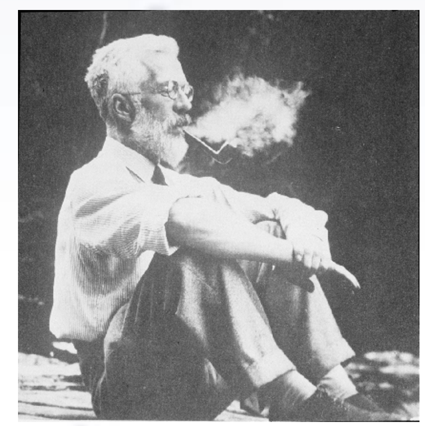
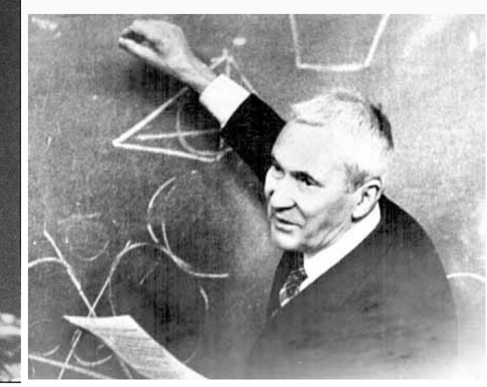
Applied Statistics

