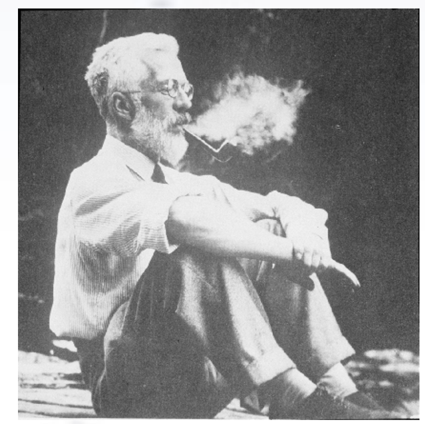
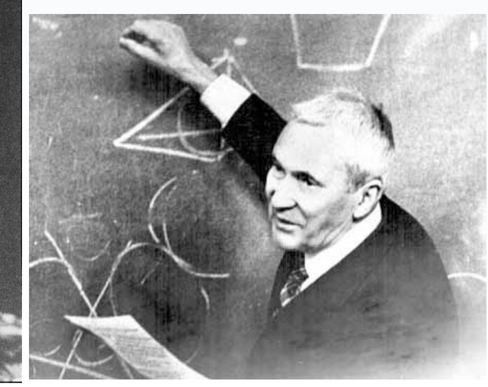
Applied Statistics

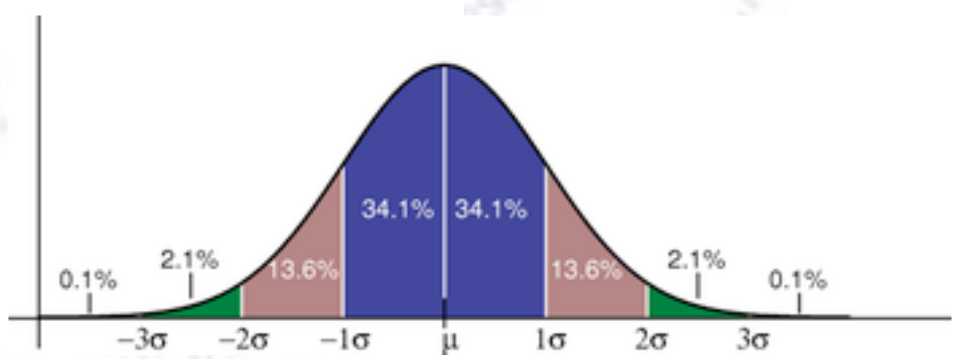
Estimating the length of the table





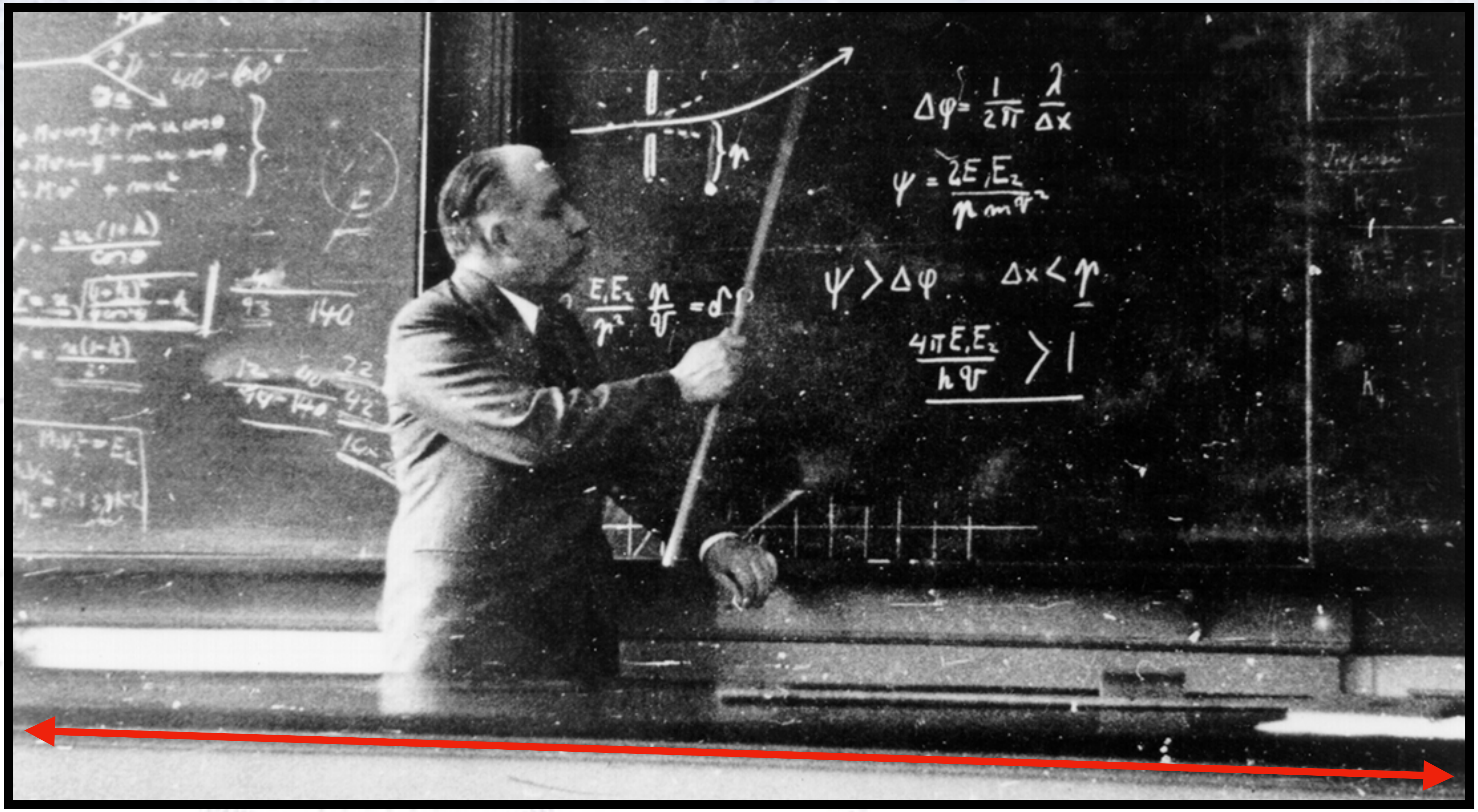



Troels C. Petersen and Mathias Luidor Heltberg (NBI)



