The Hierarchical Bayesian Inference Model

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When do we need the hierarchical Bayesian inference model?

→ Hierarchical data set

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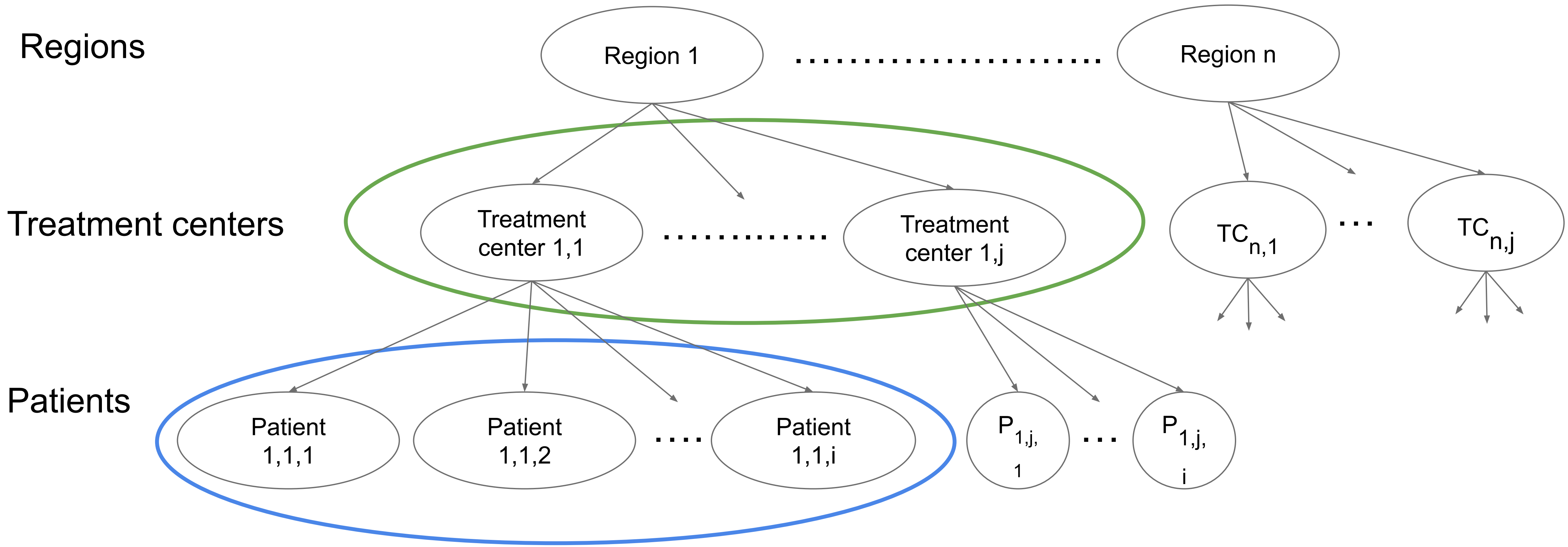
CLINICAL TRIAL

Patients clustered within treatment centers nested within regions



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Parameter of interest:
Survival probability $\theta_{n,j}$

