Compact Objects and Relativistic Astrophysics Course
Lecture on ‘Accretion Disks’

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Why Are Accretion Disks Important?

Crucial for understanding
- How stars and planets form
- What powers the brightest X-ray sources in the sky
- Why Active Galactic Nuclei (Quasars) shine
- How to use flow dynamics to map black hole space-time

Release of gravitational energy in accretion disks responsible for some of the most powerful phenomena in nature!
Imagine a cloud of gas collapsing due to its own gravity

* To a first approximation the gas is falling into a central potential
* Angular momentum is mostly conserved
* Gas can cool down faster than it can get rid of angular momentum
* Flattened, rotating structure (AKA disk!) forms...
### Typical Masses, Sizes, and Luminosities


#### Table 1.1 Central Objects in Various Objects.

<table>
<thead>
<tr>
<th>Object</th>
<th>Central “star”</th>
<th>Mass</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>YSO</td>
<td>PS/TTS</td>
<td>$\sim M_\odot$</td>
<td>$\sim R_\odot$</td>
</tr>
<tr>
<td>CV</td>
<td>WD</td>
<td>$\sim M_\odot$</td>
<td>$\sim 10^{-2} R_\odot$</td>
</tr>
<tr>
<td>LMXB</td>
<td>NS</td>
<td>$\sim M_\odot$</td>
<td>$\sim 10$ km</td>
</tr>
<tr>
<td></td>
<td>BH</td>
<td>$\gtrsim 3M_\odot$</td>
<td>$(r_g \gtrsim 10$ km)</td>
</tr>
<tr>
<td>AGN</td>
<td>SMBH</td>
<td>$\sim 10^{6-9} M_\odot$</td>
<td>$(r_g \sim 0.02-20$ AU)</td>
</tr>
</tbody>
</table>

#### Table 1.2 Accretion Disks in Various Objects.

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass</th>
<th>Size</th>
<th>Temperature</th>
<th>Luminosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>YSO</td>
<td>$\sim M_\odot$</td>
<td>$\sim 100$ AU</td>
<td>$\sim 10^{1-4}$ K</td>
<td>$\sim L_\odot$</td>
</tr>
<tr>
<td>CV</td>
<td>$\ll M_\odot$</td>
<td>$\sim R_\odot$</td>
<td>$\sim 10^{4-5}$ K</td>
<td>$\sim 10^{0-2} L_\odot$</td>
</tr>
<tr>
<td>LMXB</td>
<td>$\ll M_\odot$</td>
<td>$\sim R_\odot$</td>
<td>$\sim 10^{4-9}$ K</td>
<td>$\sim 10^{0-5} L_\odot$</td>
</tr>
<tr>
<td>AGN</td>
<td>$\lesssim 10^6 M_\odot$</td>
<td>$\sim 1$ pc</td>
<td>$\sim 10^5$ K</td>
<td>$\sim 10^{10-13} L_\odot$</td>
</tr>
</tbody>
</table>
Overview

* Observational Evidence

* Basics of Accretion Disk Physics

* Modern Accretion Disk Theory & MHD Turbulence

* Numerical Simulations
Observational Evidence
Protoplanetary Disks

HH 30
DG Tau B
Haro 6-5B
HK Tau

Disks around Young Stars
PRC99-05b • STScI OPO
C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

HST • WFPC2

from "Black Hole Accretion Disks", Kato, Fukue & Mineshige, 1998
Figure 1

Scale drawings of 16 black-hole binaries in the Milky Way (courtesy of J. Orosz). The Sun–Mercury distance (0.4 AU) is shown at the top. The estimated binary inclination is indicated by the tilt of the accretion disk. The color of the companion star roughly indicates its surface temperature.
AGN Disks

Core of Galaxy NGC 4261

Hubble Space Telescope
Wide Field / Planetary Camera

Ground-Based Optical/Radio Image

HST Image of a Gas and Dust Disk

380 Arc Seconds
88,000 LIGHTYEARS

1.7 Arc Seconds
400 LIGHTYEARS
Evidence for Disks

From ‘Accretion Power in Astrophysics’; Frank, King, & Raine, 1995
Evidence for Disks

From ‘Accretion Power in Astrophysics’; Frank, King, & Raine, 1995
Evidence for Disks

From ‘Accretion Power in Astrophysics’; Frank, King, & Raine, 1995
Black Hole States

Extreme changes in luminosity, spectra, & variability !!!
Variability at All Mass Scales!