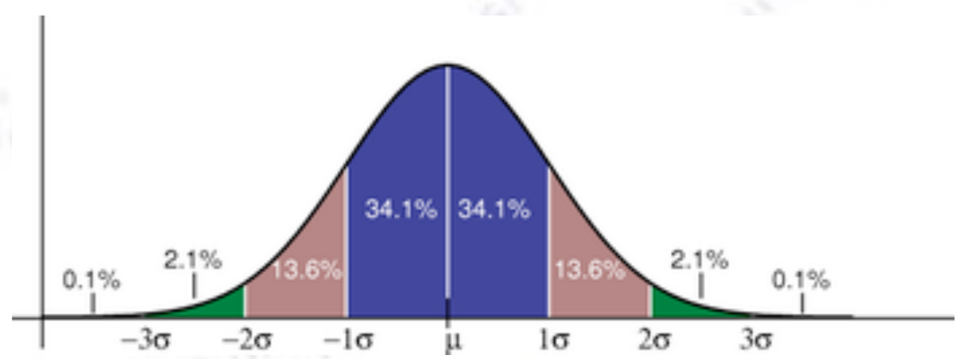
Fitting and Significance





Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

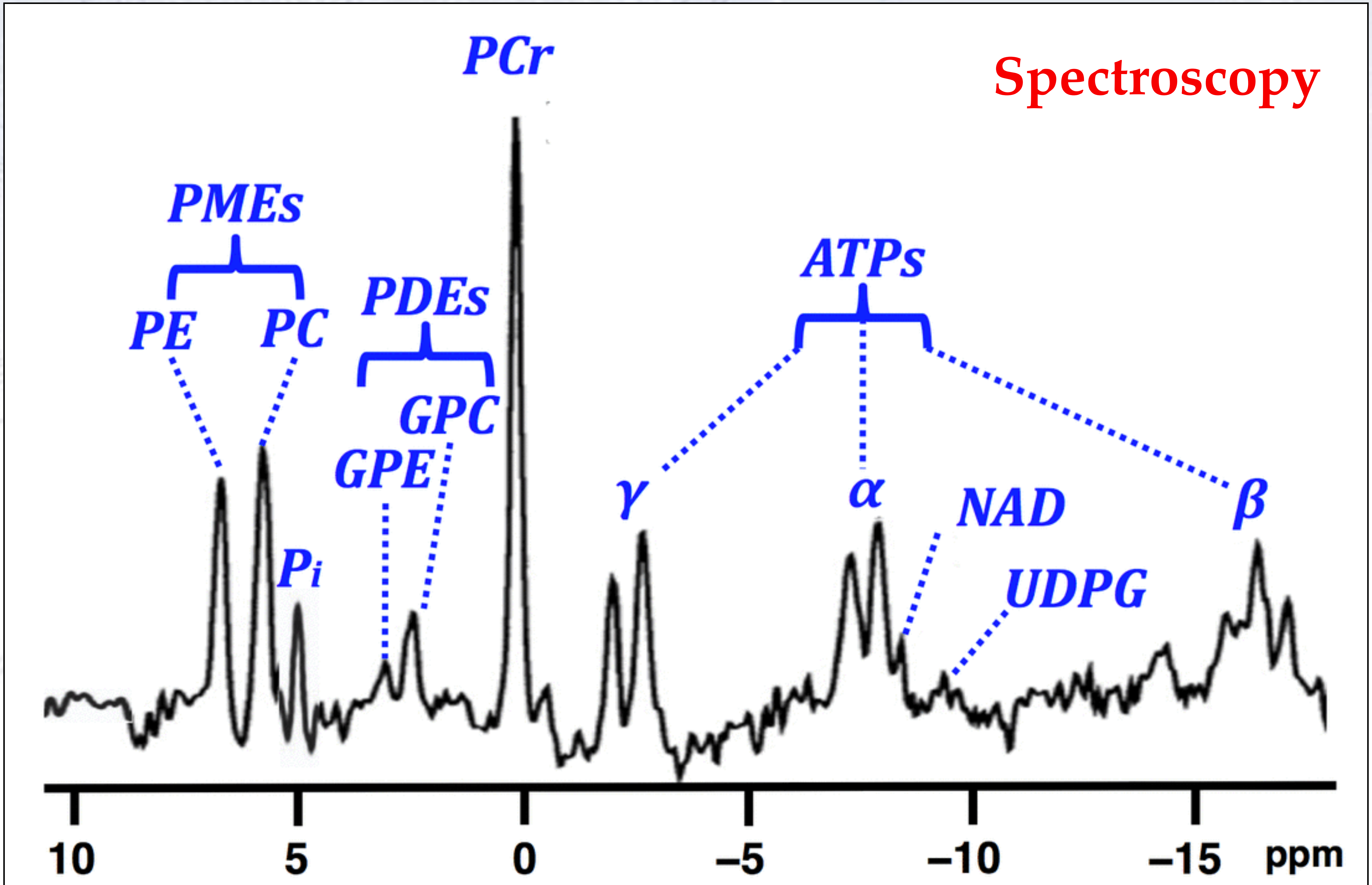
Peaks on a background

Often data boils down to a single histogram, from