"Statistics is merely a quantisation of common sense"

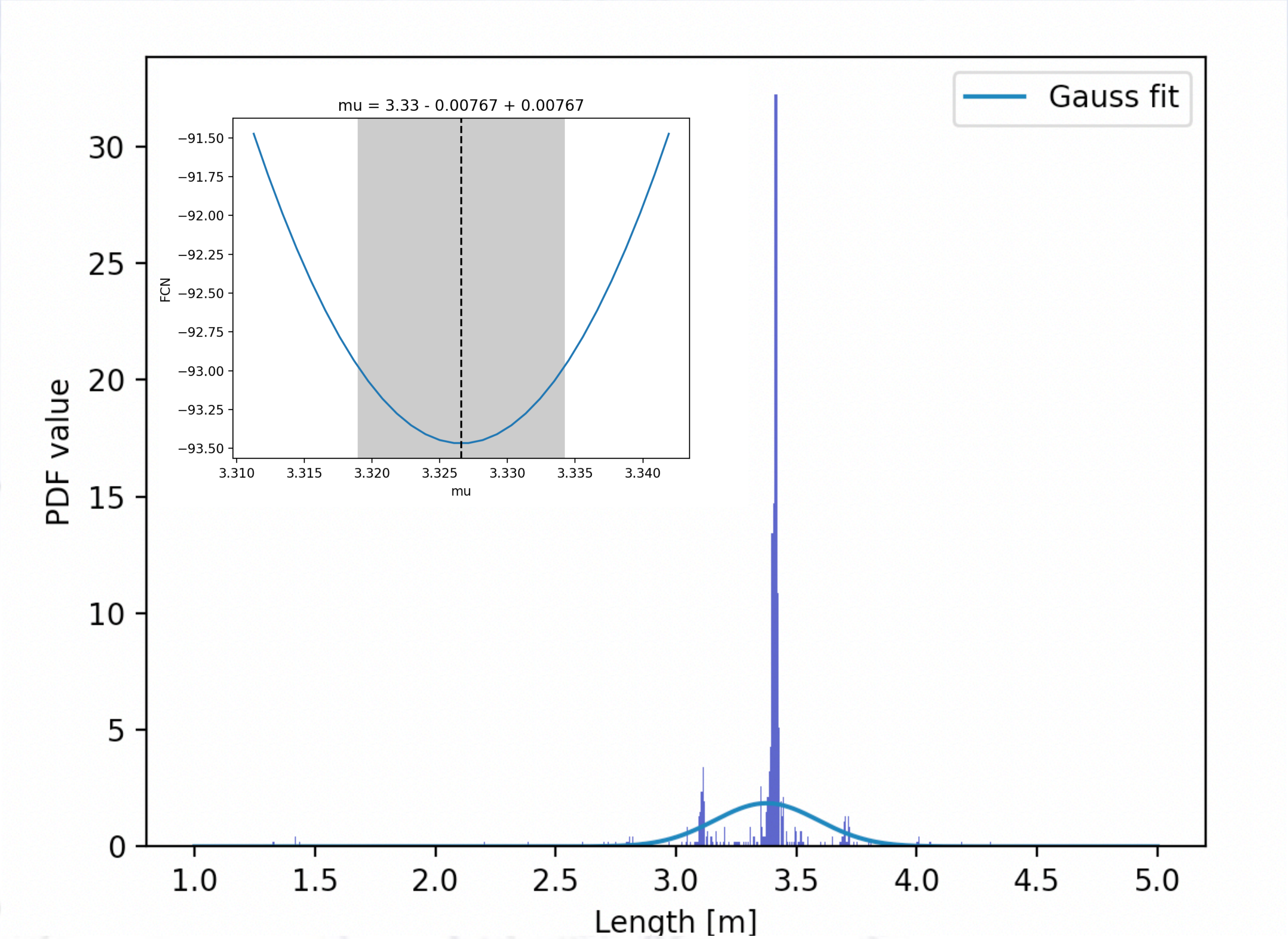
The table in auditorium A



"Everything is vague to a degree you do not realise till you have tried to make it precise."

[Bertrand Russell, 1872-1970]