Axelsson et al. 2006

McHardy et al. 2007
Relativistic Iron Lines

Broad iron lines in AGN (Fabian et al.)

Detailed modeling of line profile allows us to ‘map’ the space-time around black holes.
Basics of Accretion Disk Physics
Why Are Accretion Disk so Hard to Understand?

Gravity balanced by pressure gradient along $R$
Energy flows along $R$ too!

Magnetic fields do not seem to influence stellar structure significantly

Mostly thermal energy

Mass $\rho$ $R$
Momentum $\rho \Omega$ $\phi$
Energy $\rho \Omega^2$ $z$

Magnetic fields are essential for accretion disks to work

Non-thermal processes
Some Dynamical Considerations

If the disk were a collection of non-interacting particles there would be no accretion.

Particles in a central potential move in stable Keplerian orbits.

\[ v_K = \sqrt{\frac{GM}{R}} \]

\[ \Omega_K = \sqrt{\frac{GM}{R^3}} \]
Keplerian Disks 101

\[ \Omega(R) \]

\[ \Omega^2 = \frac{GM}{R^3} \]

\[ \Omega \sim R^{-3/2} \]

Matter
Importance of Angular Momentum Transport

Gas in the disk must lose angular momentum!!!!

\[ l(R) = v_\phi R = (\Omega R)R = \Omega R^2 \]

\[ l \sim R^{1/2} \]
Importance of Stress...

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  
Mass conservation

\[ \frac{\partial l}{\partial t} + \nabla \cdot (l \mathbf{v}) \neq 0 \]  
Angular momentum is not conserved....

\[ \frac{\partial l}{\partial t} + \nabla \cdot (l \mathbf{v}) = -\frac{1}{r} \frac{\partial}{\partial r} \left( r^2 \bar{T}_r \phi \right) \]

If there is no stress, angular momentum for fluid elements in the disk is conserved and matter does not accrete!
Solving the Angular Momentum Problem. I

Problem I: Transport of Angular Momentum

\[ \bar{T}_{r\phi} = -r \sum \nu_{\text{mol}} \frac{d\Omega}{dr} \]

\[ \nu_{\text{mol}} = \lambda_{\text{mol}} c_s \]

\[ \lambda_{\text{mol}} \sim 10^{-2} \text{cm} \]

\[ c_s \sim 10^6 \text{ cm s}^{-1} \]

\[ t_{\text{mol}} \sim \frac{R^2}{\nu_{\text{mol}}} \sim 10^8 \text{ yr} \]

\[ t_{\text{obs}} \sim 1 \text{ week} !!! \]
Reynolds numbers in accretion disks are HUGE!!!

\[ Re \sim 10^{10} - 10^{16} \]

\[ Re = \frac{VL}{\nu} \]

Courtesy of CK Chan

Guadalupe Island vortex street movie from GOES
Solving the Angular Momentum Problem. I

Solution I: Assume some kind of “turbulent” viscosity

Eddies of size ‘H’ interacting with turnover velocity ‘$\alpha c_s$’

\[
\bar{T}_{r \phi} = -r \sum \nu_{\text{turb}} \frac{d\Omega}{dr}
\]

\[
\nu_{\text{turb}} = \lambda_{\text{turb}} \nu_{\text{turb}}
\]

\[
\nu_{\text{turb}} \equiv \alpha H c_s
\]

\[
\frac{\nu_{\text{turb}}}{\nu_{\text{mol}}} \sim \frac{\alpha H}{\lambda_{\text{mol}}} \sim 10^{10}
\]

\[
t_{\text{turb}} \sim t_{\text{obs}}
\]

With this enhanced stress we can match the fast timescales observed!
The Standard Accretion Disk Model

- The gravitational field is determined by the central object
- The disk is axisymmetric
- The disk is geometrically thin and optically thick
- Hydrostatic balance holds in the vertical direction
- There are no disk winds or external torques
The Standard Accretion Disk Model

Continuity equation
\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma \bar{v}_r) = 0
\]