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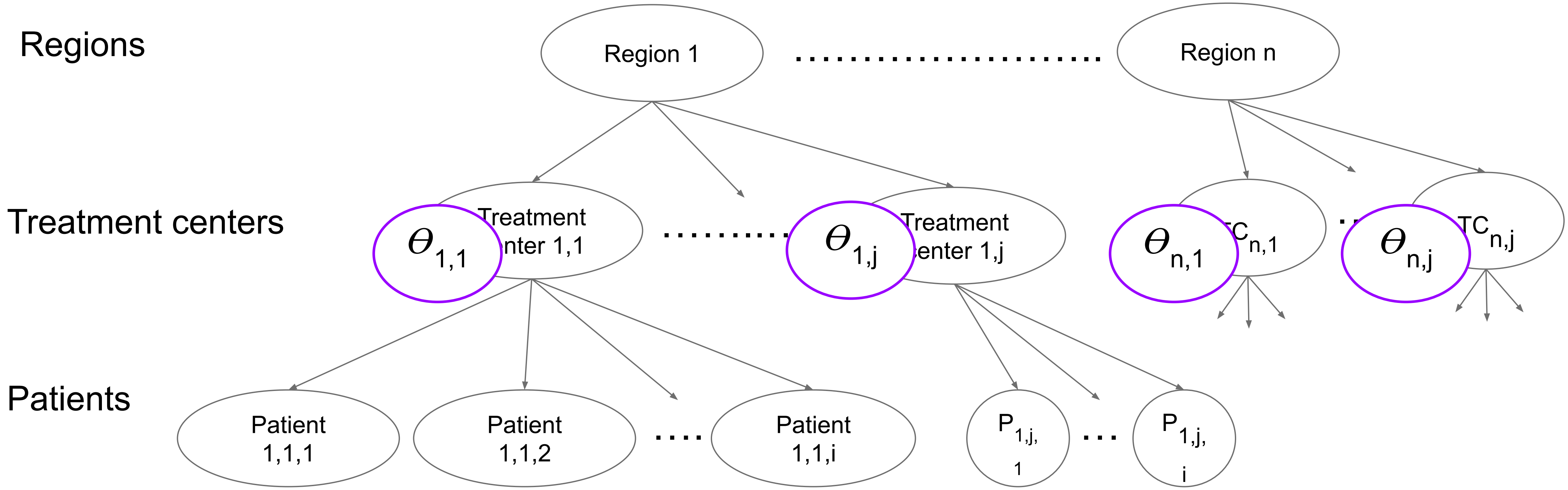

How does the HBI model differ from 'ordinary' Bayesian statistics?

'Ordinary' Bayes

$$\underbrace{P(\theta|y)}_{\text{Posterior}} \propto \underbrace{P(y|\theta)}_{\text{Likelihood}}\underbrace{P(\theta)}_{\text{Prior}}$$

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HBI model

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$$\underbrace{P(\phi, \theta|y)}_{\text{Posterior}} \propto \underbrace{P(y|\theta, \phi)}_{\text{Likelihood}}\underbrace{P(\theta|\phi)}_{\text{Prior}}\underbrace{P(\phi)}_{\text{Hyperprior}}$$

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Testing the HBI model

→ Creating a hierarchical data set

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- 75 groups ($j \in \{1,75\}$)
- 5 observations ($i \in \{1,5\}$) in each group

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Data

$$y_{ij}|\alpha_j, \beta_j \sim N(\alpha_j + \beta_j x_{ij}, \sigma^2)$$

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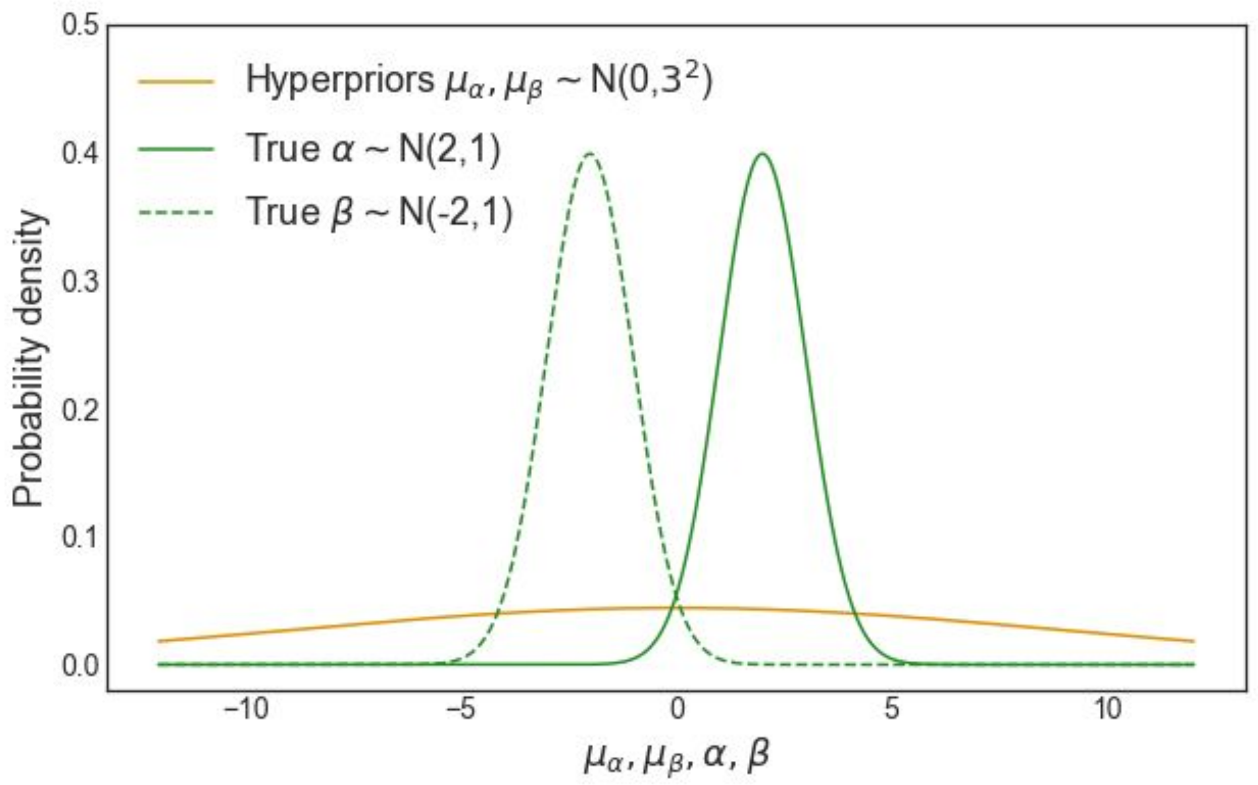

