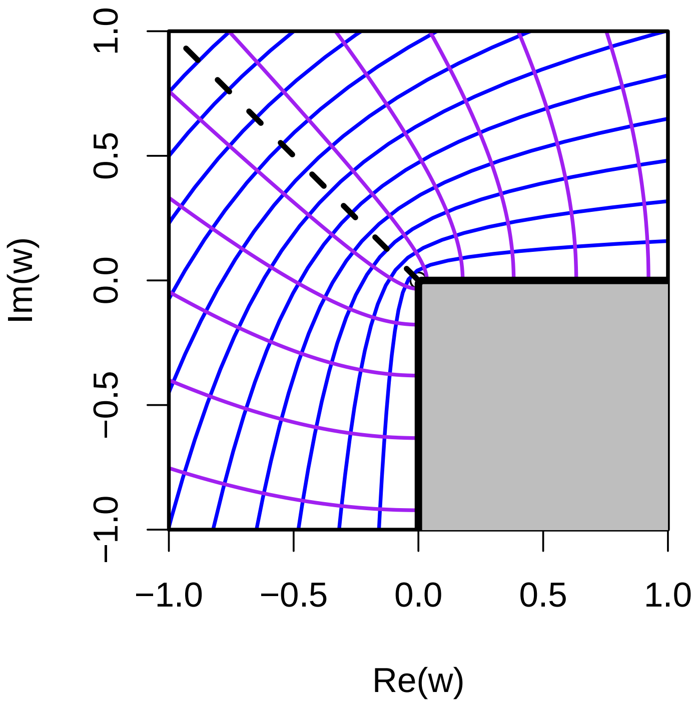
Exercises Week 10

Exercises from the book

12.6, 13.5, 13.9, 13.10, 20.1

Optional or if time permits: Potential flow around a corner



The real and imaginary parts of a holomorphic (analytic) function in the complex plane, $f = \psi + i\phi$, are always harmonic, that is they satisfy the Laplace equation

$$\nabla^2\psi = 0 \quad \text{and} \quad \nabla^2\phi = 0.$$

In terms of complex variables, we can write the in-plane Laplace equation on the form

$$0 = \nabla^2\psi = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi = 4\frac{\partial^2\psi}{\partial\bar{z}\partial z},$$

where $z = x + iy$.

In a potential flow, the velocity of the fluid is given by the gradient of a harmonic function (potential) ψ, that is

$$\mathbf{v} = \begin{pmatrix} \partial_x\psi \\ \partial_y\psi \end{pmatrix}$$

In complex variables the complex velocity v is given by the derivative of the function f

$$v = 2\frac{\partial \psi}{\partial \bar{z}} = \frac{\partial \psi}{\partial x} + i\frac{\partial \psi}{\partial y}$$

We shall now use the potential for a uniform flow with velocity k in the x-direction

$$\psi = k\text{Re}[z] \tag{1}$$

to find the flow around a corner.

- First you will have to find the conformal mapping from the domain consisting of the points $r\exp(i\theta)$ with $r > 0$ and $0 \leq \theta \leq 3\pi/2$ shown in the figure to the upper half-plane. Try with mappings on the form $\Phi(w) = w^{\alpha}$.
- Use the conformal mapping together with Eq. (1) in order to find an expression for the flow potential and the fluid velocity around the corner. Use the fact that $\psi(\Phi(w))$ is harmonic.
- Calculate the pressure along the line segment $r\exp(i3\pi/4)$ where $r > 0$ (the dashed line in the figure).