The data at first glance

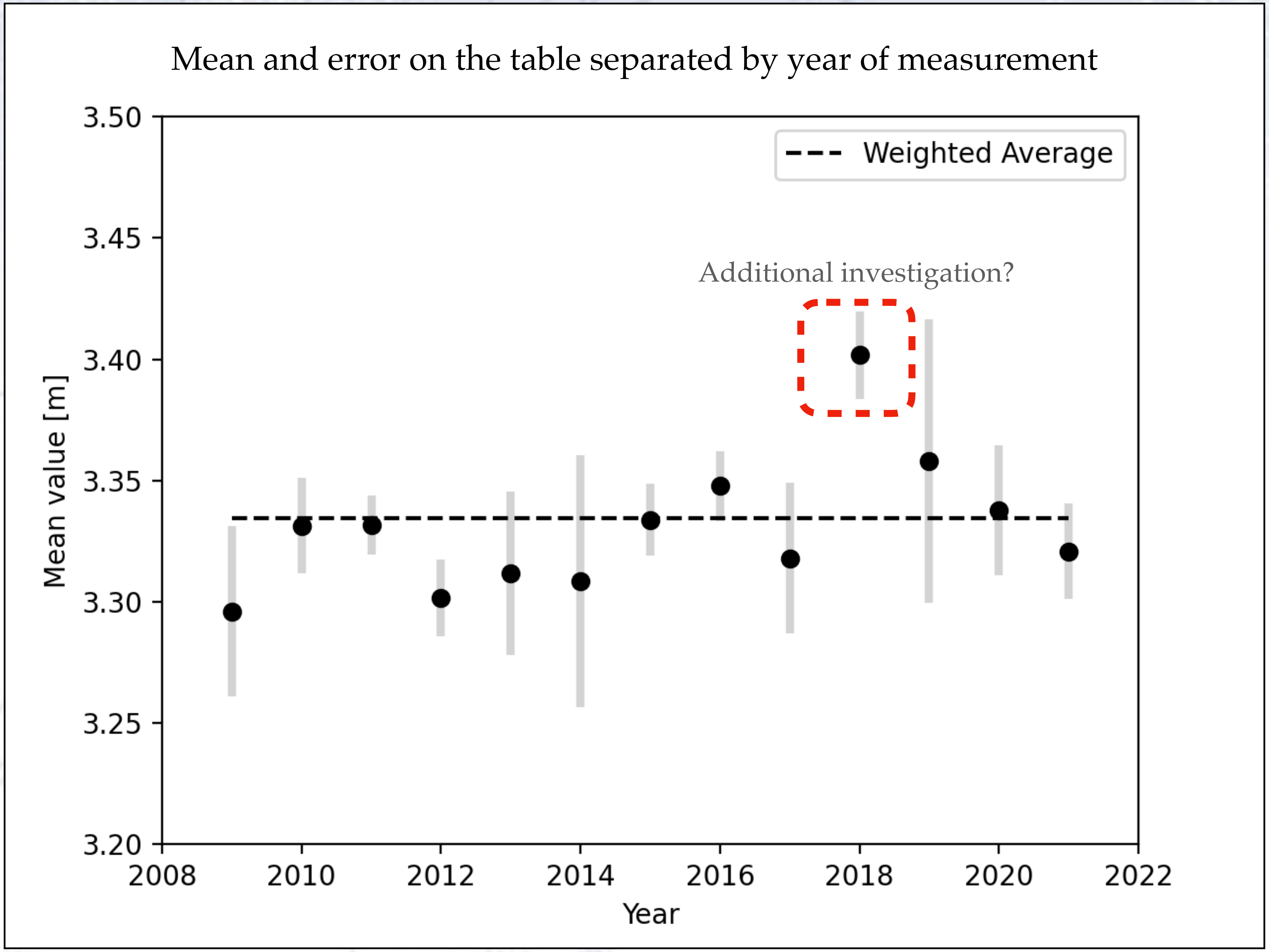


Conclusion: By only looking at the raw data we obtain the value:

$$\mu = 3.3266 \pm 0.0077 \text{ m}$$
$$\sigma = 0.2148 \pm 0.0054 \text{ m}$$
$$N = 785$$