Momentum conservation in the radial direction
\[
\frac{\partial \bar{v}_r}{\partial t} + \bar{v}_r \frac{\partial \bar{v}_r}{\partial r} - \frac{\bar{l}^2}{r^3} = -\frac{GM}{r^2} - \frac{1}{\Sigma} \frac{\partial P}{\partial r}
\]

Angular momentum conservation
\[
\frac{\partial \bar{l}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{l} \bar{v}_r) = -\frac{1}{r} \frac{\partial}{\partial r} (r^2 \bar{T}_{r\phi})
\]

Energy balance
\[
2 \sigma T_{\text{eff}}^4 = \left( -\frac{\partial \ln \Omega}{\partial \ln r} \right) \Omega \bar{T}_{r\phi}
\]

Equation of state
\[
P = c_s^2 \Sigma + 2H \frac{a}{3} T_c^4
\]

\[\Sigma, \bar{v}_r, \bar{v}_\phi, P, H, c_s, T_{\text{eff}}, T_c, \tau, \bar{\kappa}\]
Wide Range of Physical Conditions

From ‘Accretion Power in Astrophysics’; Frank, King, & Raine, 1995
\[ \Sigma = 5.2 \alpha^{-4/5} \dot{M}_{16}^{7/10} M_{1}^{1/4} R_{10}^{-3/4} f^{14/5} \text{ g cm}^{-2}, \]

\[ H = 1.7 \times 10^{8} \alpha^{-1/10} \dot{M}_{16}^{3/20} M_{1}^{-3/8} R_{10}^{9/8} f^{3/5} \text{ cm}, \]

\[ \rho = 3.1 \times 10^{-8} \alpha^{-7/10} \dot{M}_{16}^{11/20} M_{1}^{5/8} R_{10}^{-15/8} f^{11/5} \text{ g cm}^{-3}, \]

\[ T_{c} = 1.4 \times 10^{4} \alpha^{-1/5} \dot{M}_{16}^{3/10} M_{1}^{1/4} R_{10}^{-3/4} f^{6/5} \text{ K}, \]

\[ \tau = 33 \alpha^{-4/5} \dot{M}_{16}^{1/5} f^{4/5}, \]

\[ v = 1.8 \times 10^{14} \alpha^{4/5} \dot{M}_{16}^{3/10} M_{1}^{-1/4} R_{10}^{3/4} f^{6/5} \text{ cm}^{2} \text{ s}^{-1}, \]

\[ v_{R} = 2.7 \times 10^{4} \alpha^{4/5} \dot{M}_{16}^{3/10} M_{1}^{-1/4} R_{10}^{-1/4} f^{-14/5} \text{ cm s}^{-1}, \]

with \( f = \left[ 1 - \left( \frac{R_{*}}{R} \right)^{1/2} \right]^{1/4}. \)

We can now calculate the emitted spectrum...
Assume energy generated by stresses responsible for angular momentum transport is locally radiated away.

Disk can be divided in regions

1. Outer: $P_{\text{gas}}$ and free-free

2. Middle: $P_{\text{gas}}$ and $e$ scattering

3. Inner: $P_{\text{rad}}$ and $e$ scattering

From ‘The Physics of Compact Objects’; Shapiro & Teukolsky; 1983
Summary of Standard Accretion Disk Model

Since the early 70’s we have used this ‘viscosity’ prescription to remove angular momentum from the disk...

\[ \bar{T}_{r\phi} = \alpha P \left( -\frac{d \ln \Omega}{d \ln r} \right) \]

We can calculate the global structure of the disk

\[ \Sigma(r), \ P(r), \ T(r), \ v_r(r), \ ... \]
Modern Accretion Disk Theory & MHD Turbulence
Problem II: Origin of Turbulence?
- Keplerian flows are very STABLE to hydrodynamic perturbations (Hawley et al. 1999, Yi et al. 2006)
- Convection? (Stone & Balbus 1996, Lesur & Ogilvie 2010)

Solution II: Magnetic Fields

* Velikhow & Chandrasekhar (early 60’s)
* Balbus & Hawley (early 90’s)
* Mechanism to disrupt laminar flows
* Numerical simulations confirm development of MHD turbulence
Turbulent Magnetized Accretion Disks: MRI