which conclusions are drawn.

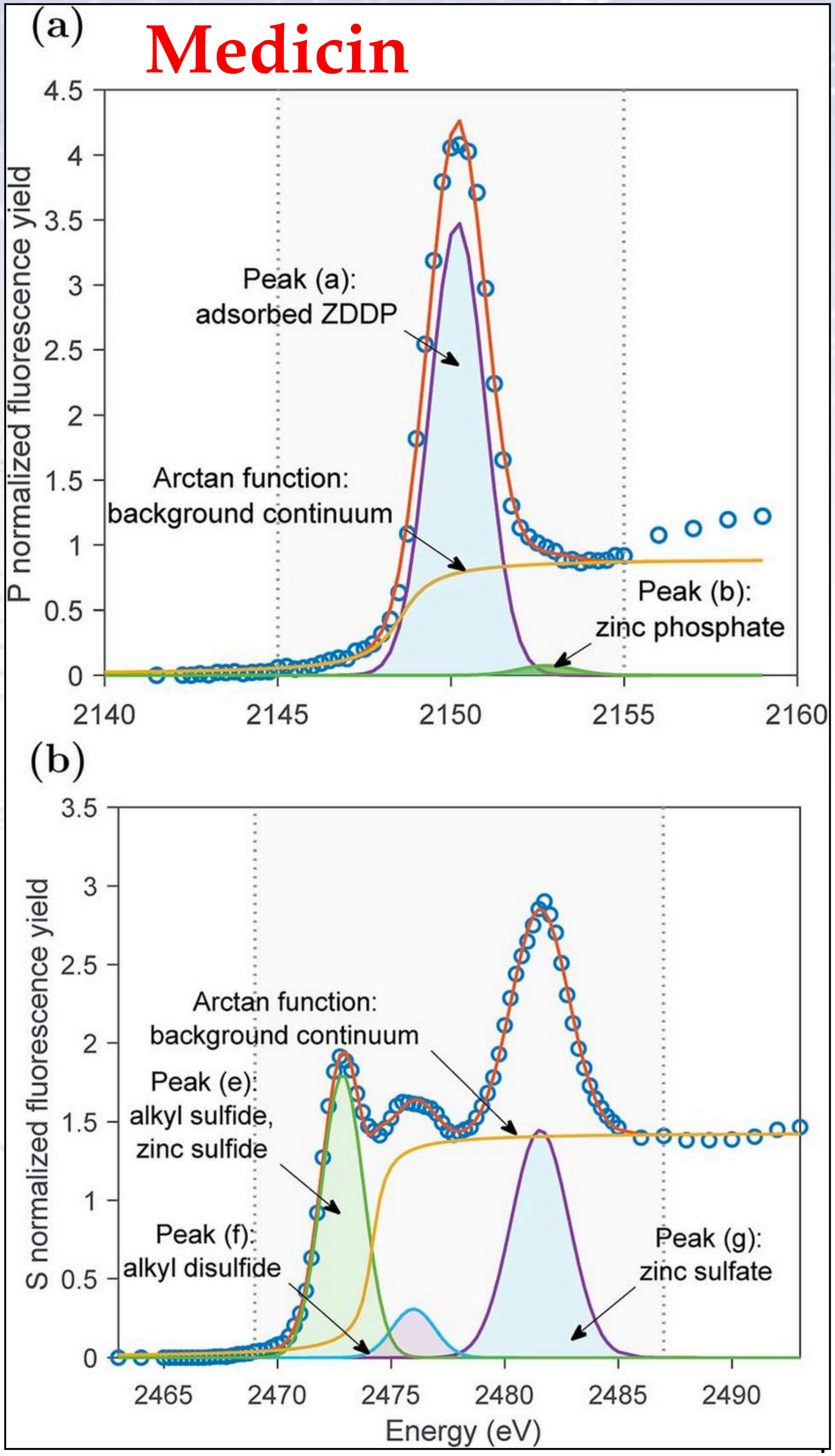
Though more complex representations might display sharper results, one often seeks simplicity, when making conclusions and attempting to convince others.

