Dynamically controlled toroidal and ring-shaped magnetic traps

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We present traps with toroidal (T^2) and ring-shaped topologies based on adiabatic potentials for radio-frequency-dressed Zeeman states in a ring-shaped magnetic quadrupole field. Simple adjustment of the radio-frequency fields provides versatile possibilities for dynamical parameter tuning, topology change, and controlled potential perturbation. We show how to induce toroidal and poloidal rotations, and demonstrate the feasibility of preparing degenerate quantum gases with reduced dimensionality and periodic boundary conditions. The great level of dynamical and even state-dependent control is useful for atom interferometry.

I. INTRODUCTION

Cold neutral atoms are particularly well suited as a generic model for many-body systems as they allow tuning of parameters that are not accessible in other systems such as the interparticle interaction strength. Studying the properties of quantum gases in confining geometries where the dimensionality of the system can be chosen or even be adjusted is at the focus of current research [1]. The dimensionality of the confined cloud plays a key role for important properties such as long-range order [2,3] and superfluidity [4–7] of the gas. This is also true for the topology of the trap, and hence large interest in multiply connected trapping potentials such as rings has recently emerged.

In first experiments with nontrivial trapping topologies, ultracold and Bose-condensed atoms have been confined in ring-shaped potentials [8–10]. Most of these experiments rely on the production of a ring-shaped magnetic quadrupole field to trap the atoms. The ring of zero field in the center of this trap causes a loss of atoms due to Majorana spin flips to untrapped states. These leaks have been plugged using time orbiting ring traps (TORTs) [9], out-of-plane current carrying wires [10], or stadium-shaped traps reducing the ring of zero to four points [11]. Also electrostatic fields have been proposed as a possible solution for a chip-based trap [12]. For chip-based approaches [12–14] the symmetry-breaking potential corrugation caused by lead wires must typically also be addressed.

In this paper, we show how adiabatic potentials for radio-frequency- (rf-) dressed states [15–21] can be used in a circular configuration. These dressed traps have recently been shown to provide long lifetimes, allow for evaporative cooling [22,23], and have been used to coherently split matter waves [17,24].

We present an experimentally feasible method that results in a trap with new flexibility going well beyond that of a TORT trap, using a similar setup but with an oscillating field of much lower amplitude at higher frequencies. Reaching beyond simple ring geometries, toroidal surfaces can be

formed and smoothly transformed to a (multiple) ring geometry. One- and two-dimensional (1D and 2D) quantum gases with periodic boundary conditions are accessible as well as the crossover regimes between confinement of different dimensionality (3D to 2D and 1D, and 2D to 1D). Moreover, we show that the trapped gases can be dynamically set into various types of motion by simple experimental techniques. These include toroidal and poloidal rotations on torus surfaces and independently controlled acceleration along circular paths.

II. TRAP DESIGN

Our trapping configuration is based on magnetic microstructures, which have been proposed and demonstrated for atom trapping using both current-carrying wires [25–27] and permanent magnetic structures [28–30]. We start from a ring-shaped magnetic quadrupole field, generated by two concentric rings of magnetized material with out-of-plane magnetization **M**; see Fig. 1. The advantages here are high-field gradients (up to 10^3 T/m) and the absence of lead wires that break the symmetry of the trap. For small heights h of the magnetic layer, the field sources become equivalent to two pairs of concentric, counterpropagating, linelike currents $I = Mh$ around the edges of the material. They produce a magnetic quadrupole field with a ring-shaped line of zero field.
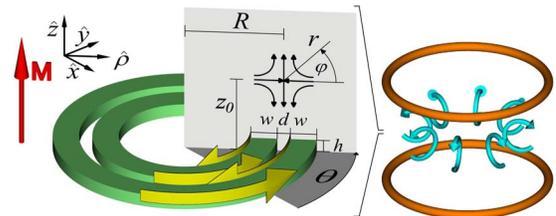

FIG. 1. (Color online) Schematic view of a quadrupole ring produced by two rectangular rings of uniform magnetization $M \cdot \hat{\mathbf{z}}$, mean radius R, widths w, distance d, and height h. Additional coordinates are given by the distance r from the ring of zero field and the toroidal and poloidal angles θ and φ, respectively. An additional pair of external coils is used to produce a circularly polarized, axially symmetric rf field.

