

Fig. 2. Period–luminosity diagram for the nearest 12 field high–amplitude δ Sct stars (Groups 1 and 2). Hipparcos data with 1σ error bars (in π) are marked by diamonds. Note that the symmetric error in π gives asymmetric error bars in M_V. Lines gives P–L relations (full line: our Eq. (4), dashes: Eq. (8) including Lutz–Kelker corrections, dots: van Leeuwen (1998); cf. Table 2 and text)

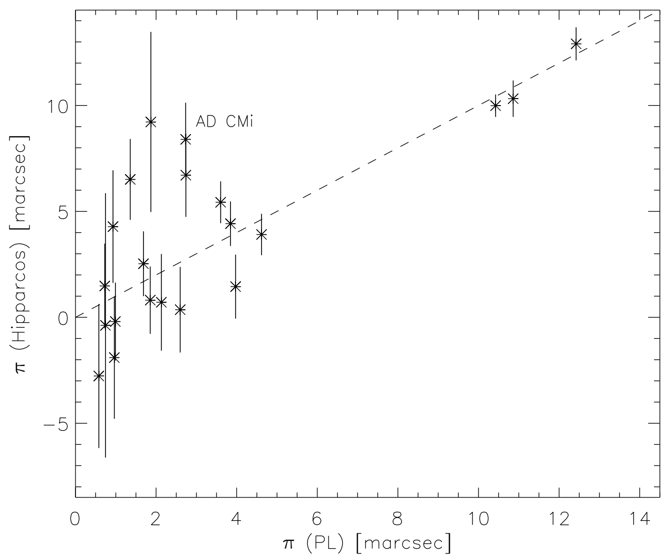

Fig. 3. Comparison of observed Hipparcos parallaxes marked with 1σ error bars with "true" parallaxes calculated by means of the Hipparcos P–L relation. For a homogeneous sample of stars a scatter distribution around the identity line (dashed line) is expected, and no clear deviation from this case seems to be present here

an unbiased, statistical study. This very difficult problem has been studied in many papers in the literature (see e.g. Lutz & Kelker [1973], Reid [1997], and references therein).

4.1. Improved statistics and Lutz–Kelker corrections

Lutz & Kelker (1973) discussed the use of trigonometric parallaxes for the calibration of luminosity systems. Considering simplified samples they showed that in most cases systematic errors are present for *all* stars (not only for stars with observed parallax below a lower limit), and that the error depends upon the relative error, s_π, not upon the size of the observed parallax π. The value of the Lutz–Kelker correction is of course small for small s_π, and increases to e.g. -0.02, -0.11 and -0.28 mag for s_π = 0.05, 0.10 and 0.15, respectively. The correction is well defined only up to s_π = 0.175, where the value is -0.43 mag. For higher s_π the relevant correction probably increases very quickly.

For our derivation of the Hipparcos P–L relation Lutz–Kelker corrections seem to be relevant. For the stars of highest weight, AI Vel and SX Phe, the correction amounts to only -0.02 mag. But for V474 Mon with $s_\pi = 0.08$ the correction is -0.07 mag, for VZ Cnc with $s_\pi = 0.18$ we reach the limit of -0.43 mag, and for the two low–weight stars larger but not precisely known values should be used. Using for these stars the value -0.60 mag, we obtain the Hipparcos mean P–L relation including Lutz–Kelker corrections:

$$M_V = -4.10 \log \Pi_0 - 2.34 = -4.10 \log (\Pi_0/0.1) + 1.76 \quad (8)$$

with rms–scatter ± 0.09 mag for the weights given in Table 1. In Fig. 2 it is seen that this relation deviates very little from Eq. (4) except for the longest periods where no high–quality Hipparcos data are available.

Van Leeuwen (1998) gives a statistically correct derivation of the Hipparcos P–L relation, using the parallaxes directly in a nonlinear lsq–solution based upon the same data as our Eqs. (4) and (8). His solution is reproduced in Table 2 and shown in Fig. 2. It is intermediate between our Eqs. (4) and (8). Thus our attempt to apply Lutz–Kelker corrections seems to result in an over–correction. We emphasize that the differences between these three Hipparcos P–L relations are so small that our conclusions with respect to comparisons with other P–L relations from the literature are not changed at all. The reason why the P–L relations shown in Fig. 2 are almost identical is of course that the three data–points of heigh weight have very small statistical corrections.

For the issue of a possible existence of an AD CMi group we need corrections to the individual M_V–values used in Fig. 2. However, Lutz–Kelker corrections have precise meaning only in a statistical sense, so the following considerations are meant for illustrating and understanding the AD CMi problem; they can not be regarded as precise arguments. From Table 1 it is seen that we now need M_V–corrections up to $s_\pi = 0.62$, which is much higher than the limit of 0.175. We find an interesting clue considering the pre–Hipparcos parallaxes of AI Vel and SX Phe: $\pi = 0.028 \pm 0.011$ arcsec, i.e. $s_\pi = 0.39$ and $\pi = 0.023 \pm 0.008$ arcsec, $s_\pi = 0.35$, respectively. Considering the Hipparcos values of Table 1 as "true" values, we find M_V–corrections to the old values of -2.2 and -1.2 mag, respectively. With an M_V–correction of ≈ -2 mag at $s_\pi = 0.4$, we can argue that values of ≈ -3 mag at $s_\pi = 0.6$ are not unreasonable. Therefore we cautiously argue that the apparently well separated AD CMi group in Fig. 2 may very well appear due to statistical M_V–corrections. Thus the Hipparcos parallaxes give no convincing argument for the existence of a low–luminosity HADS group with physical properties deviating from those of normal low–amplitude δ Sct stars.