In such histograms, the most prominent and easily interpretable feature is **a peak**. Due to finite detector resolution these are mostly Gaussian, and so fitting one or more **Gaussian peaks on a background is a very typical case**.

Fitting peaks - outside physics!



Fitting peaks… outside physics!



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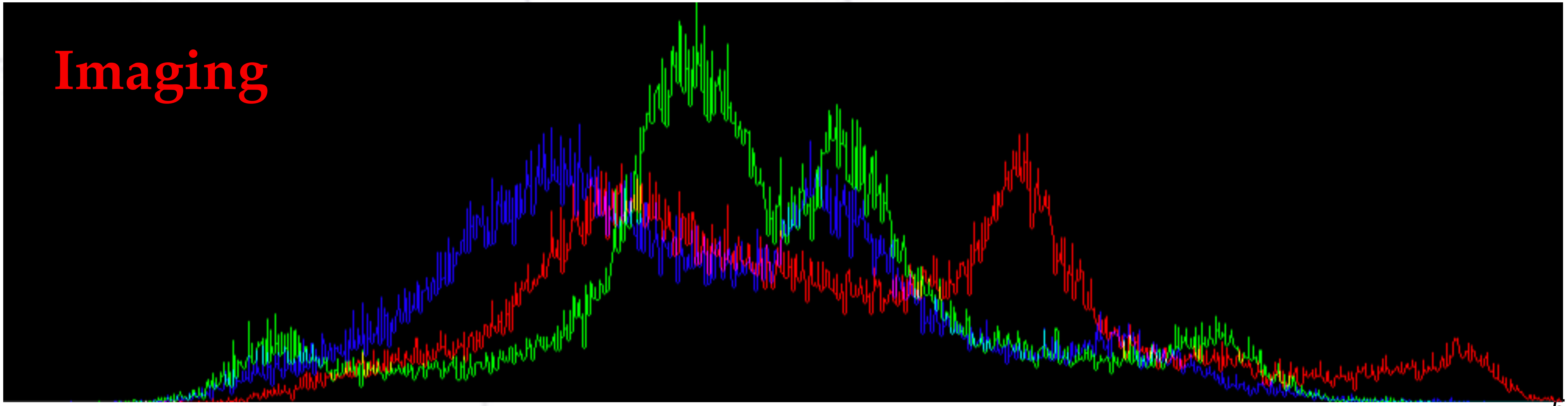
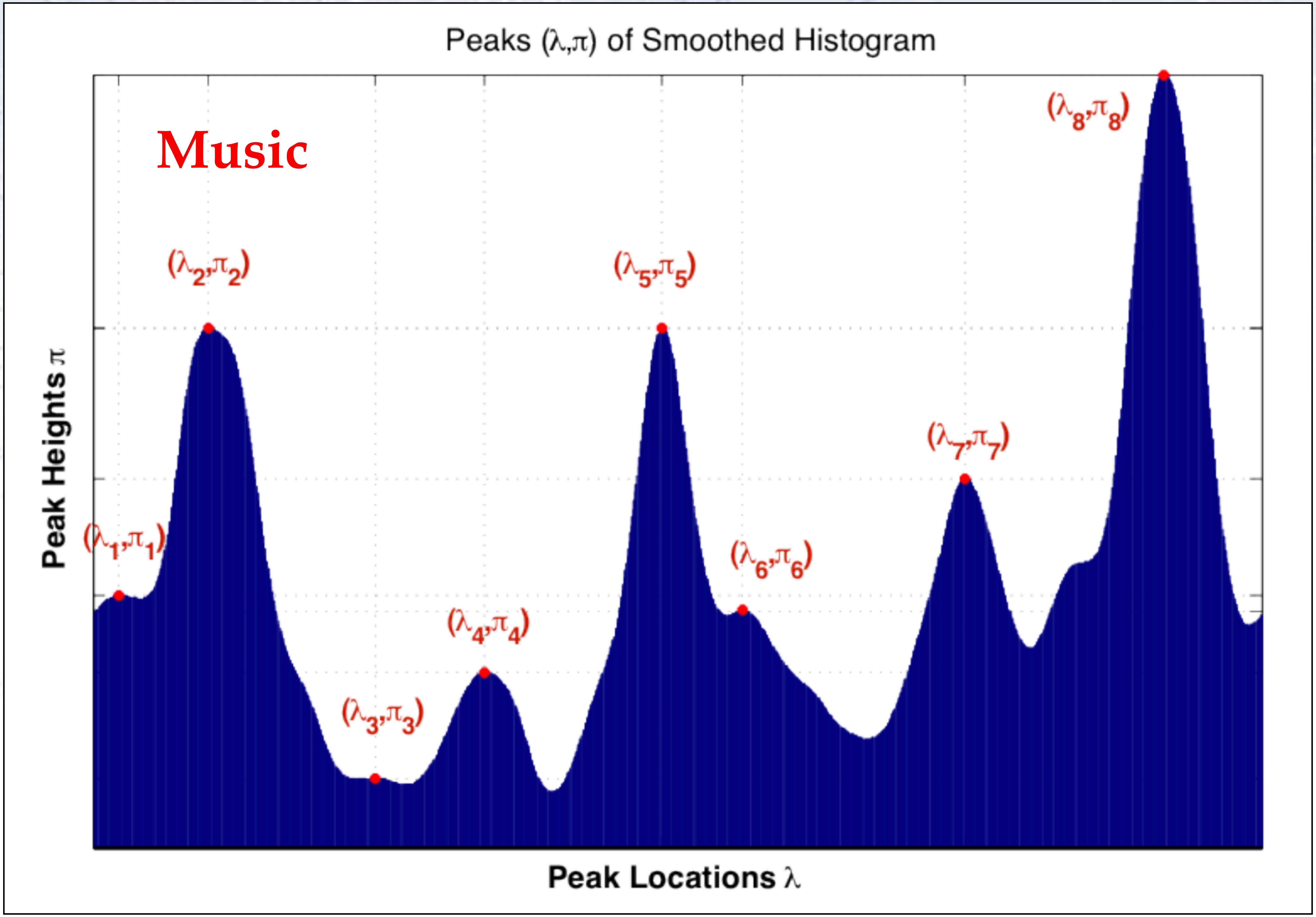


Fitting peaks - outside physics!



