# Dimensionality Reduction 

Principal Component Analysis (PCA)<br>t-Stochastic Neighbor Embedding (t-SNE)

Uniform Manifold Approximation and Projection (UMAP)

## Quick review

Wednesday May 3, 2023

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We've learned several useful methods already. What sorts of things are we now good at?

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## Principal Component Analysis (PCA)

https://setosa.io/ev/principal-component-analysis/


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## PCA in 3D

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## PCA in 17D

https://setosa.io/ev/principal-component-analysis/

Alcoholic drinks
Beverages
Carcase meat
Cereals
Cheese
Confectionery
Fats and oils
Fish
Fresh fruit
Fresh potatoes
Fresh Veg
Other meat
Other Veg
Processed potatoes
Processed Veg
Soft drinks
Sugars

| England | N Ireland | Scotland | Wales |
| :---: | :---: | :---: | :---: |
| 375 | 135 | 458 | 475 |
| 57 | 47 | 53 | 73 |
| 245 | 267 | 242 | 227 |
| 1472 | 1494 | 1462 | 1582 |
| 105 | 66 | 103 | 103 |
| 54 | 41 | 62 | 64 |
| 193 | 209 | 184 | 235 |
| 147 | 93 | 122 | 160 |
| 1102 | 674 | 957 | 1137 |
| 720 | 1033 | 566 | 874 |
| 253 | 143 | 171 | 265 |
| 685 | 586 | 750 | 803 |
| 488 | 355 | 418 | 570 |
| 198 | 187 | 220 | 203 |
| 360 | 334 | 337 | 365 |
| 1374 | 1506 | 1572 | 1256 |
| 156 | 139 | 147 | 175 |

## PCA in 17D

https://setosa.io/ev/principal-component-analysis/


## OK, so how can we find the right basis?

1. Standardization

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2. Compute covariance matrix

$$
\left[\begin{array}{ccc}
\operatorname{Cov}(x, x) & \operatorname{Cov}(x, y) & \operatorname{Cov}(x, z) \\
\operatorname{Cov}(y, x) & \operatorname{Cov}(y, y) & \operatorname{Cov}(y, z) \\
\operatorname{Cov}(z, x) & \operatorname{Cov}(z, y) & \operatorname{Cov}(z, z)
\end{array}\right]
$$

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3. Compute eigenvectors and eigenvalues


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\end{array}\right]
$$

3. Compute eigenvectors and eigenvalues
4. Discard vectors that are not important enough


Example: Handwritten Digits

$$
\begin{array}{llllllllllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
\hline 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9
\end{array}
$$

## Group "similar" things together

Principal Component Analysis


Example: Handwritten Digits

$$
\begin{array}{llllllllllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
\hline 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9
\end{array}
$$

## Some things aren'† linear!

Wikimedia Commons


## Group "similar" things together

Pezzotti 2019



## Group "similar" things together

"Fashion MNIST" datasets, t-SNE


## Group "similar" things together



## Example: Separate Short and Long GRBs

R. Mallozzi, updated Aug 2018 at https://gammaray.msfc.nasa.gov/batse/grb/duration/


## t-SNE map for Swift light curves

Kragh Jespersen et al. 2020


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Kraah Jespersen et al. 2020


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Kragh Jespersen et al. 2020



## Embeddings, colored by duration



## Embeddings, colored by duration



## Hardness distribution



## Redshift distribution

Kragh Jespersen et al. 2020
redshift


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## Possible subgroupings?

Kragh Jespersen et al. 2020


## Structure is durable, not location!

Steinhardt, Mann, Rusakov, and Kragh Jespersen 2023

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

# Objects can "jump" 

Steinhardt, Mann, Rusakov, and Kragh Jespersen 2023


## Example: Photometry

## Example: Photometry

## Example: Galaxy spectra

SDSS/Galaxy Zoo


## Spectra have similar features!

Zaroubi et al. 2013


## Maybe limited information is enough



## Our goal:



## Two Fundamental Assumptions for Photometry

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COSMOS2015 catalog, objects at $\mathrm{z} \approx 1$


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COSMOS2015 catalog, objects at $\mathrm{z} \approx 1$


## Approach 2: Color space map using all bands

## Two Three Fundamental Assumptions for Photometry

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3. We can map objects from the full, n-dimensional space with all bands to a smaller one with many neighbors, and the other two assumptions will continue to hold.

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3. We can map objects from the full, n-dimensional space with all bands to a smaller one with many neighbors, and the other two assumptions will continue to hold.

## Approach 2: First, make a t-SNE map reducing to two dimensions



## Approach 2: Similar galaxies are nearby

COSMOS2015 catalog, objects at $\mathrm{z} \approx 1$


## Approach 2: Similar galaxies are nearby

WARNING: positions are neither fixed nor meaningful. Topology is meaningful.


# Approach 2: Arranging by photometry also calculates other useful things! 

COSMOS2015 catalog, objects at $\mathrm{z} \approx 1$
$\log _{10}\left(M_{*}\right)$


- $\mathrm{S} / \mathrm{N}_{\text {MIPS }}<5$
- $\mathrm{S} / \mathrm{N}_{\text {MIPS }}>5$

projector.tensorflow.org

| DATA |
| :--- | :--- | :--- |
| Stensors found |
| Word2Vec 10 K |

## Run

For faster results, the data will be sampled down to 5,000 points.

Learn more about UMAP

## (?)

-.... 7 A $\mid$ Points: $10000 \mid$ Dimension: $200 \mid$ Selected 101 points


| Show All Data | Isolate 101 points | Clear selection |
| :---: | :---: | :---: |
|  |  | by |
| Search |  | word |
| neighbors (? |  | 10C |
| distance | COSINE | EUCLIDEAN |

Nearest points in the original space:

| chicago | 0.40 |
| :--- | :--- | :--- |
| massachusetts | 0.40 |
| philadelphia | 0.41 |
| atlanta | 0.49 |
| harvard | 0.50 |
| london | 0.50 |
| illinois | 0.51 |
| baltimore | 0.51 |
| maryland | 0.54 |
| york | 0.55 |
| cincinnati | 0.55 |
| toronto | 0.55 |
| seattle | 0.56 |
| brooklyn | 0.57 |
| miami | 0.58 |
| cambridge | 0.58 |
| pennsylvania | 0.58 |
| pittsburgh | 0.58 |
| detroit | 0.59 |
| california | 0.59 |
| sox | 0.60 |
| kansas | 0.60 |