However we can do better than this…

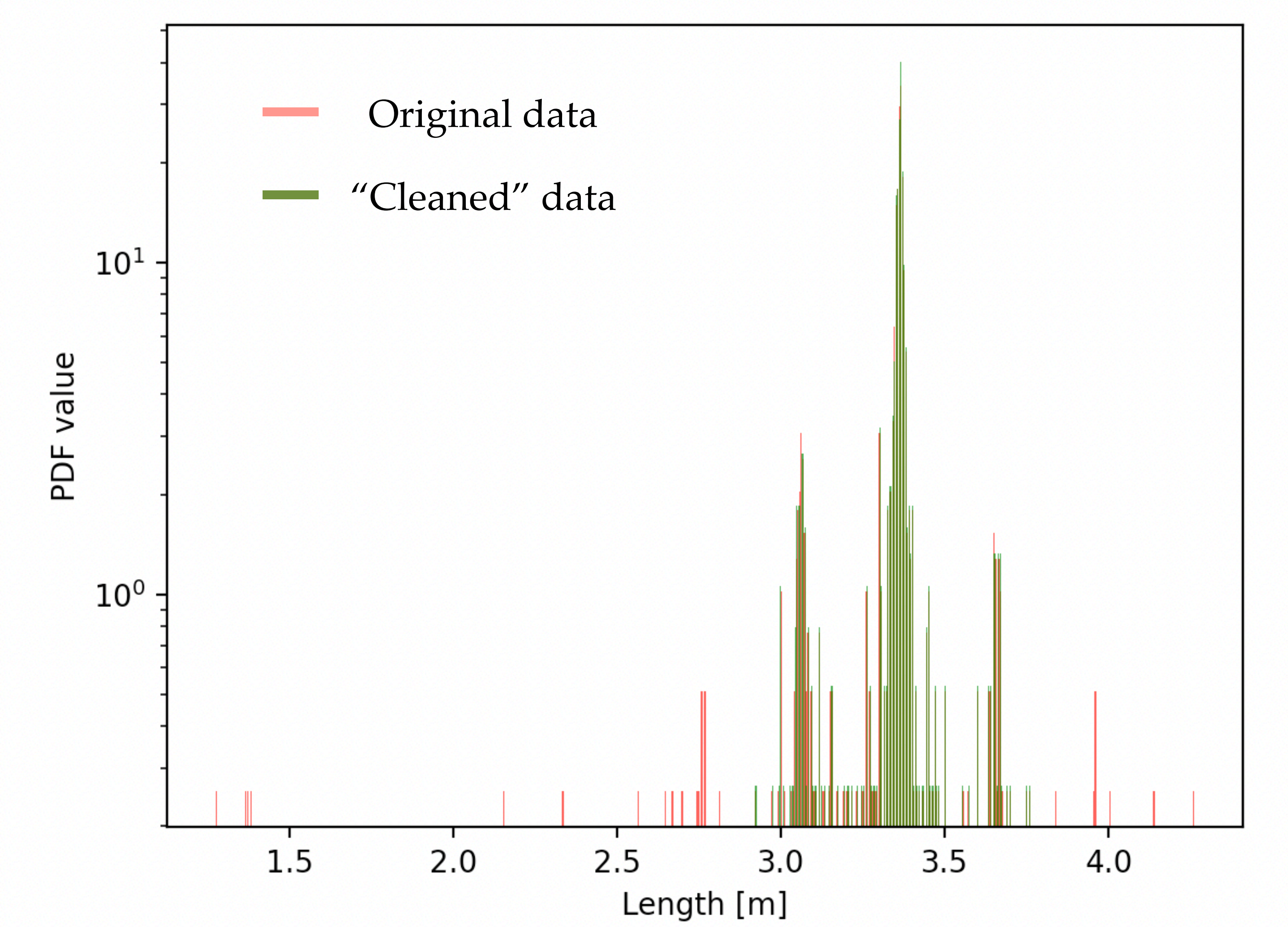
Crosschecking



These measurements seem to be in OK agreement - however the 2018 seems to be off…

First rejection criteria

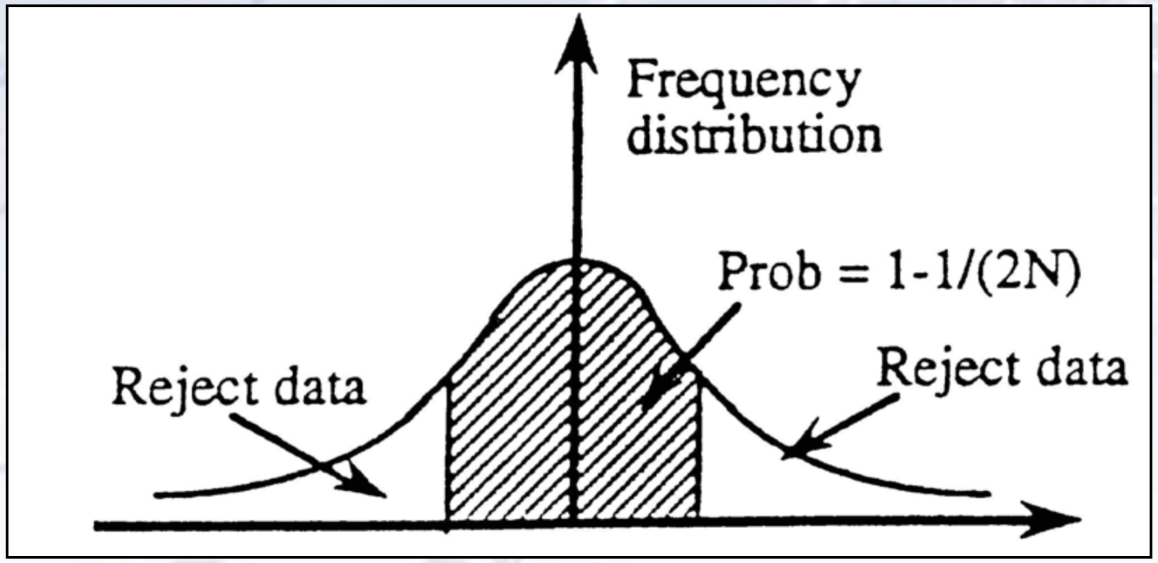
We will start by removing the "worst" points. Here I remove the points that are more than 4 std's away from the mean.



With this we reject 24 points - so the data sample consist of 761 points.

Removing data points

Removing improbable data points is formalised in **Chauvenet's Criterion,** though many other methods exists (see Peirce, Grubbs, etc.)



The idea is to assume that the distribution is Gaussian, and ask what the probability of the farthest point is. If it is below some value, which is to be determined ahead of applying the criterion, then the point is removed, and the criterion is reapplied until no more points should be removed.