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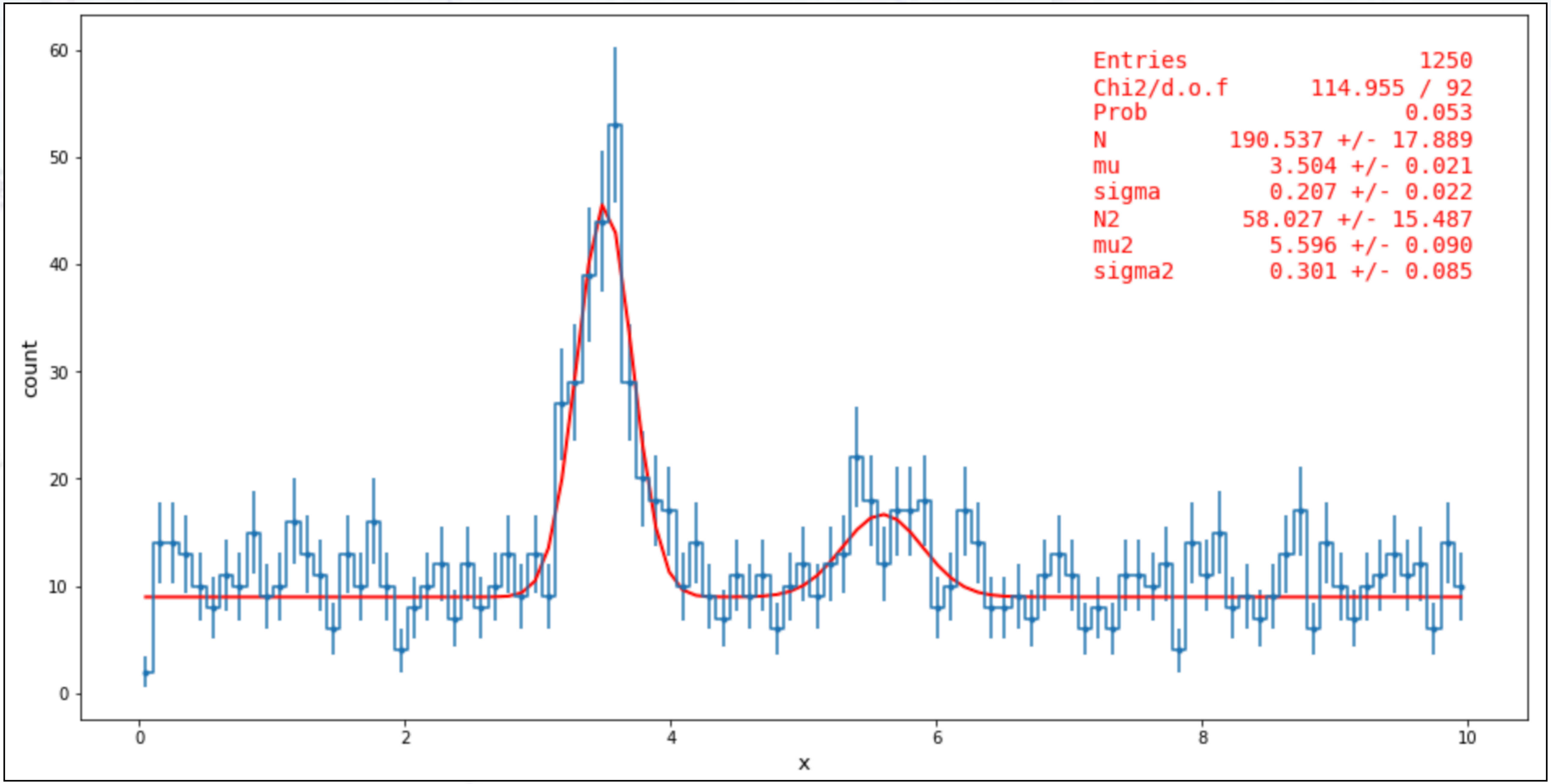
In such histograms, the most prominent and easily interpretable feature is a peak. Due to finite detector resolution these are mostly Gaussian, and so fitting one or more **Gaussian peaks on a background is a very typical case**.

Of course one should make sure to give good starting values, or else the fit won't find the peak(s).

But what to do if one is searching for peaks?
How can one evaluate, if the peaks are real or just fluctuations of noise?
And can a combined/simultaneous fit to several peaks improve the result?

Peaks on a background

The (local) significance of a peak can be determined by considering how far away from zero the normalisation is. In the below case, the two peaks have significance ?