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We assume that the radii of the rings are large compared to their widths and separation, $R \gg w, d$. In this case, the field in the vicinity of the magnetic rings can be approximated by a two-dimensional quadrupole field with field gradient q, centered above the mean radius R at height $z_0 = \frac{1}{2}\sqrt{d(2w+d)}$. Its axes are parallel and perpendicular to the surface.

Cold atoms in a low-field-seeking spin state can be confined in this field, but will suffer losses due to nonadiabatic spin-flip transitions near the line of zero field. These can be avoided using adiabatic, radio-frequency-induced potentials. A theoretical description was developed in [15,16]. Resonant coupling between Zeeman levels leads to a modification of the dressed-state energies, producing local potential minima near positions where the coupling field is resonant with the atomic Larmor frequency. As was noted in [17] and recently discussed in [18] for a cylindrical case, the polarization of the fields must be taken into account. This important design parameter introduces new possibilities, on which we put emphasis in this paper.

For an atom with total angular momentum F, the Hamiltonian in a weak magnetic field can be approximated by $H(t) = g_F \mu_B \hat{\mathbf{F}} \cdot \mathbf{B}(t)$, with the Bohr magneton μ_B and the Landé factor g_F. The magnetic field consists of a static contribution $\mathbf{B}_{dc}$ as given by the ring-shaped quadrupole field (Fig. 1) plus an rf contribution $\mathbf{B}_{rf}(t) = \mathrm{Re}[\mathbf{B}_{rf} \exp(-i\omega t)]$. At each point we take the direction of the static field as the local quantization axis, $\mathbf{B}_{dc} = B_0 \hat{\mathbf{e}}_0$.

The vector $\mathbf{B}_{rf}$ is complex valued and can be decomposed in a basis of orthonormal, spherical polarization vectors $(\hat{\mathbf{e}}_0, \hat{\mathbf{e}}_+, \hat{\mathbf{e}}_-)$, corresponding to π, σ^+, and σ^- polarizations, respectively: $\mathbf{B}_{rf} = \beta_0 \hat{\mathbf{e}}_0 + \beta_+ \hat{\mathbf{e}}_+ + \beta_- \hat{\mathbf{e}}_-$. We transform the Hamiltonian to a frame rotating about $\hat{\mathbf{e}}_0$ at the rf frequency ω and make a rotating-wave approximation (RWA) by neglecting remaining time-dependent terms. The resulting effective Hamiltonian in the rotating frame has the eigenvalues

$$E_{m_F} = m_F \mu_B g_F \sqrt{(B_0 - \hbar\omega/g_F\mu_B)^2 + |\beta_+|^2/2}. \quad (1)$$

It is crucial that in the RWA only the σ^+ field component (relative to the static field) appears in the effective Hamiltonian.

For a ring-shaped quadrupole field, the resonance condition is met on a toroidal surface of constant field modulus $B_0 = qr_0 = \hbar\omega/g_F\mu_B$. In Fig. 2 the trapping potential along the z direction at $\rho = R$ is depicted. Atoms can be trapped at the avoided crossings between dressed energy levels, which occur on the surface of resonance. The remaining potential on this surface is determined by the rf component β_+, referenced to the local direction of the static field.