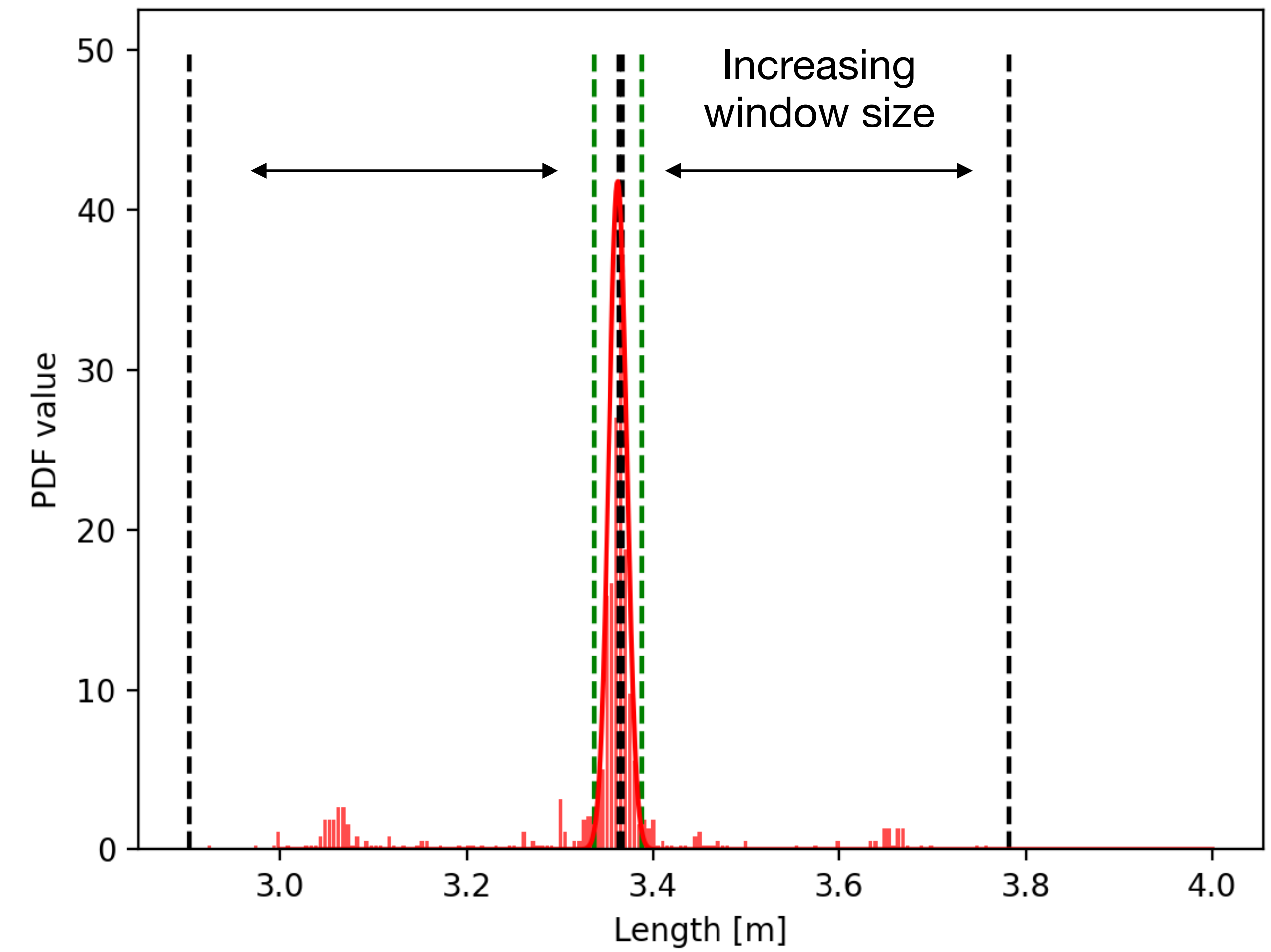
We choose to say, that if the outermost point in the Gaussian case has **less than 5% chance of being this far out** (taking the total number of points into account), then I reject it.

However, **ALWAYS keep a record of your original data**, as it may contain more effects than you originally thought.

Is the central peak gaussian?

In order to apply the criterion however, we should have something that is somewhat gaussian.

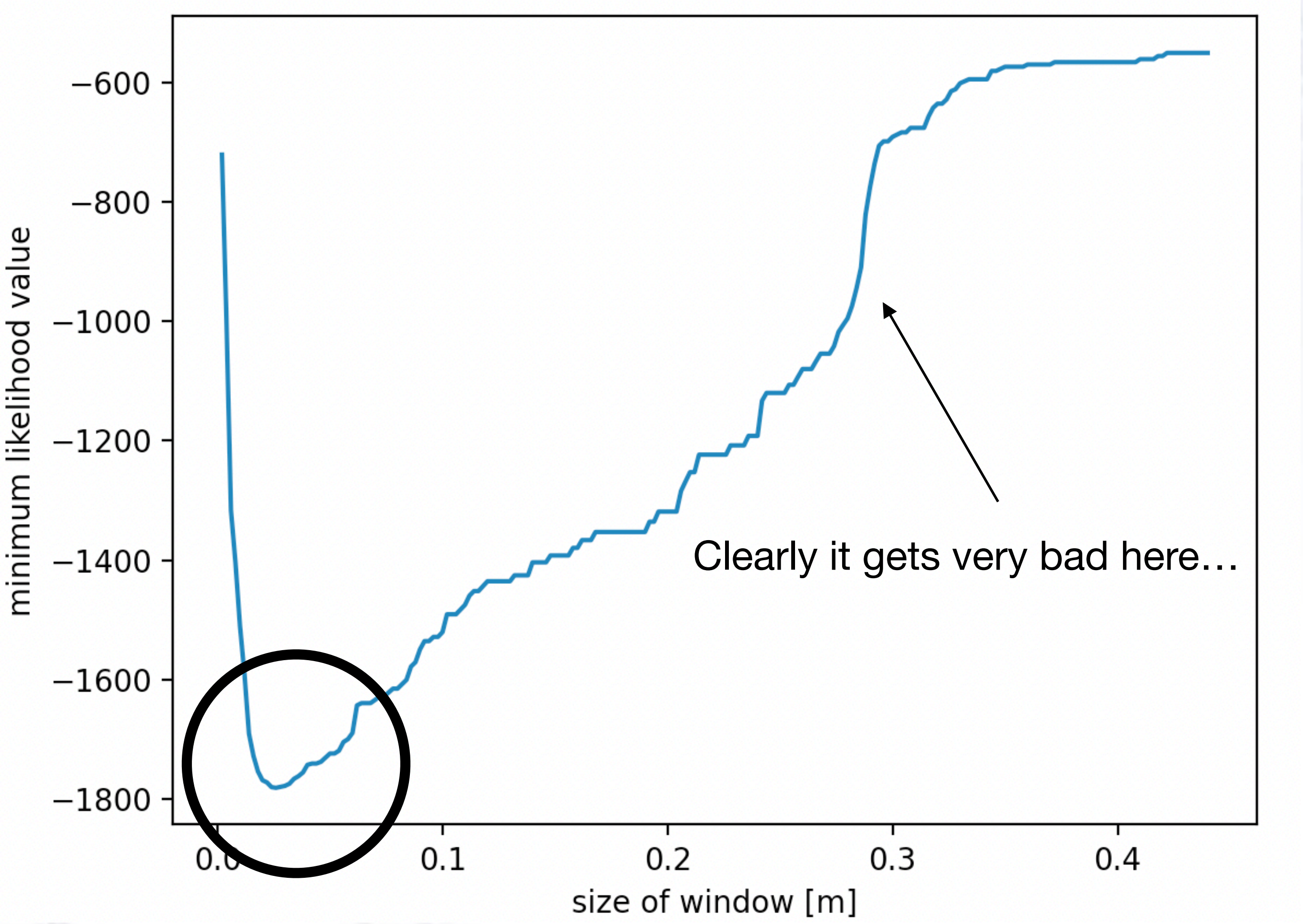
We therefore want to start by identifying the "most gaussian" part of the distribution be increasing the window size of the fitting



