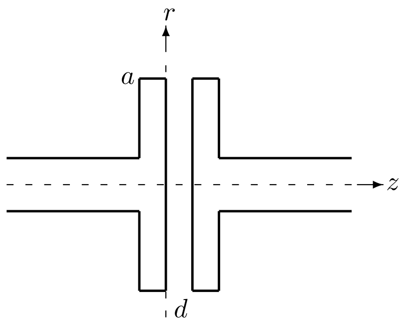
Extra problems for chapter 19

December 18, 2004

Problems



19.11 A simple hydraulic clutch connecting two rotating shafts consists of two circular plates of radius a and variable distance d bathed in an incompressible oil of mass density ρ_0 and viscosity η. One shaft can exert a turning moment on the other through viscous forces, and in the limit of $d \to 0$ the moment becomes so large that two shafts are solidly connected. For simplicity we consider the only case where one shaft is rotating with constant angular velocity Ω while the other is fixed. Edge effects may be disregarded everywhere. For numeric calculations one may use $a = 10$ cm, $\rho_0 = 0.8$ g/cm^3, $\eta = 0.5$ Pas, and $\Omega = 100$ rpm $= 100 \cdot 2\pi/60$ s^{-1}.

(a) Find the condition for creeping flow between the plates. What limitation does this condition set on d.

(b) Show that the fields (either in cylindrical or Cartesian coordinates)

$$\boldsymbol{v} = r\Omega\frac{z}{d}\boldsymbol{e}_\phi = (-y, x, 0)z\frac{\Omega}{d} \qquad\qquad p = 0 \tag{19.39}$$

satisfy the Navier-Stokes equations for creeping flow in the gap (disregarding gravity) as well as the boundary conditions on the plates.

(c) Calculate the vorticity field and the stress tensor in the gap.

(d) Calculate the couple (turning moment of force) on the fixed shaft. How small must the distance between the plates be for the couple to be 100 Nm? Determine the dissipated power deposited in the oil.

Answers

19.11 **(a)** The velocity at the rim of the plates is $U = a\Omega$ so that the Reynolds number becomes

$$\mathsf{Re} = \frac{Ud}{\nu} = \frac{\rho_0 \Omega a d}{\eta} \ . \tag{19.40}$$

The condition $\mathsf{Re} \ll 1$ translates into

$$d \ll \frac{\eta}{\rho_0 \Omega a} \approx 0.6 \ \mathsf{mm} \tag{19.41}$$

(b) In Cartesian coordinates it is immediately clear that $\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0$ and $\boldsymbol{\nabla}^2\boldsymbol{v} = \boldsymbol{0}$, such that the creeping flow equations (19.1) are fulfilled. The no-slip conditions are also trivially fulfilled for $z = 0$ and $z = d$, because the fluid rotates as a solid for any given z.

(c) The vorticity field becomes in Cartesian

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v} = (-x, -y, 2z)\frac{\Omega}{d} = (-r\boldsymbol{e}_r + 2z\boldsymbol{e}_z)\frac{\Omega}{d} \tag{19.42}$$

The stress tensor becomes in Cartesian

$$\boldsymbol{\sigma} = \eta\frac{\Omega}{d}\begin{pmatrix} 0 & 0 & -y \\ 0 & 0 & x \\ -y & x & 0 \end{pmatrix} = (\boldsymbol{e}_z\boldsymbol{e}_\phi + \boldsymbol{e}_\phi\boldsymbol{e}_z)\eta\frac{\Omega}{d} \tag{19.43}$$

(d) The turning moment on the fixed plate is

$$\mathcal{M}_z = \int_0^a r\sigma_{\phi z}\, 2\pi r\, dr = \frac{\pi\eta\Omega a^4}{2d} \ . \tag{19.44}$$

Solving for the distance one gets $d \approx 8\ \mu\mathsf{m}$. The dissipated power is $P = \mathcal{M}_z\Omega \approx 1\ \mathsf{kW}$.