In order to form a trap, β_+ must not vanish in any point on the resonant surface. Such points, like the zeros in quadrupole magnetic traps, limit the trap lifetime and have to be avoided. It is not sufficient to use any rf field that provides field components perpendicular to the static field, as, e.g., a rotating field in the $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ plane fails. A good way to drive the resonance is by a phase-shifted superposition of a spherical quadrupole field, radially polarized along $\hat{\boldsymbol{\rho}}$, and a uniform field with linear $\hat{\mathbf{z}}$ polarization. A single pair of external coils can generate this field without breaking the axial symmetry.
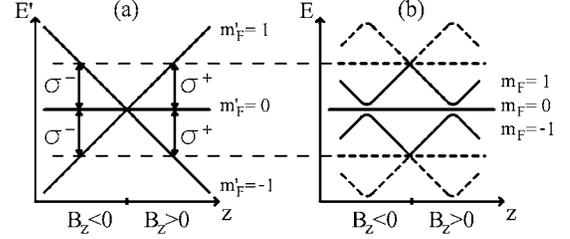

FIG. 2. Schematic view of rf-induced potentials: (a) The energy splitting between atomic Zeeman levels depends linearly on position in a magnetic quadrupole field. (b) The energy levels appear shifted in the rotating frame and exhibit avoided crossings at resonant rf coupling. Note that across $B_z = 0$ the effective quantum number m_F changes sign and is undefined at zero field. Atoms traversing this region can undergo transitions to different spin states in the adjacent dressed-state manifolds (dashed lines). For negative B_z or negative g_F the resonance frequency is also negative, inverting the rotational sense of the required coupling field when referenced to the z direction.

The coils can be placed symmetrically above and below the chip surface. An interesting possibility to generate phase shifted radial fields is to incorporate conducting metal rings below the surface and induce rf currents. In general, the rf field is elliptically polarized in the $\hat{\boldsymbol{\rho}}, \hat{\mathbf{z}}$ planes. Denoting its circular components by complex amplitudes a and b, it is expressed in global, Cartesian coordinates by

$$\mathbf{B}_1 = \frac{a}{\sqrt{2}}\begin{pmatrix} \cos\theta \\ \sin\theta \\ i \end{pmatrix} + \frac{b}{\sqrt{2}}\begin{pmatrix} \cos\theta \\ \sin\theta \\ -i \end{pmatrix}. \quad (2)$$

In the vicinity of the quadrupole ring the static field direction is $\hat{\mathbf{e}}_0 = (-\cos\theta\cos\varphi, -\sin\theta\cos\varphi, \sin\varphi)$. We construct a right-handed orthonormal triplet by choosing the direction tangential to the ring, $\hat{\mathbf{e}}_1 = (-\sin\theta, \cos\theta, 0)$, and $\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_0 \times \hat{\mathbf{e}}_1$. We define

$$\hat{\mathbf{e}}_+ = \frac{(\hat{\mathbf{e}}_1 + i\hat{\mathbf{e}}_2)}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\sin\theta - i\cos\theta\sin\varphi \\ \cos\theta - i\sin\theta\sin\varphi \\ -i\cos\varphi \end{pmatrix}, \quad (3)$$

and obtain the coupling component

$$\beta_+ = \hat{\mathbf{e}}_+^* \cdot \mathbf{B}_1 = -\frac{a}{2}e^{-i\varphi} + \frac{b}{2}e^{i\varphi}. \quad (4)$$

The potential on the surface of resonance is given by

$$E_{m_F}(\varphi) = \frac{\mu}{2\sqrt{2}}\sqrt{|a|^2 + |b|^2 - 2|a||b|\cos[2(\varphi - \varphi_0)]}, \quad (5)$$

where $2\varphi_0 = \arg b - \arg a$ and $\mu = m_F g_F \mu_B$.