Peaks on a background

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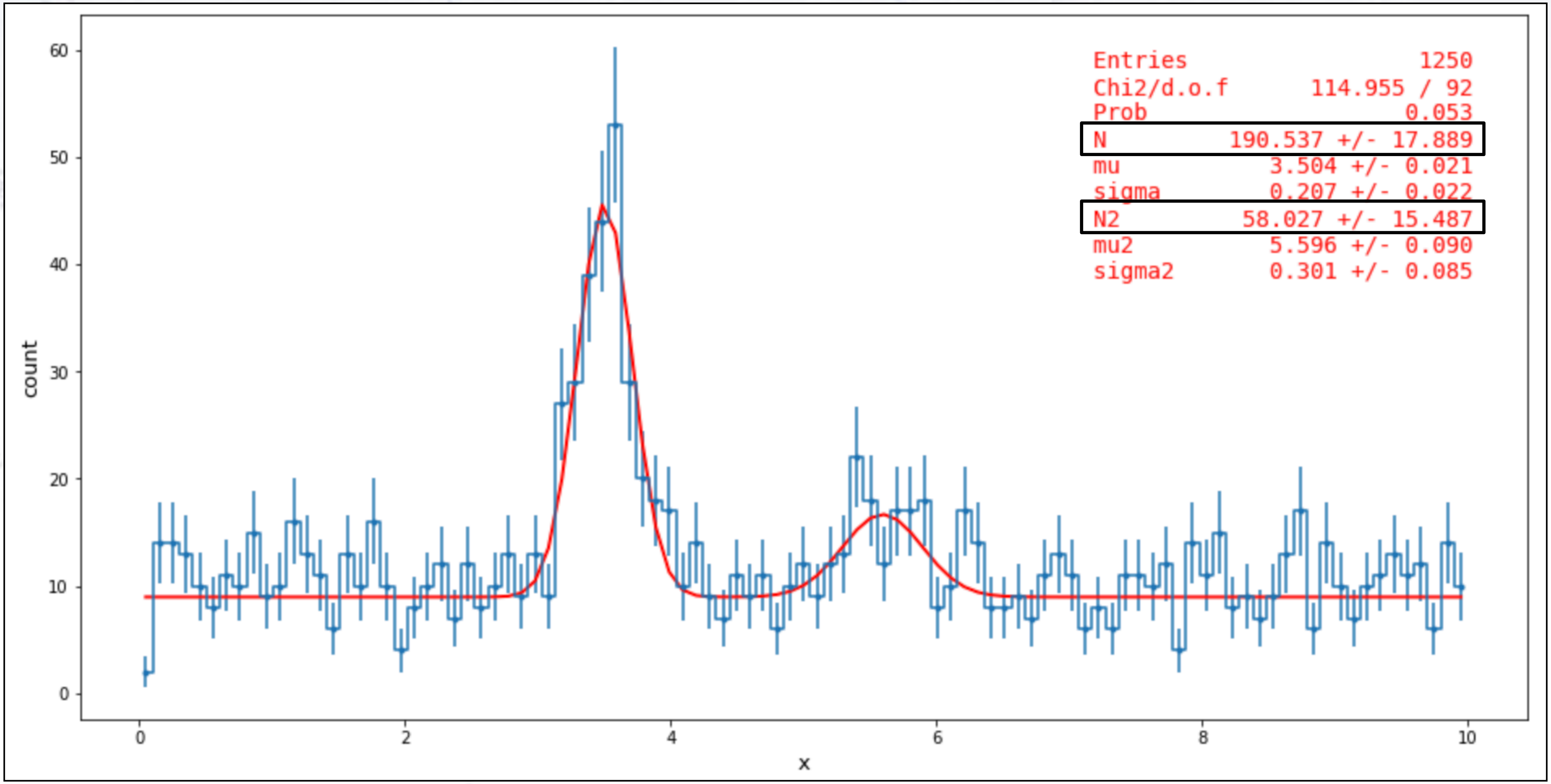