Differentially rotating magnetized plasmas are unstable to the Magnetorotational Instability (MRI) (Velikhov & Chandrasekhar, 60’s; Balbus & Hawley, 90’s)

We need to understand the dynamics of magnetic fields in differentially rotating plasmas!!!
A Piece of Physics: The Magnetorotational Instability
Acoustic Waves

Restoring force: pressure gradients

\[ \delta P \]

\[ \omega^2 = k^2 c_s^2 \]

\[ c_s^2 = \gamma \frac{P}{\rho} \]

Alfven Waves

Restoring force: magnetic tension

\[ \delta B \]

\[ \omega^2 = k^2 v_A^2 \]

\[ v_A^2 = \frac{B^2}{4\pi\rho} \]
MHD Waves

Magnetic Field

\[ B_z \]

\[ r \]

\[ z \]

\[ \phi \]
MHD Waves

Magnetic Tension

$B_z \delta B_{\phi}$

Magnetic Field

$\phi$

$r$

$z$
MHD Waves

\[ B_z \delta B_{\phi} \]

Magnetic Field
MHD Waves

$B_z \delta B_\phi$

Magnetic Field
MHD Waves

$B_z \delta B_\phi$

Magnetic Field
MHD Waves

\[ B_z \delta B_\phi \]

Magnetic Field
MHD Waves

$B_z \delta B_\phi$

Magnetic Field
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$B_z \delta B_\phi$

Magnetic Field
MHD Waves

\[ B_z \delta B_{\phi} \]

Magnetic Field
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\[ B_z \delta B_\phi \]
MHD Waves

$B_z \delta B_\phi$

Magnetic Field
MHD Waves

\[ B_z \delta B_\phi \]

Magnetic Field
MHD Waves

Magnetic Field

$B_z \delta B_\phi$

$r$

$z$

$\phi$

$\phi$

$r$
MHD Waves

Magnetic Field

$B_z \delta B_\phi$
MHD Waves

\[ B_z \delta B_\phi \]

\[ \omega^2 = k^2 v_A^2 \]

\[ v_A^2 = \frac{B^2}{4\pi\rho} \]
Magnetorotational Instability

To the black hole

\( \frac{d\Omega}{dr} \)

\( B_z \)

Differential rotation

Magnetic Field
Magnetorotational Instability

Differential rotation

Magnetic Field

$B_z \delta B_\phi$
Magnetorotational Instability
Magnetorotational Instability

Differential rotation

Magnetic Field
Magnetorotational Instability

Differential rotation

Magnetic Field
Magnetorotational Instability

Differential rotation

Magnetic Field
Magnetorotational Instability

Differential rotation
Magnetic Field
Magnetorotational Instability

Differential rotation

Magnetic Field
Magnetorotational Instability

Magnetic Torques

\[ B_z \delta B_{\phi} \]

\[ B_z \delta B_r \]

Differential rotation

Magnetic Field

Magnetorotational Instability
Magnetorotational Instability

Differential rotation

Magnetic Field

Magnetorotational Instability

\[ k^2 v_{A_z}^2 < -2 \Omega^2 \frac{d \ln \Omega}{d \ln r} \]

\[ v_A^2 = \frac{B^2}{4\pi \rho} \]
MHD Equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + (\rho \mathbf{v} \cdot \nabla) \mathbf{v} = -\rho \nabla \Phi - \nabla \left( P + \frac{B^2}{8\pi} \right) + \left( \frac{B}{4\pi} \right) \cdot \nabla B \]

\[ \frac{\partial B}{\partial t} + (\nabla \cdot \mathbf{v}) B - (B \cdot \nabla) \mathbf{v} + (\mathbf{v} \cdot \nabla) B = 0 \]

\[ P = P_0 \left( \frac{\rho}{\rho_0} \right)^\Gamma \]
Local -shearing box- MHD Equations

