Pressure discontinuity

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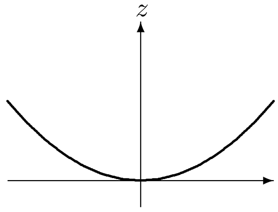
The paragraph named **Pressure** in subsection **Boundary conditions** on p. 236 should be replaced by the following (inserted below the paragraph named **Velocity**).



Local coordinate system at a particular point of the interface.

Pressure: Newton's third law requires the stress vector $\boldsymbol{\sigma} \cdot \boldsymbol{n}$ to be continuous across any physical interface between two materials (in the absence of surface tension). On page 117 it was remarked that this requirement will not in general guarantee the continuity of pressure, and on page 161 an counterexample from elasticity theory was given. As will now be shown, it is the difference in viscosity between moving incompressible fluids that generates a pressure jump across their interface.

> To demonstrate the existence of the discontinuity in a given point of the interface, we choose a local coordinate system with origin in this point and the z-axis along the normal to the interface. As we approach the interface along the normal from either side by letting $z \to \pm 0$, the no-slip condition guarantees the continuity of the all the components of the velocity field. The derivatives $\nabla_x v_x$ and $\nabla_y v_y$ which do not affect z are consequently also continuous at the interface. For incompressible fluids the divergence condition $\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$ furthermore implies that $\nabla_z v_z = -\nabla_x v_x - \nabla_y v_y$ on both sides of the interface, and thus we conclude that $\nabla_z v_z$ must also be continuous, and in general non-vanishing. Newton's third law secures that the normal stress $\sigma_{zz} = -p + 2\eta \nabla_z v_z$ is always continuous across the interface, in spite of the jump in viscosity. Consequently, the pressure p must be discontinuous at the interface to compensate for the jump in viscosity, in fact $\Delta p = 2\Delta\eta \, \nabla_z v_z$. An important exception is the interface between a solid at rest and an incompressible fluid. Here the velocity field has to vanish at the interface and from the divergence condition we get $\nabla_z v_z = 0$ such that the normal stress in this case is given by the pressure alone independently of the viscosity.