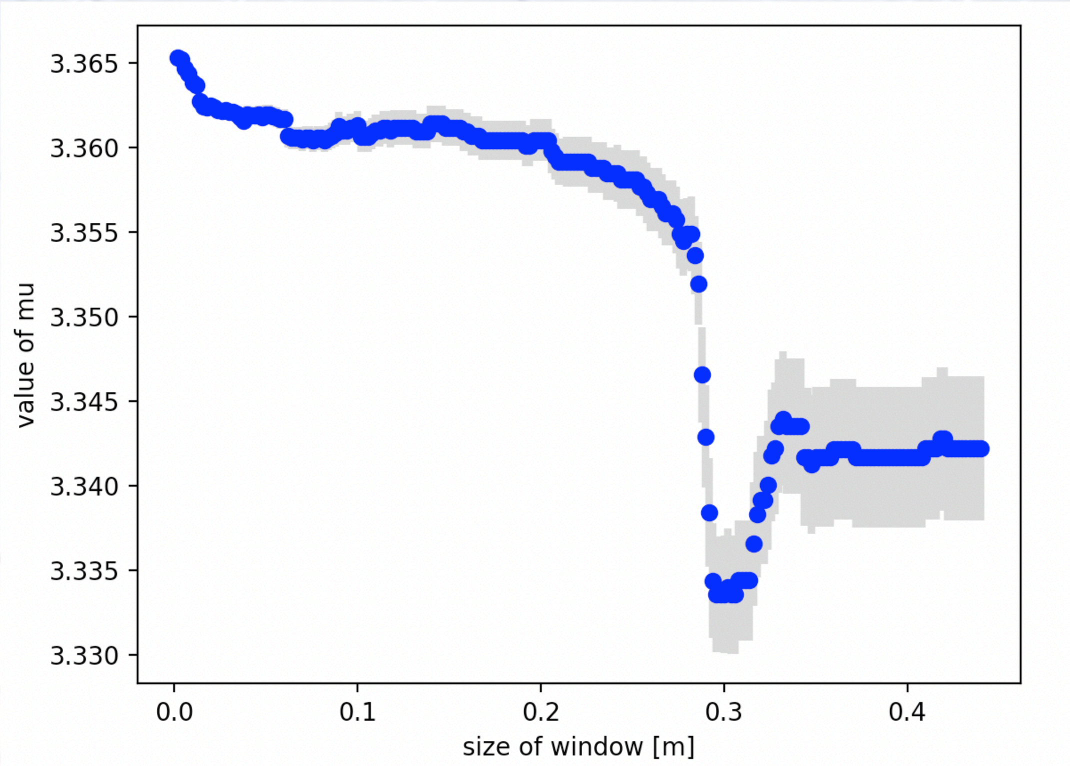
Finding the most gaussian window

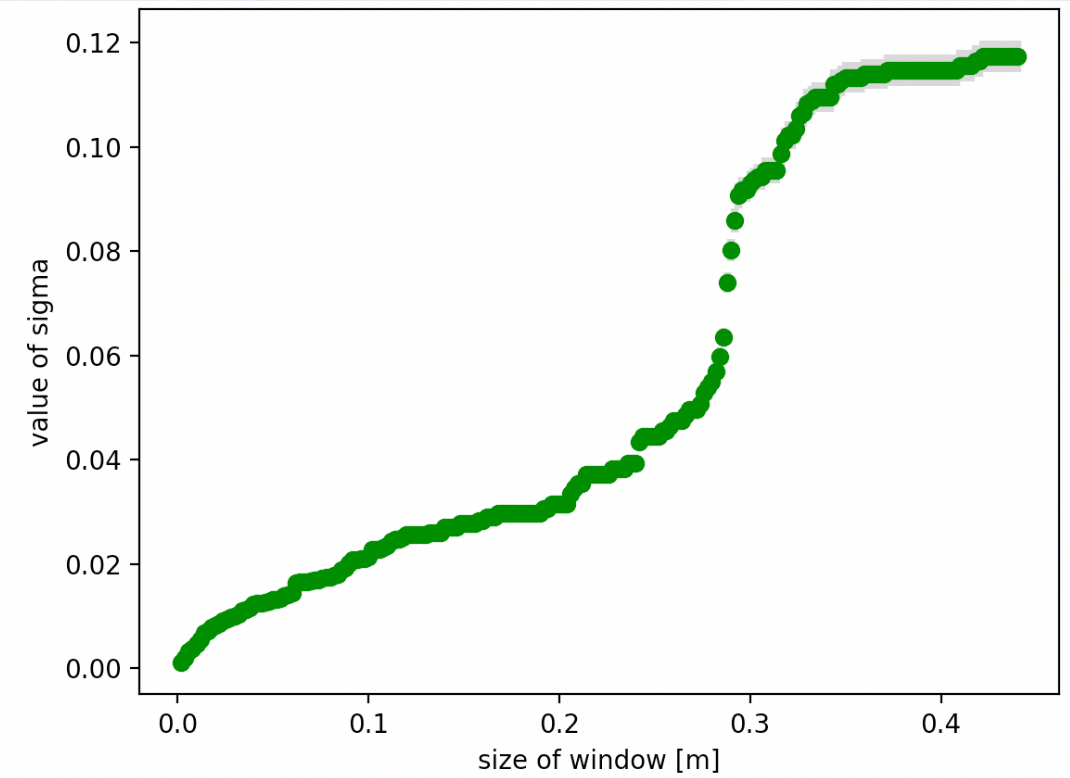
Here we search for the value of the maximal likelihood as a function of the window size.

