Hurricanes and butterflies

Thomas C. Halsey and Mogens H. Jensen

Chaotic systems can be characterized by the swirling patterns of 'strange attractors'. A powerful method to determine their behaviour has been validated for the most famous case, the Lorenz attractor.

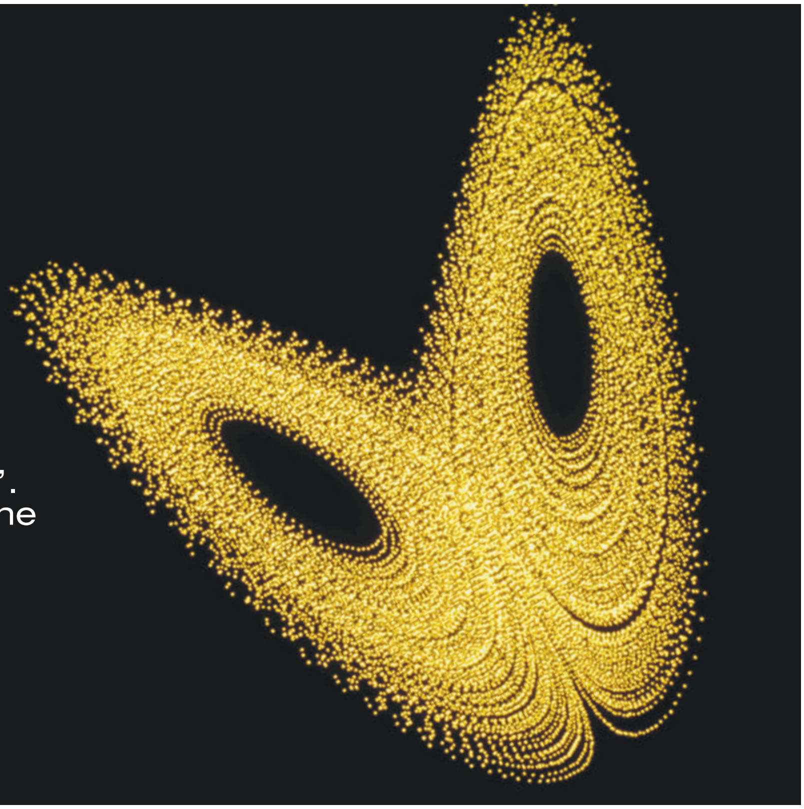

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Figure 1 **The Lorenz attractor, tracing the phase-space trajectory for a simple model for atmospheric convection. The trajectory is quite densely bunched in some regions and is quite sparse in others, which makes this 'strange attractor' multifractal.**

Chaotic systems are famous for their sensitive dependence on initial conditions. Small changes in the original variables describing a system, induced perhaps by the flapping wings of a butterfly, can result in large changes in the outcomes — such as the magnitude of property damage in Florida, in the wake of a hurricane. History only happens once, so it would seem impossible to determine *a posteriori* that the path of a particular hurricane could have been altered if only the butterflies had been more cooperative.

However, writing in *Physical Review Letters*, Sam Gratrix and John N. Elgin[1] provide grounds for optimism: they have developed a powerful new method to determine from experimental observation of a system whether it is chaotic, and, if it is, what the precise quantitative nature of that chaos is. Their method is based on fractal geometry. Fractals are structures or curves that remain rough or heterogeneous on all length scales and are characterized by their 'fractal dimension'. The coastline of England is such a curve, as pointed out by Benoît Mandelbrot, because the coastline can be regarded as rough at least down to the scale of the individual sand grains on the beaches.

Dynamical systems are generally analysed in terms of their behaviour not in real space but in the phase space spanned by the variables describing the system — for a hurricane, these include values of atmospheric pressure, humidity and velocity, on a grid sufficiently fine to determine the future course of the hurricane. Depending on the number of such variables, the phase space for a particular system will have a corresponding number of dimensions. As they evolve, however, chaotic systems settle on some structure of a lower dimension than their corresponding phase space, structures called 'strange attractors'. These are fractals, and perhaps the most famous strange attractor is the Lorenz attractor (Fig. 1), discovered in 1963 by Edward Lorenz[2] for a model of atmospheric convection (hurricanes again).