If all HADS are normal δ Sct stars following the P–L relation Eq. (4) we can calculate realistic ("true") parallax values, which we denote π(PL), from the precisely known pulsation periods. Eq. (4) gives M_V, which together with the observed V gives π(PL) using Eq. (1). Here we can neglect interstellar absorption, because the relevant distances are small and we are not interested in high accuracy. Figure 3 shows for all stars of Table 1 the observed Hipparcos parallaxes with errors bars indicated as function of these "true" parallaxes. Do we find a reasonable distribution around the identity line? It is seen that in almost all cases the stars are within $\pm 2\,\sigma_\pi$ from the identity line. Only AD CMi (3.2 σ_π) and DY Her (2.7 σ_π) have larger deviations. And the discrepancy of 3.2 σ_π for AD CMi is less impressive than the corresponding 5.4 σ_{M_V} mentioned above. In our view Fig. 3 confirms that the HADS may be a homogeneous group of variable stars.

Antonello and Mantegazza (1997) also consider the possible existence of a low–luminosity HADS group indicated by the Hipparcos parallaxes, emphasizing the SX Phe stars. They make simulations of simplified samples of stars and calculate the observed distribution of M_V as function of s_π. They find that for $s_\pi > 0.15$ the probability of deriving too high M_V–values increases very steeply with s_π, and conclude that this effect offers a possible explanation for the deviating apparently low–luminosity stars, in agreement with our discussion.

5. Discussion and conclusion

We use Hipparcos parallaxes of six HADS to construct a P–L relation that is independent of all previous ones. The accuracy of an absolute magnitude M_V predicted from an observed fundamental mode period Π_0 is estimated to be about ± 0.1 mag, both by calculation of formal standard errors and by comparisons with several P–L relations based on photometric calibrations to M_V taken from the literature.

There are several reasons why we cannot expect precise P–L relations: First, any P–L relation is an average relation for a sample of stars, which (perhaps) obey a precise P–L–C relation, if the sample is homogeneous. Therefore a color term should be included, as it is well known from similar analyses of classical Cepheids (e.g. Feast and Catchpole [1997], Sasselov et al. [1997], and references therein).

Second, our calibrating stars include both pop. I and pop. II variables as most P–L relations in the literature, in particular anchoring the low–period part on pop. II stars (SX Phe). From stellar evolution theory one might expect systematic differences between the two populations, which could invalidate the statistics. SX Phe with by far the shortest period of the stars in Group 1 of Table 1, is also the only star of large metal deficiency with [Fe/H] given to -1.3 by McNamara (1992) and -1.7 by Nemec et al. (1994). RS Gru has [Fe/H] $= -0.5$, while the remaining four calibrating variables are normal pop. I stars with [Fe/H] ≈ 0.0. Our calibrating sample seems to follow the average $\log \Pi_0 -$ [Fe/H] relation very well (see e.g. Fig. 4 of McNamara 1995). Therefore, we can expect the Hipparcos P–L relation to give an average P–L–C–[Fe/H] relation for HADS.

Third, Antonello & Mantegazza (1997) find that differences between M_V derived from photometric indices and those from Hipparcos parallaxes are sometimes large, mainly due to photometric effects of metallicity and rotational velocity. This indicates that rotation plays a role for low–amplitude δ Sct stars, which has not been studied for HADS.

Clearly, all these effects should be included in improved P–L relations, which will require much more stars for reliable calibrations. Such data for HADS will not become available until results from the next astrometric satellite are ready in perhaps 5–10 years. At present, the best possibility for improving the Hipparcos P–L relation seems to be to include data for medium–amplitude δ Sct stars with $0.1 < A_V < 0.3$, where improved data including mode identifications are becoming available (e.g. Solano & Fernley 1997).

In connection with fundamental distance determination the importance of the Hipparcos P–L relation is that the distance scale defined by this relation is independent from those of the primary distance indicators, classical Cepheids and RR Lyrae stars. Thus, comparisons of HADS observations with this relation give distances that may provide new checks of fundamental distance determinations to e.g. globular clusters, the Galactic bulge and the Magellanic Clouds. Minniti et al. (1997) use the MACHO database to discuss δ Sct stars in the Galactic bulge. They stress that when a firm P–L relation is established, these variables could yield an independent distance to the Galactic center as accurate as that measured using RR Lyrae stars. Petersen & Høg (1998) use comparisons of OGLE observations of SX Phe stars in ω Centauri with the Hipparcos P–L relation to obtain distance estimates to this globular cluster in good agreement with earlier distance determinations in the literature. We conclude that in a few years HADS may provide valuable, independent checks of astrophysically important distances.

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