This means that if we consider this window, we will work with data as gaussian as it comes for this distribution.



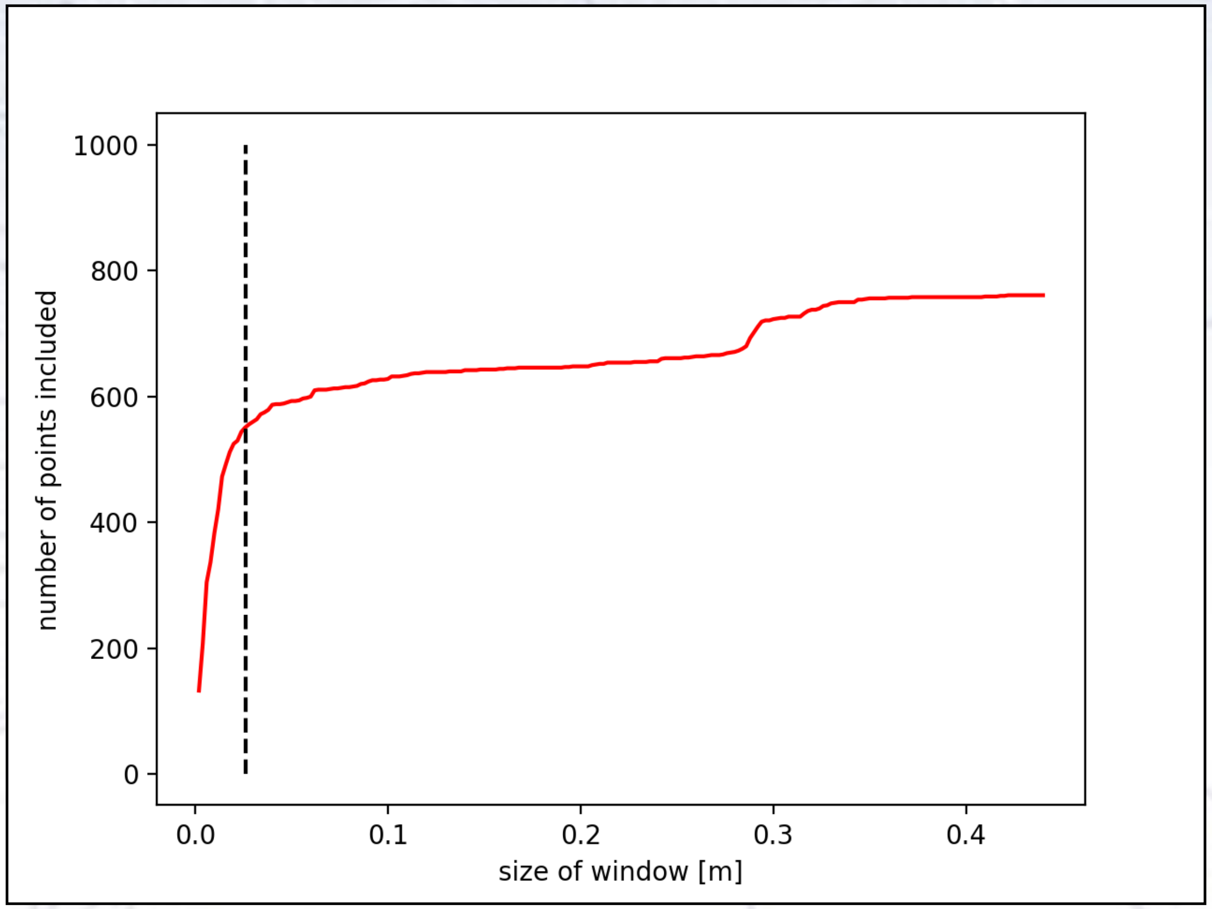
Is the central peak gaussian?