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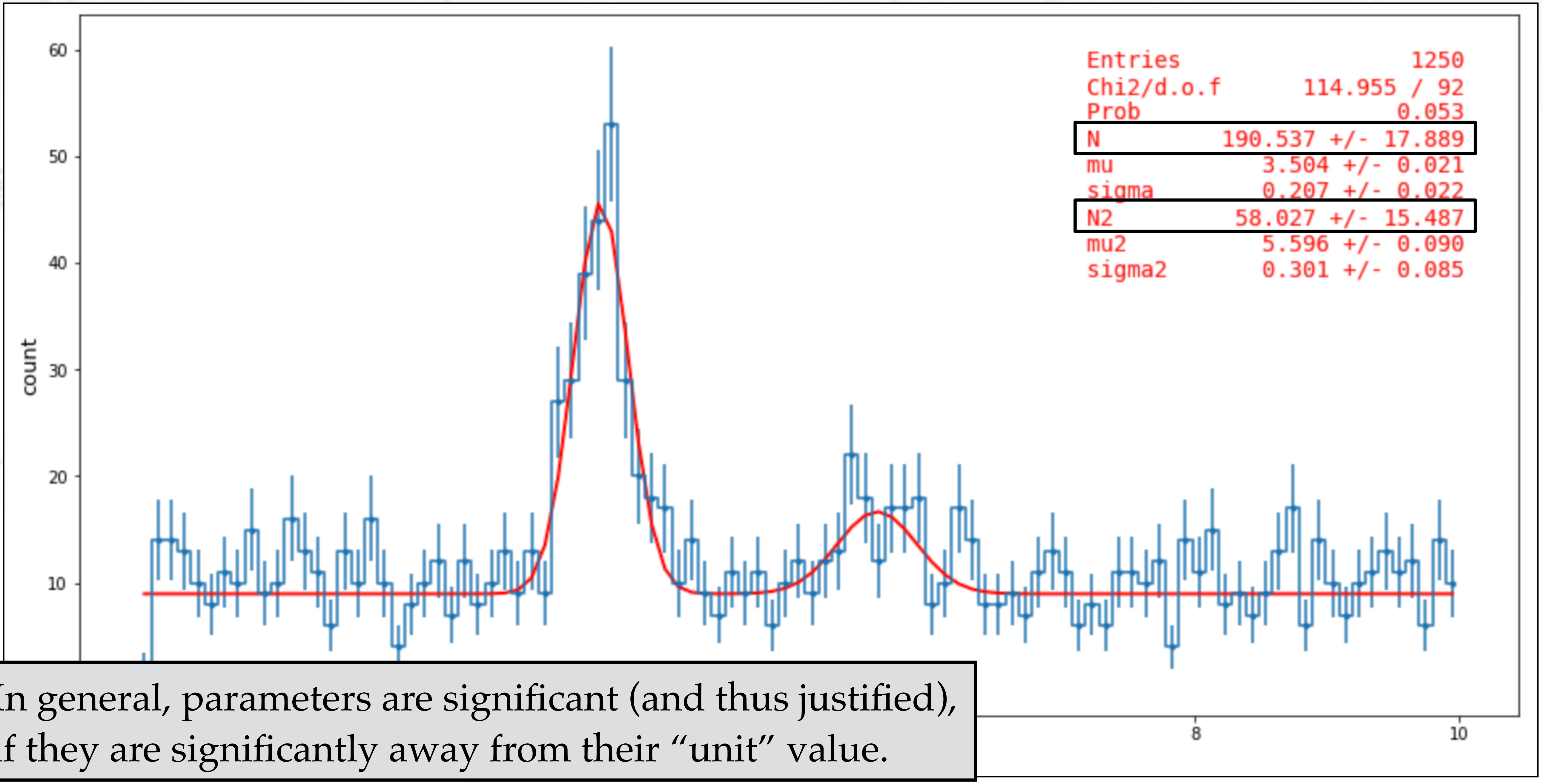

In general, parameters are significant (and thus justified), if they are significantly away from their "unit" value.