We assume $R \gg r_0$ and thus approximate a and b to be independent of r and φ. A special case is obtained for a circular field, $a = 0$ or $b = 0$. The potential, Eq. (5), then becomes independent of both φ and θ, so the trap extends over the entire toroidal surface; see Fig. 3(a). This opens up the possibility to trap cold atoms in a two-dimensional, boundaryless geometry.
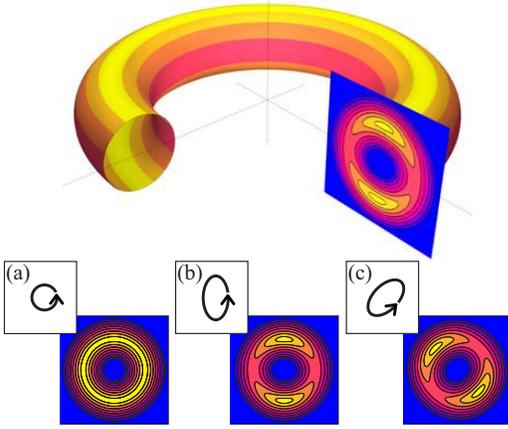
FIG. 3. (Color online) Visualization of the adiabatic trapping potential. On the top, the potential on the resonant torus for vertically split rings is shown together with a cut parallel to a ρ, z plane. On the bottom, ρ, z potentials are shown for (a) circular rf ($b=0$) and elliptical rf ($a>b$) with (b) $\varphi_0=\pi/2$ and (c) $\varphi_0=\pi/4$ (increasing potential from light to dark).

More generally, if neither a nor b vanishes, the field is elliptically polarized. In this case, the potential has minima for two poloidal angles. The trap thus splits poloidally into two opposite rings, as shown in Fig. 3. The orientation of this splitting is determined by φ_0.

For convenience, we redefine the local coupling amplitude in the poloidal minimum as $c=||a|-|b||/2\sqrt{2}$ and define $\epsilon=|b/a|$. The trapping frequencies in the radial and poloidal directions then become

$$\omega_r = \frac{q}{\sqrt{c}}\sqrt{\frac{\mu}{m}}, \quad \omega_\varphi = \frac{2}{r_0}\sqrt{\frac{c\epsilon}{(1-\epsilon)^2}}\sqrt{\frac{\mu}{m}}, \quad (6)$$

where m is the mass of an atom.

III. ADIABATICITY REQUIREMENTS

The condition that the internal atomic state adiabatically follow the external motion is violated at points where the rf field is resonant but the coupling component β_+ is too small. For example, for $|a|=|b|$—i.e., linear rf polarization—the coupling will vanish in the two rings. To avoid Landau-Zener transitions, the energy splitting E_{m_F}/m_F must be large compared to the trap frequency ω_r. We thus demand that $E_{m_F}/m_F \geq \alpha\hbar\omega_r$, with the factor $\alpha\gg 1$, and get the adiabaticity condition

$$\frac{c^3}{q^2} \geq \frac{\alpha^2\hbar^2 m_F}{\mu_B g_F m}. \quad (7)$$

In addition, it is necessary to stay in the RWA validity regime; i.e., the rf frequency must be high compared to the energy splitting. Thus we choose $\hbar\omega \geq \delta E_{m_F}/m_F$ with the

numerical factor $\delta>1$. This is equivalent to $qr_0 \geq \delta c$. Both relations together impose a size restriction[1]

$$qr_0^3 \geq \frac{\alpha^2\delta^3\hbar^2 m_F}{\mu_B g_F m}. \quad (8)$$

IV. LOW-DIMENSIONAL TRAPS

As a first application we discuss the feasibility to realize two-dimensional traps for degenerate quantum gases. One requirement to be met is that the thermal motion of the atoms be limited to the radial ground state, $\hbar\omega_r \geq \zeta k_B T$ with $\zeta>1$. Together with the adiabaticity condition, the achievable temperature sets lower limits to the fields:

$$q \geq \sqrt{\frac{\alpha m}{m_F}}\frac{(\zeta k_B T)^{3/2}}{g_F\mu_B\hbar}, \quad c \geq \frac{\alpha\zeta k_B T}{g_F\mu_B}. \quad (9)$$