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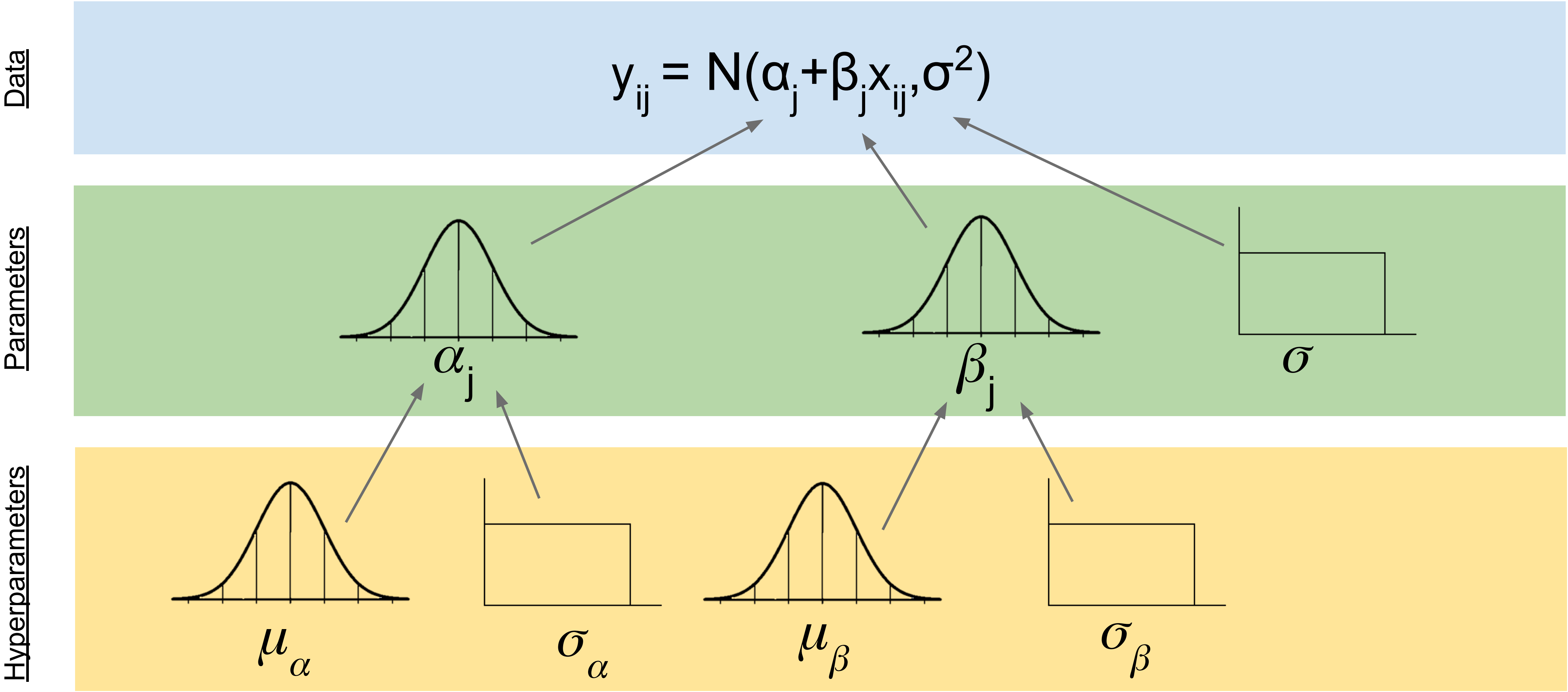