But strange attractors are not simple fractals like the coast of England. They are 'multifractals', whose quantitative properties vary from point to point in an intricate manner, with the result that they are characterized by a range of fractal dimensions. Imagine that the strange attractor in Figure 1 is covered with small boxes; then consider what percentage of its time the system finds itself in any particular box. The distribution of times for the boxes is extremely wide — from boxes that are almost never visited by the system dynamics, to boxes in which it spends a disproportionately large amount of time. So it seems elementary to determine whether or not a particular dynamical system is chaotic: simply reconstruct its trajectory through phase space, cover that trajectory with boxes, measure the amount of time spent in each box, and then determine whether or not the multifractal structure you have computed is consistent with chaos.

This box-counting method to diagnose multifractality has been applied to a wide variety of systems over the past 15 years. Many systems known on other grounds to be multifractal have had their multifractality confirmed in this way. Unfortunately, many systems known on other grounds not to be multifractal have also had their 'multifractality' confirmed in this way. Alas, box-counting proves little about multifractality, and much about the truth of a famous Mark Twain saying regarding statistics and lies.

Nevertheless, there is an alternative route to the determination of multifractal properties. Mathematicians know that the strange attractor can actually be constructed from the union of all periodic trajectories of a system, provided that trajectories of arbitrarily long periods are included (over a short observation time, these trajectories might not be obviously periodic, just as it takes at least 28 days' worth of observations to conclude that the Moon does, indeed, revolve around the Earth). Using an ingenious method to categorize these long trajectories, Gratrix and Elgin[1] have reconstructed in great detail both the Lorenz attractor and its multifractal properties.

Of course, in nature we rarely have the opportunity to observe a dynamical system for long enough to find enough trajectories to rebuild the strange attractor in such detail. However, the results from the periodic trajectory study can be compared with a much simpler approach, based on recurrence times. Recurrence times are simply defined:

consider a point on the attractor, and ask how long it will be before a trajectory starting at that point returns, not exactly to that point (as in the periodic trajectory calculation), but to within some certain distance of that point. Although a method was developed in the mid-1980s to find multifractal properties based on the properties of the recurrence times[3], this method had not been applied to strange attractors, and had not been verified against more rigorous methods (an important step, given the unfortunate history of box-counting).

These gaps have now been filled by Gratrix and Elgin[1], by developing the recurrence-time method for the Lorenz attractor and by verifying it against the periodic trajectory method. Because calculations based on recurrence times should be relatively straightforward for experimentalists, and as we now have reason to believe that they will be more reliable than box-counting results, we can confidently await a new series of experimental demonstrations of the chaotic properties of a variety of natural systems.

But will this solve the problem of the butterfly and the hurricane? The Lorenz attractor lives in a three-dimensional phase space; a hurricane lives in a phase space with an enormous number of dimensions. After several decades of work on chaos, we still do not understand the extent to which systems with such large numbers of degrees of freedom (typically turbulent systems) can be understood using the same concepts as for chaotic systems, which are relatively simple in comparison. So if a hurricane destroys your beach house, the verdict against the butterfly is: not proven. ■

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Human longevity

The grandmother effect

Kristen Hawkes

Why do women live long past the age of child-bearing? Contrary to common wisdom, this phenomenon is not new, and is not due to support for the elderly. Rather, grannies have a lot to offer their grandchildren.

Those who think postmenopausal women make little difference in the story of human populations will be surprised by the report of Lahdenperä and colleagues on page 178 of this issue[1]. The authors have unearthed firm evidence in support of the 'grandmother hypothesis', according to which a grandmother has a decidedly beneficial effect on the reproductive success of her children and the survival of her grandchildren.