With $\zeta T=1 \mu$K, $\alpha=10$, and ^{87}Rb atoms in the $|F, m_F\rangle = |2,2\rangle$ state, the required fields are $q>90$ T m^{-1}, which is easily achievable with microstructured permanent magnetic material, and $c>30 \mu$T. Note that c plays a similar role as the minimum field in a Ioffe-Pritchard magnetic trap, typically $\sim 100 \mu$T. Neglecting angular momenta, we estimate the lifetime in the trap by a simple integration of the Landau-Zener spin-flip probability over the radial velocity distribution. For the given numbers we find a negligible loss rate. Under our assumptions and contrary to the behavior in more common traps with confinement along two or three dimensions, the lifetime should strongly decrease for higher temperatures as all atomic trajectories necessarily cross the trap minimum twice per radial oscillation period. The estimated lifetime reaches 1 s for $T\approx 100 \mu$K, allowing for a final evaporative cooling stage in this trap.

The second requirement is that atomic collisions do not excite radial motion. This is ensured when the chemical potential $g_{2d}n_{2d} \approx (2\hbar)^{3/2}(\pi\omega_r/m)^{1/2}a_s N/A$ is lower than $\hbar\omega_r$, where a_s is the s-wave scattering length.

A unique feature of two-dimensional systems is the occurrence of superfluidity in a cold temperature phase without spontaneous symmetry breaking. The underlying microscopic mechanism of Berezinskii-Kosterlitz-Thouless type is currently under investigation in harmonically trapped cold atomic clouds [7]. A toroidal trapping geometry as discussed here allows one to approximate the homogeneous case more closely. While a true Bose-Einstein condensate in two dimensions is still possible in harmonically confined two-dimensional gases, this is no longer true if the potential is homogeneous in both dimensions [3]. In an infinite system, the Berezinskii-Kosterlitz-Thouless transition temperature is proportional to the atomic density: $T_{BKT}\approx(\pi\hbar^2/2mk_B)N/A$ [4,5]. Here we equate the superfluid density ρ_s and the total density ρ_t. The difference between ρ_s and ρ_t and recent measurements on the critical temperature in a harmonically trapped system are discussed in Ref. [34]. To achieve a high

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[1]This relation separates the setup from a TORT; a continuous transition between the trap types is impossible.

enough transition temperature at reasonable atom numbers, the toroidal surface $A \approx 4\pi^2 R r_0$ must be chosen as small as possible. The minimal radius r_0 depends on the static field gradient q and is given by Eq. (8). The radius R cannot be chosen arbitrarily small as the circular rf amplitude a is limited by its radial quadrupole component. We find that it is favorable to use lower static field gradients q. This requires larger r_0 ($\sim q^{-1/3}$), but allows for smaller R as it reduces the required rf amplitude a ($\sim q^{2/3}$). The limit to this strategy is given by the decreasing trap frequency ω_r. Defining the radial rf gradient $p = |a|/(\sqrt{2}R)$ and requiring $\hbar\omega_r \gtrsim \zeta k_B T_{BKT}$, we find the optimum static field gradient

$$q = \frac{\zeta N p}{16\pi\alpha\delta m_F}. \tag{10}$$

With $N = 10^5$ ^{87}Rb atoms in the $|F, m_F\rangle = |2, 2\rangle$ state, $\alpha = 10$, $\delta = \zeta = 5$, and $p = 1$ T m^{-1}, a transition temperature of $T_{BKT} \approx 215$ nK can be achieved at torus sizes of $R \approx 65$ μm and $r_0 \approx 1.6$ μm with $q \approx 100$ T/m and $\omega = 2\pi \times 1.1$ MHz. With these parameters, we estimate the chemical potential to be well below the trap frequency, $g_{2d}N/A < \hbar\omega_r$. For an accurate description, the finite size of the system must be taken into account. But since the poloidal circumference is still more than 40 times longer than the healing length of the gas $\xi = \hbar/(2mg_{2d}n_{2d})^{1/2} \approx 225$ nm, we expect the above parameters to be a good approximation of the crossover to superfluidity.

