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Hidden Markov Models

The smoothing and Baum-Welch methods

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Journal article

A Hidden Markov Model of atomic quantum jump dynamics in an optically probed cavity

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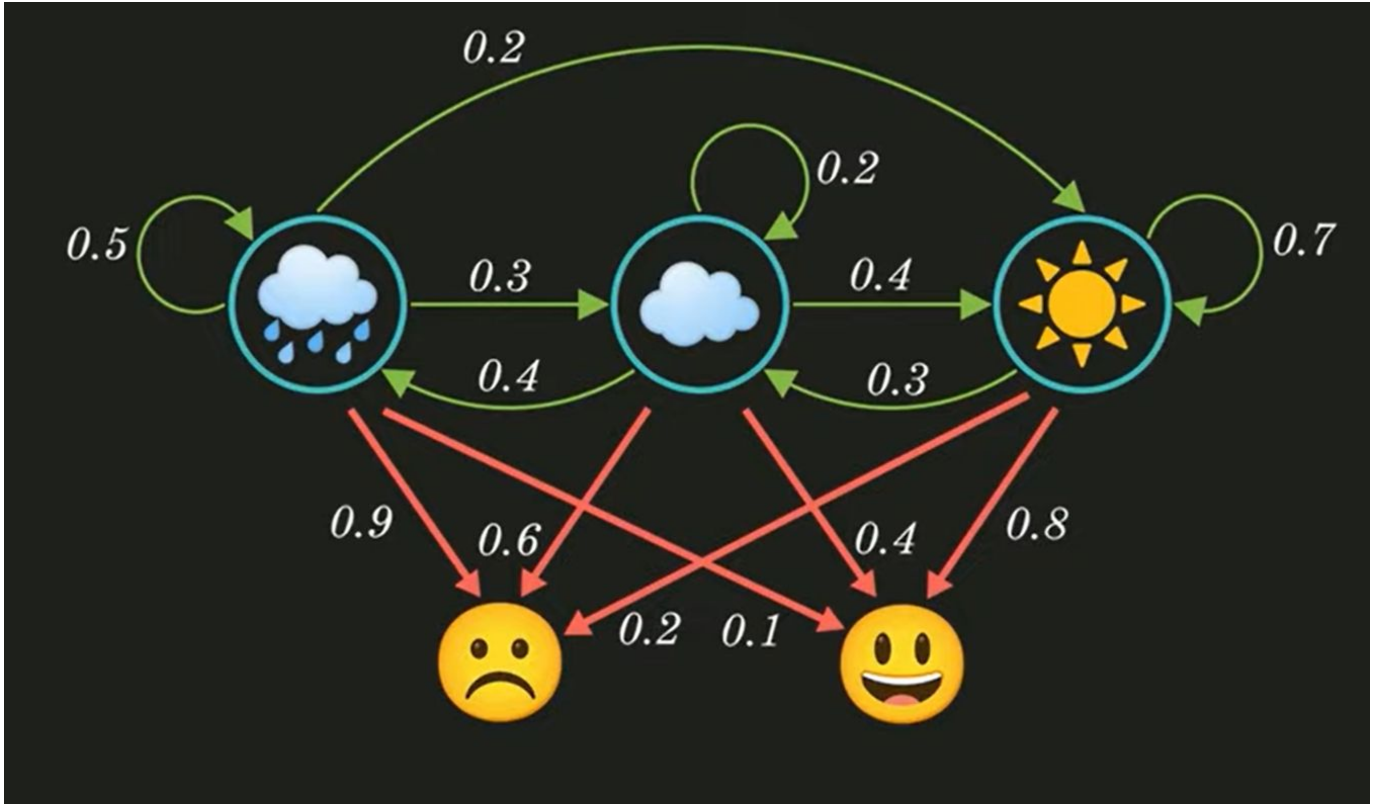
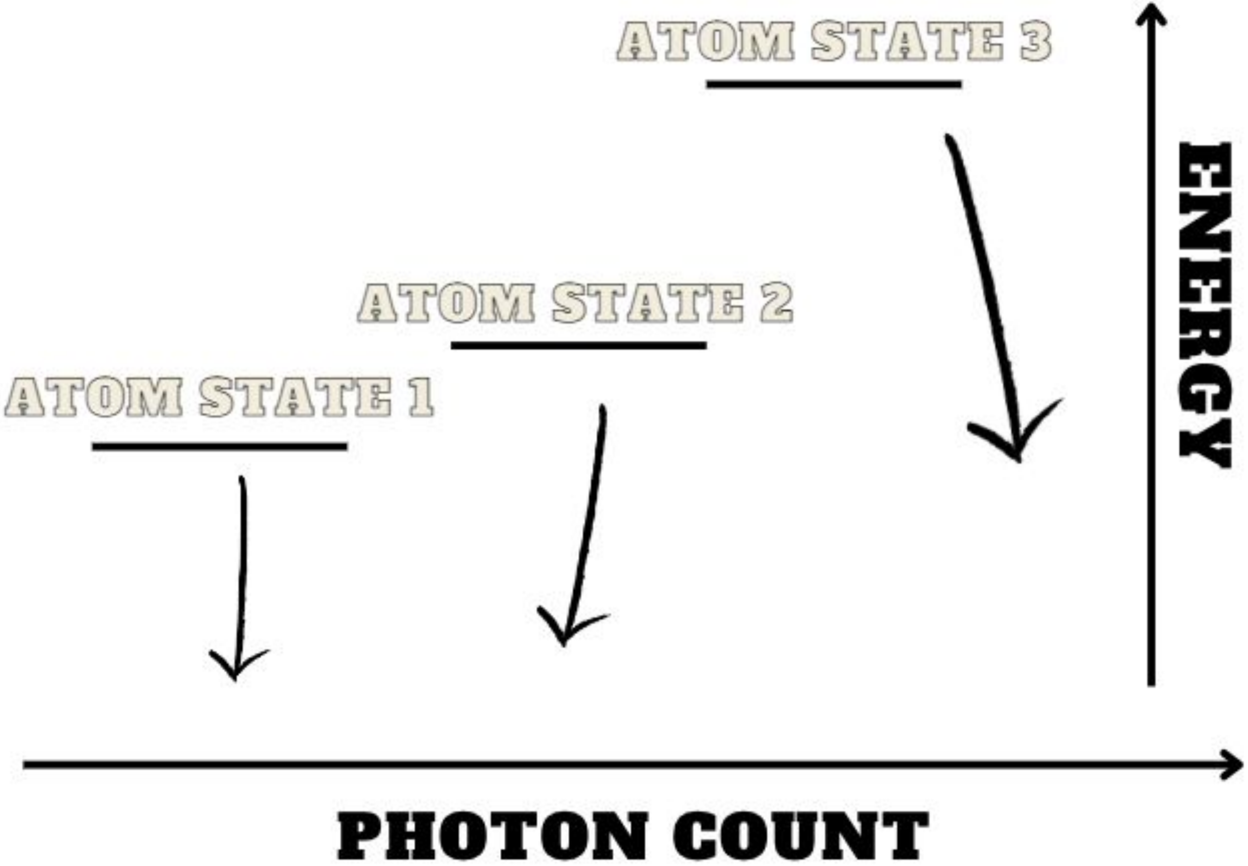
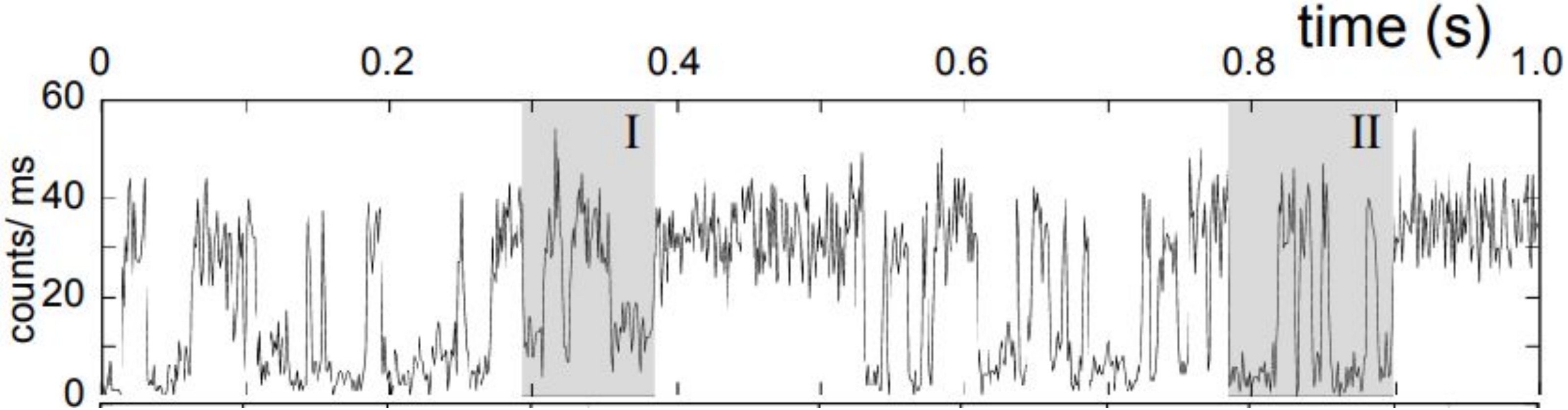
We analyze the quantum jumps of an atom interacting with a cavity field. The strong atom-field interaction makes the cavity transmission depend on the time dependent atomic state, and we present a Hidden Markov Model description of the atomic state dynamics which is conditioned in a Bayesian manner on the detected signal. We suggest that small variations in the observed signal may be due to spatial motion of the atom within the cavity, and we represent the atomic system by a number of hidden states to account for both the small variations and the internal state jump dynamics. In our theory, the atomic state is determined in a Bayesian manner from the measurement data, and we present an iterative protocol, which determines both the atomic state and the model parameters. As a new element in the treatment of observed quantum systems, we employ a Bayesian approach that conditions the atomic state at time t on the data acquired both before and after t and we show that the state assignment by this approach is more decisive than the usual conditional quantum states, based on only earlier measurement data.