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -2\Omega_0 \times \mathbf{v} + q\Omega_0^2 \nabla (r - r_0)^2 \]

\[ - \frac{1}{\rho} \nabla \left( P + \frac{\mathbf{B}^2}{8\pi} \right) + \frac{(\mathbf{B} \cdot \nabla)\mathbf{B}}{4\pi \rho} + \nu \nabla^2 \mathbf{v} \]

\[ \frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{v} + \eta \nabla^2 \mathbf{B} \]

\[
\begin{align*}
\text{Re} & \equiv \frac{v_{Az}^2}{\nu \Omega_0} \equiv \frac{1}{\nu} \\
\text{Rm} & \equiv \frac{v_{Az}^2}{\eta \Omega_0} \equiv \frac{1}{\eta} \\
\text{Pm} & \equiv \frac{\nu}{\eta}
\end{align*}
\]

Use background field and rotation rate to define characteristic scales in the problem.
Dynamical equations = 0

Dynamical equations for $\delta = 0$

Neglect 2nd order terms

Expand in Fourier Series

$\delta A \sim \delta \rho, \delta P, \delta v, \delta B \rightarrow \sum A_k \exp[-i(k \cdot x + \omega t)]$

Obtain the *dispersion relation* $p(k, \omega) = 0$

Find the *eigenfrequencies* $\omega = \omega(k)$
Growth Rates with Dissipation

\[(k^2 + \sigma_\nu \sigma_\eta)^2 + \kappa^2 (k^2 + \sigma_\eta^2) - 4k^2 = 0\]

\[\sigma_\nu = \sigma + \nu k^2\]

\[\sigma_\eta = \sigma + \eta k^2\]

Pessah & Chan, 2008

Various limits studied by Sano et al. 1998, Lesaffre & Balbus 2007, Lesur & Longaretti 2007, and many others...
Angular Momentum Transport: MRI & Turbulence

Angular momentum conservation

\[
\frac{\partial l}{\partial t} + \nabla \cdot (l \mathbf{v}) = - \frac{1}{r} \frac{\partial}{\partial r} \left[ r^2 (R_{r\phi} - M_{r\phi}) \right]
\]

Reynolds Stress

\[ R_{r\phi} = \langle \rho \delta v_r \delta v_\phi \rangle \]

Maxwell Stress

\[ M_{r\phi} = \frac{\langle \delta B_r \delta B_\phi \rangle}{4\pi} \]

MRI and MHD turbulence lead to correlated fluctuations of the PROPER sign!!!

Sano et al. 2004
Saturation of MRI & MHD Turbulence

Insets from Sano et al. 2004
Numerical Simulations
Importance & Limitations of Numerical Simulations

- Mean Field Models
- Disk Spectrum
- BH Variability
- Numerical Simulations
- Inside Horizon
- BH Growth
- Viscous Orbital
- Saturation Region
- Hubble Time
- 1Gyr
- 1yr
- 1sec
- 0.1AU
- 1AU
- 1000AU
- $10^7M_\odot$
MHD Turbulent Transport vs. alpha-viscosity

Shakura & Sunyaev (‘70s): transport due to turbulence

\[ \bar{T}_{r\phi} = \alpha P \left( -\frac{d \ln \Omega}{d \ln r} \right) \]

Pessah, Chan, & Psaltis, 2006
Pessah, Chan, & Psaltis, 2008
Shakura & Sunyaev, 1973

Standard prescription does not capture physics correctly!

Need to understand turbulent MHD angular momentum transport from first principles...
Current Focus and Future Prospects

Better understanding of micro-physics. Thermodynamics.

Synergy between analytical and numerical work.

More realistic global simulations.
Global 3D MHD Simulations

Beckwith et al. 2011
It is usually argued that stresses vanish at the inner disk. Simulations show stresses DO NOT vanish inside ISCO.

Implications:

* Inner disk structure
* Emission inside ISCO

* Spin measurements
* Spin evolution
Accretion Disks Big Picture Recap

Fundamental Problem in Accretion Disks:
How to get rid of angular momentum...

Since early 70’s we have dealt with this problem by using a ‘viscosity’ prescription proposed by Shakura & Sunyaev (1973)

Balbus & Hawley (1991): magnetic fields are key!
Differentially rotating, magnetized plasmas are unstable to MRI
Ensuing turbulence removes angular momentum from disk!

Massive analytical and numerical efforts to understand MHD turbulence and simulate accretion disks

From J. Hawley’s website

From J. Stone’s website
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