For this kind of geometry to be implemented it is essential to compensate gravity over the vertical extension of the torus, $2r_0$. The resulting potential variation over this scale must be small compared to the chemical potential. In the above example the gravitational potential difference amounts to $\Delta U_{gr} \approx h \times 7$ kHz, while the chemical potential is only $g_{2D}n_{2D} \approx h \times 1$ kHz. This problem can be solved in various ways.

It is straightforward to show that an rf field parallel to $\hat{\mathbf{e}}_1$, as produced by a straight central wire perpendicular to the plane, could be used to cancel the vertical potential difference. To implement such a solution, however, is technologically demanding. A simplified approach could be based on a short-circuited coaxial cable mounted vertically below the chip surface. A second solution is the use of weak electrostatic fields. We estimate that applying voltages on the order of ≈ 1 V to a micrometer-sized structure at a distance of ≈ 10 μm creates potential gradients comparable to gravitation [31]. Another solution is to reflect a far-detuned laser beam under grazing incidence off the metallic chip surface. The optical dipole force at the height of the torus compensates gravity already at moderate laser intensity. The grazing incidence is required for maximal linearity, because it creates a long-wavelength standing-wave pattern above the chip surface. By varying the incident angle, the optical lattice period can be chosen such that the torus's height z_0 is at the first or second inflection point of the dipole potential. A feasible example is to reflect a red-detuned laser ($\lambda = 1064$ nm) at a grazing angle of 40 mrad off the chip surface (13.3-μm lattice period, $z_0 = 10$ μm). An intensity of $I = 920$ mW/mm^2 suffices to compensate gravity with a negligible photon scat-

tering rate ($\approx 10^{-3}$ s^{-1}). The remaining nonlinear vertical potential variation amounts to $\Delta U \approx h \times 200$ Hz. Focusing the laser onto a size somewhat larger than the torus [full width at half maximum (FWHM) $w_0 \sim 700$ μm] ensures homogeneous illumination ($\Delta U_{dipole} < h \times 200$ Hz) and requires a laser power of $P \sim 500$ mW, which can be reduced using astigmatic focusing. Note that the given example is adapted to a trap below the chip. Compensation of gravity for a trap above the chip surface requires a blue-detuned laser or a smaller incidence angle to reach the first inflection point of the dipole potential.

It is possible to reduce the dimensionality further and create one-dimensional rings. In this case also the poloidal degree of freedom must be frozen, $\hbar\omega_\varphi \gg k_B T, g_{1d}n_{1d}$. At reasonable rf amplitudes of $a, b \approx 10^{-4}$ T, this can be achieved with the radius r_0 chosen in the micron range. It enables studies of superfluidity in one-dimensional gases [6].

In macroscopic ring geometries it has not yet been possible to create a complete ring-shaped condensate as potential corrugations were larger than the chemical potential of the atomic cloud [9]. Recently, the corrugations near microscopic permanent magnetic trapping structures have been studied in detail, showing similar magnitudes [32]. For microscopic traps progress in the fabrication process might be expected and the problem is less severe because larger chemical potentials can be reached with tighter confinement. In addition, the rf dressing of the potential reduces the corrugations in the effective potential experienced by the atoms, because atoms are trapped at positions of constant field modulus. A perturbing field therefore displaces the trap, but the potential minimum is determined by the coupling to the rf field. The coupling is altered by a tilt of the static field with respect to the polarization of the rf field. In our case, only the corrugating field component tangential to the toroidal direction ΔB_θ modifies the potential minimum. We find that for small perturbations the potential corrugation is given by $\Delta E \approx \mu c \left(\frac{\Delta B_\theta}{q r_0} \right) = \mu \Delta B_\theta / \delta$. Compared to the undressed case, the effect of corrugations is suppressed by a factor of δ.