Comparing the HBI model to other models

→ Overview of alternative models

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1) Completely pooling

↓

Assume that α and □ are equal for all groups.

↓

Corresponds to running a single regression on the whole data set

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2) No pooling

↓

Assign each α and their own wide, flat prior.

↓

Corresponds to running a single regression on each group.

Comparing the HBI model to other models

→ Markov Chain Metropolis-Hastings sampler: Obtaining estimates of α and □

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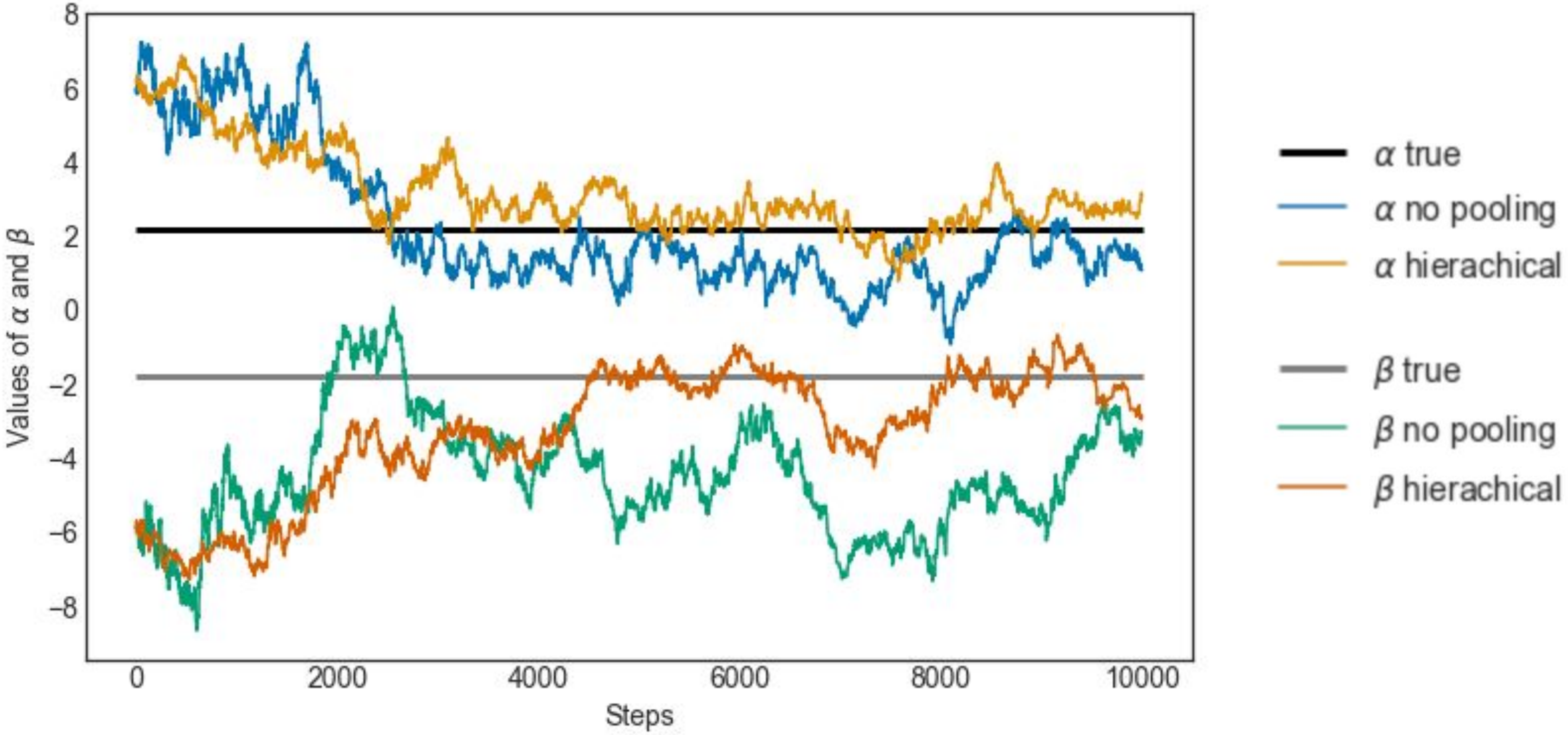
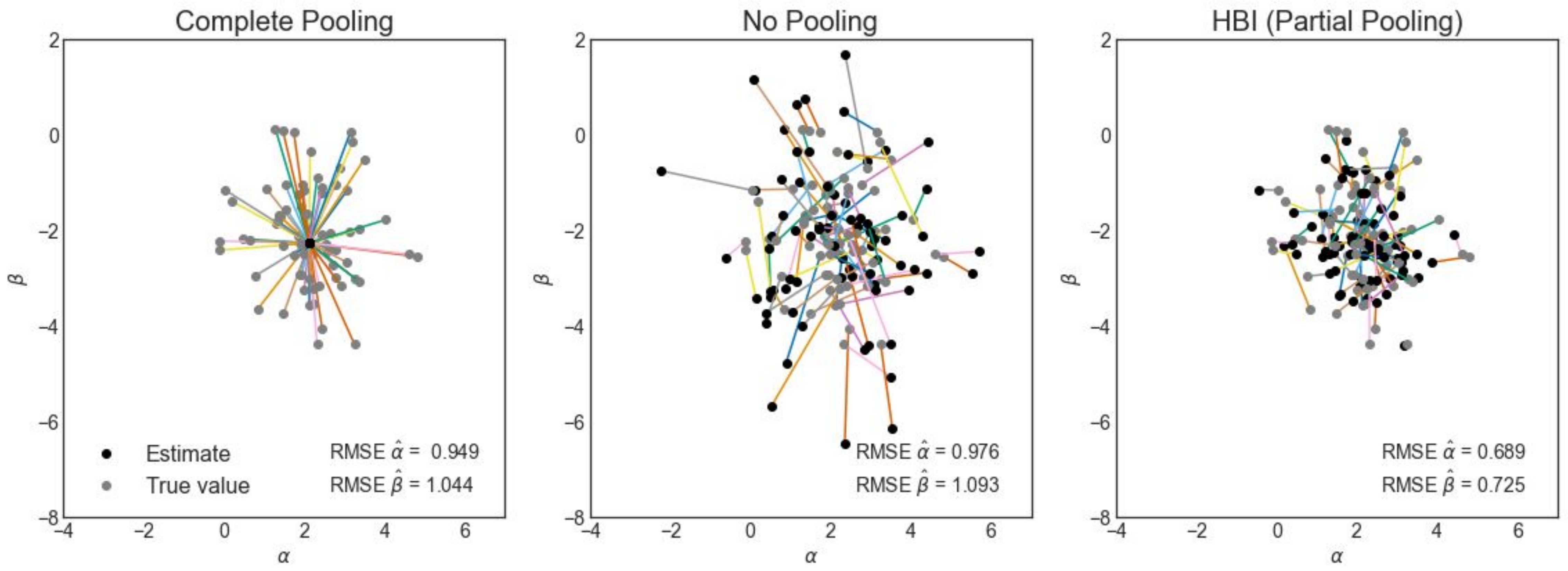


Comparing the HBI model to other models

→ Comparing estimates of α and □ to the true values



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→ Lets the data shape the prior itself by introducing the hyperprior

Summarizing: Hierarchical Bayesian Inference Model

*The HBI model provides a flexible framework for statistical modeling that can capture **variability across groups** and **improve the accuracy and precision of the parameter estimates***

- → Allows sharing of statistical strengths between the groups of data by assuming parameters come from common distributions
- → The prior is affected by the data itself by introducing the hyperprior
- → Estimates are less sensitive to noise as the prior structure pulls the estimates towards the population distribution (shrinkage)

Thank you for you attention :)