Hidden Markov Models




ATOM STATE 3
ATOM STATE 2
ATOM STATE 1
ENERGY
PHOTON COUNT

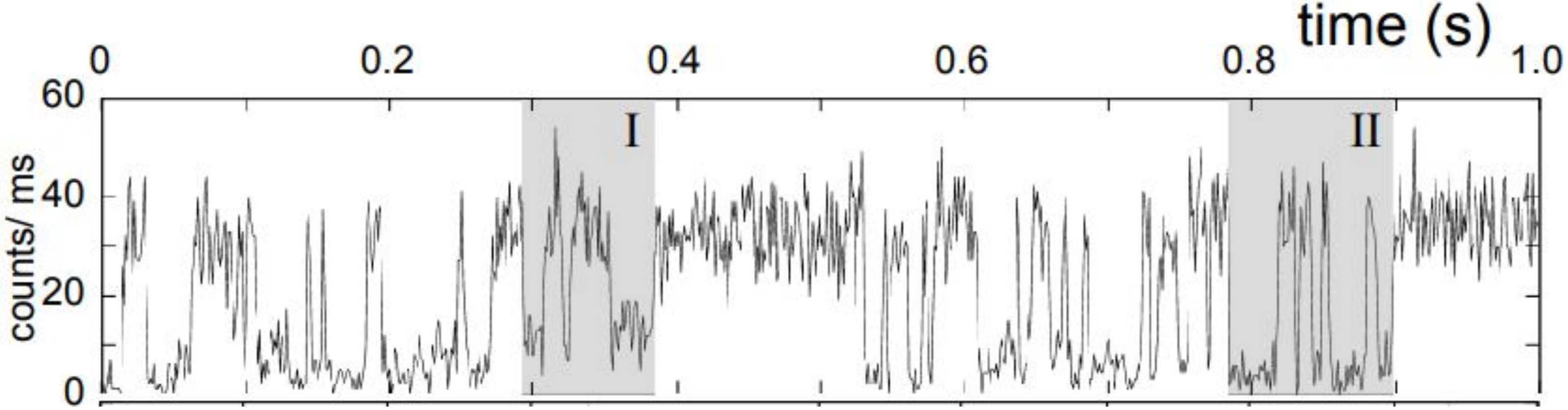
Smoothing



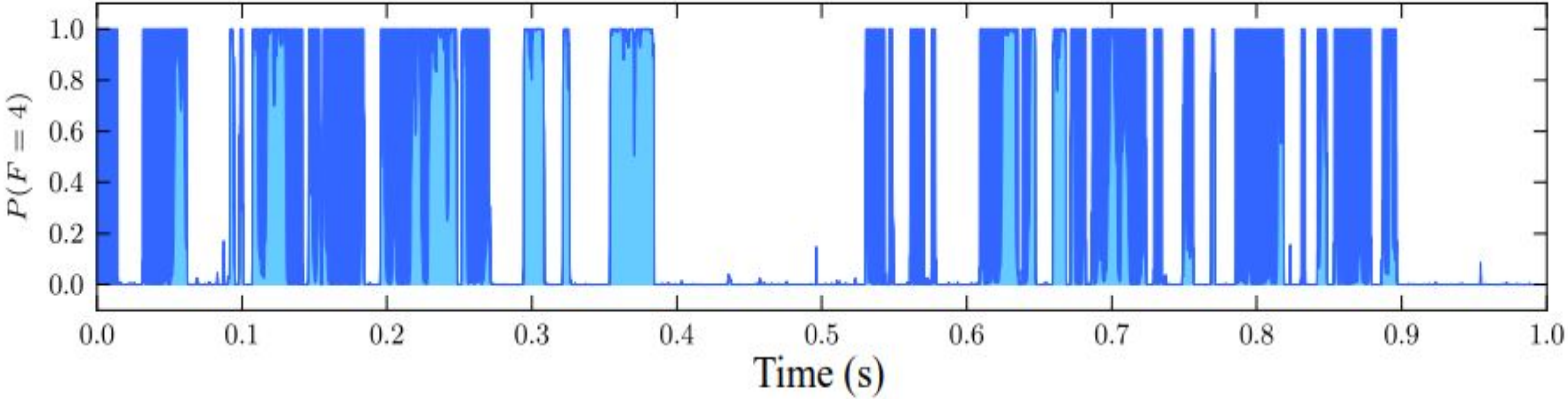
- Given transition and signal probabilities
- Goal: find the posterior probability of being in state i at time t given a set of observations:

$$P(X_t = i | s_1, ..., s_N)$$

Data


time (s)
0
0.2
0.4
0.6
0.8
1.0
60
40
20
0
counts/ ms
I
II

Result


1.0
0.8
0.6
0.4
0.2
0.0
$P(F = 4)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Time (s)

Baum-Welch algorithm

- Goal: Given a number of states and a series of observations, find the optimal transition probabilities and signal probabilities for an HMM
- Start with random model parameters and define:

$$\gamma_t(i, j) \equiv P\left(X_t = i, X_{t+1} = j \mid s_1, \ldots s_N\right)$$

- Using the chain rule: $P(X = i, X_{+1} = j) = P(X_{+1} = j|X = i)P(X = i)$

$$P_{est}(X_{+1} = j|X = i) = \frac{\sum_t \gamma_t(i, j)}{\sum_t(\sum_j \gamma_t(i, j))}$$

Baum-Welch algorithm

- Goal: Given a number of states and a series of observations, find the optimal transition probabilities and signal probabilities for an HMM

- Using chain rule $P(X = i, X_{+1} = j) = P(X_{+1} = j | X = i) P(X = i)$

- New guess at transition probabilities:

$$P_{est}(X_{+1} = j | X = i) = \frac{\#\text{Times model is in } i \text{ and then } j}{\#\text{Times model is in } i}$$

Baum-Welch algorithm

- Signal probabilities is re-estimated by probabilistically counting the number of times the observation *s* occurs in state *i*:

$$P(s|X = i) = \frac{\sum_t P(X_t = i|s_1, \dots s_N)\delta(s - s_t)}{\sum_t P(X_t = i|s_1, \dots s_N)}.$$

- Baum-Welch algorithm is guaranteed to converge to a local maximum.

Baum-Welch algorithm

- Signal probabilities is re-estimated by probabilistically counting the number of times the observation *s* occurs in state *i*:

$$P_{est}(s|X=i) = \frac{\#\text{Times model is in } i \text{ and } s \text{ is observed}}{\#\text{Times model is in } i}$$

- Baum-Welch algorithm is guaranteed to converge locally

Conclusion

- In a Hidden Markov Model we model the system state development from an observed signal
- The Smoothing method calculates a posterior distribution from the full signal.
- The Baum-Welch algorithm estimates state transition and signal probabilities.
- In this example the Smoothing and Baum-Welch algorithms were used to find a posterior distribution of the states of an atom in time.