The question of human longevity has deeper evolutionary importance than many think. It is often assumed that the steady increase in life expectancy over the past century and a half[2] has resulted in a larger proportion of older people than ever before. But, until the past few decades, increases in life expectancy reflected reductions in infant and juvenile mortality, and made little difference to the fraction of women past child-bearing age. As shown in Fig. 1, it is levels of fertility, not life expectancy (mortality), that shift the proportion of elders in a population. Even when life expectancy is well below 40 years, most girls who survive childhood live past their child-bearing years. In both historical and hunter-gatherer populations, a third or more of women are usually beyond the age of 45.

This large proportion of older people has fundamental implications for all human social organizations. Its unusual character is highlighted by comparisons with other primates. For example, among our closest living relatives, chimpanzees, female fertility declines at about the same age as in people, from a peak before age 30 to virtually zero at age 45 (ref. 3). But chimpanzee survival rates fall along with fertility, so that in the wild less than 3% of the adults are over 45 (ref. 4).

We might assume that the large fraction of elders in human populations reflects a characteristically human social safety net. But natural selection generally favours the flow of help from older to younger kin (Fig. 2), so we should be sceptical that a species-wide pattern of care for older people explains human longevity[5]. Developments in evolutionary life-history theory suggest that, instead of help *for* older members of the population, it is help *from* postmenopausal grandmothers that accounts for the age structures of human societies.

Mammalian life histories fall along a fast–slow continuum[6]. At one end, maturation is quick, fertility is high and adults die young. At the other, maturity is delayed, reproduction is slow, and adults usually live long enough to grow old[7]. The most successful model for explaining this cross-species variation[8] shows that adult lifespans can determine the other life-history traits. The relationship between average adult lifespan and average age at maturity is much the same across the living primates, including humans[9]. Chimpanzees are at the slow

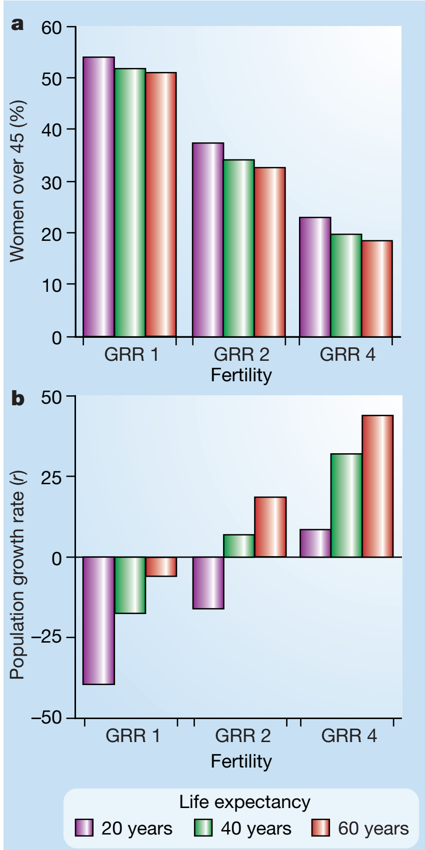

Figure 1 **Population profiles for women — females over 15 years of age — for different variables.** a, **The proportion of women over 45 for different life expectancies (20, 40 and 60 years) and fertility levels (expressed as gross reproduction rate, GRR, which is equivalent to the average number of daughters for women who survive the fertile years). When life expectancies are the same, higher fertilities make younger cohorts larger and the proportion of elders smaller (compare all the purple bars, all the green bars, all the orange bars). When fertility levels are the same (each cluster of bars), the fraction of elders varies little even across a tripling of life expectancies. Life expectancies vary widely with differing levels of infant and juvenile mortality. The point to stress is that, at the same fertility level, populations with shorter life expectancies do not have fewer elders: in fact, they have slightly more women past fertile ages.** b, **Population growth rate (r = annual rate of growth/1,000) accompanying each combination of fertility and life expectancy. The very high proportions ($>$50%) of women past 45 occur in sharply declining populations, and the very small proportions ($<$20%) in swiftly growing populations. Growth rates that differ from zero cannot be sustained for long. (Data from ref. 17.)**