V. CONTROLLING ATOMIC MOTION

As a second application we consider the possibilities for controlling atomic motion by slowly varying and weakly perturbing the rf fields. As the relative phase of a and b controls the poloidal angle φ_0 of the trap minima, a small frequency difference between the two components can be used to induce rotation of atoms around the quadrupole center. Such stirring in the poloidal direction can be used to excite vortex loops that lie inside the toroidal surface.

It is also possible to produce and rotate trap minima in the toroidal direction by additional fields. Split rings can even be controlled independently from each other. As an example, we discuss counterrotating, vertically split rings. After inducing such a splitting with $\mathbf{B}_1$, the rotations can be introduced using a weaker, linearly polarized rf field, parallel to the plane of the torus. Decomposed into circular components, such a field can be written in global coordinates as
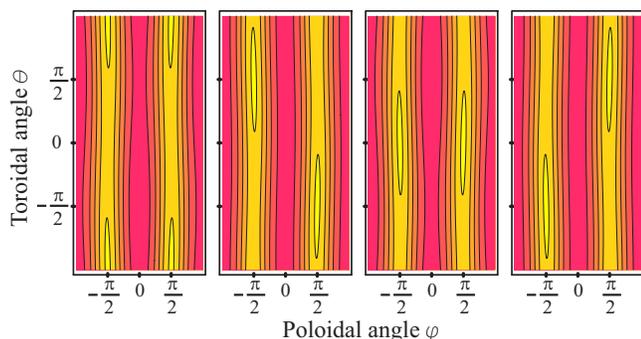
FIG. 4. (Color online) Contour plots of the trapping potential in unfolded θ, φ coordinates on the resonant torus for vertically split, counterrotating rings. Potentials are shown for $\arg u = \arg v = 0$, $\pi/2$, π, and $3\pi/2$ (increasing potential from light to dark).

$$\mathbf{B}_2 = \frac{u}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} + \frac{v}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}. \tag{11}$$

Using Eq. (3), the corresponding coupling element in local coordinates is given by

$$\beta_+ = \frac{u}{2}(1 + \sin \varphi)ie^{i\theta} - \frac{v}{2}(1 - \sin \varphi)ie^{-i\theta}. \tag{12}$$

Due to the direction of the static field $\mathbf{B}_{dc}$, these components do not couple at either the top or the bottom of the torus, depending on their sense of rotation. On the opposite side, they interfere with the cylindrical field $\mathbf{B}_1$, leading to maxima and minima along the toroidal angle θ. As the direction of the interference is opposite for u and v, a linearly polarized field ($|u| = |v|$) with a frequency different from that of $\mathbf{B}_1$ will induce equal but counterpropagating rotations of the two rings. The resulting potentials are shown in Fig. 4. It is interesting to mention that this configuration can be used for state-dependent control of atomic motion [23], because the rotational sense of the coupling field depends on the sign

of the Landé factor g_F. Due to the symmetry, the field $\mathbf{B}_1$ creates a state-independent potential for states with the same modulus $|g_F|$, while the circular components of field $\mathbf{B}_2$ swap roles for opposite signs of g_F. We note that the field $\mathbf{B}_2$ can also be used to cancel potential differences due to a tilt in the gravitational field.

VI. CONCLUSION AND OUTLOOK

The possibility to split the torus into two rings with a dynamically tunable barrier enables tunneling (Josephson oscillations) and superconducting-quantum-interference-device- (SQUID-) type experiments. Several rf frequencies [19] can be used to create independently controlled, concentric atom shells, providing a tool for interferometric analysis and for the creation of exotic lattices.

In conclusion, we have presented a class of rf dressed magnetic traps that offer great flexibility and a high degree of dynamical control. This versatility arises from the combination of a static ring-shaped quadrupole field with rf fields of suitable polarizations. It enables studies of low-dimensional gases with periodic boundary conditions, creation of vortex loops, persistent currents, and solitons. Coherent splitting of matter waves [17] together with independent control of atomic motion in split rings, as well as state-dependent control of atomic motion, makes this trap type particularly interesting for rotation sensing [33].

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