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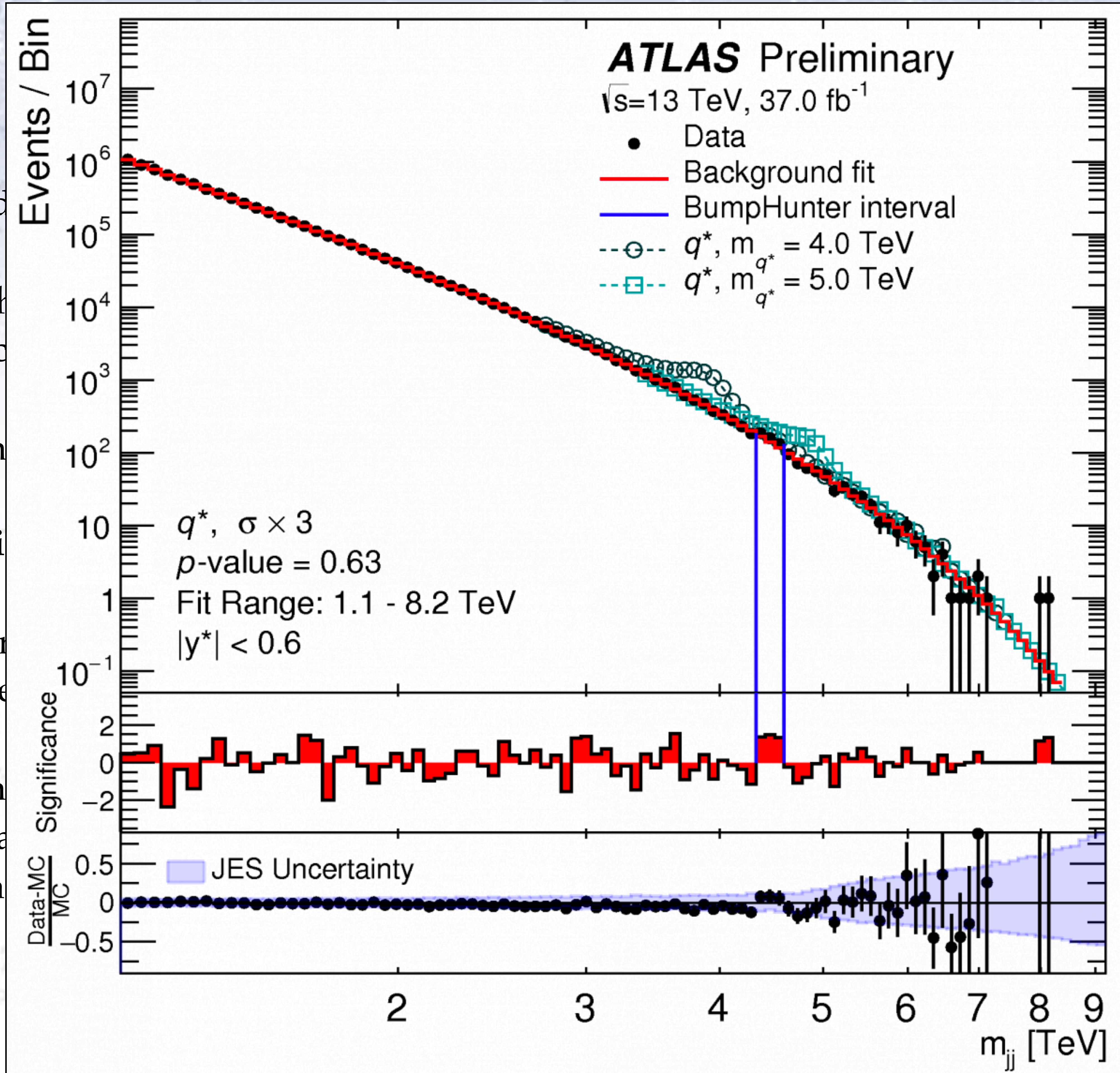

Three attempts at answers

a) What to do if one is searching for peaks?
First of all, establish a good (non-peaking) background model (crucial!).
Secondly, determine the expected signal resolution.
Only then, start fitting the background with a potential peak with fixed width.

b) How can one evaluate, if the peaks are real or just fluctuations of noise?
The significance of the (floating) amplitude tells you the local significance.
As you might be searching in many places, this reduces your certainty to the global significance:

$$p_{\mathrm{global}} = 1 - (1 - p_{\mathrm{local}})^N \simeq N\, p_{\mathrm{local}}$$

c) And can a combined/simultaneous fit to several peaks improve the result?
Yes! If you are fitting several peaks, which you expect to have overlapping parameters (e.g. same width), then fitting with a reduced number of parameters makes the fit more constrained, accurate, and convergent.

Note, that in the last case, this is why a "calibration channel" is great to have.