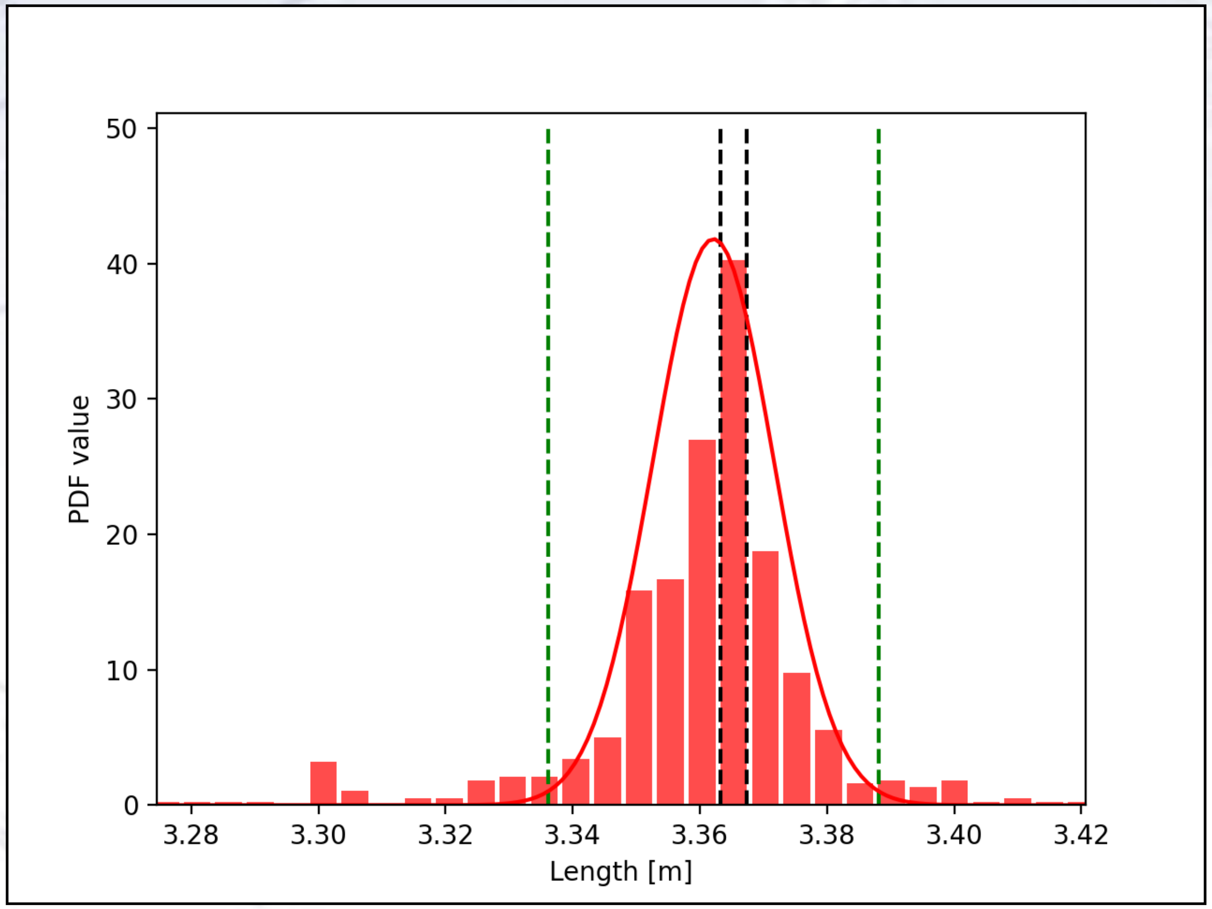




As we follow how the likelihood changes, we can extract the parameter estimates and their uncertainties as extracted from the parameter fits.

Is the central peak gaussian?





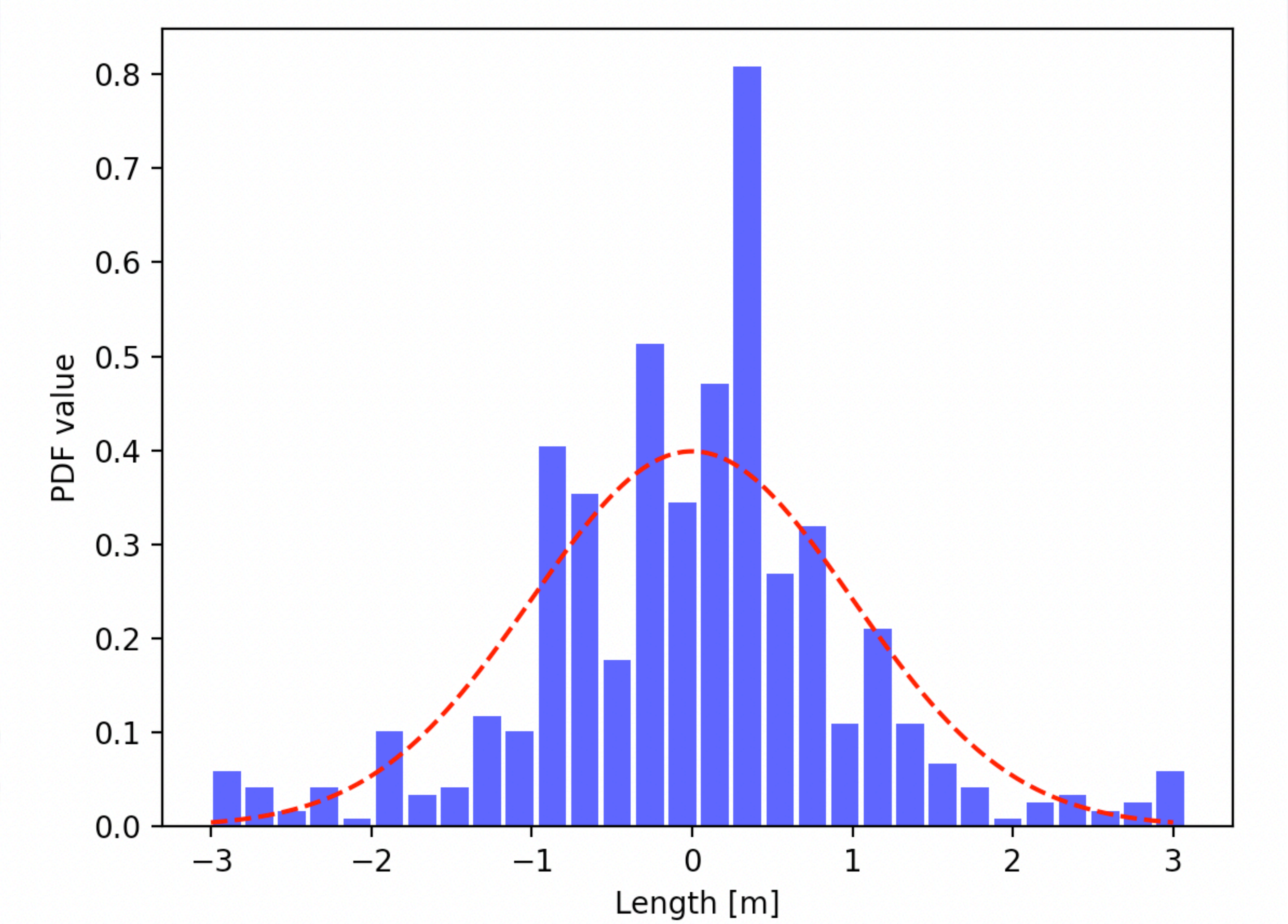
At this optimal likelihood, we note that we include 571 datapoints out of the original 761.

Drawing the green lines, to show our range of fitting, we can now use this standard deviation and reject points more than 4 sigmas away.

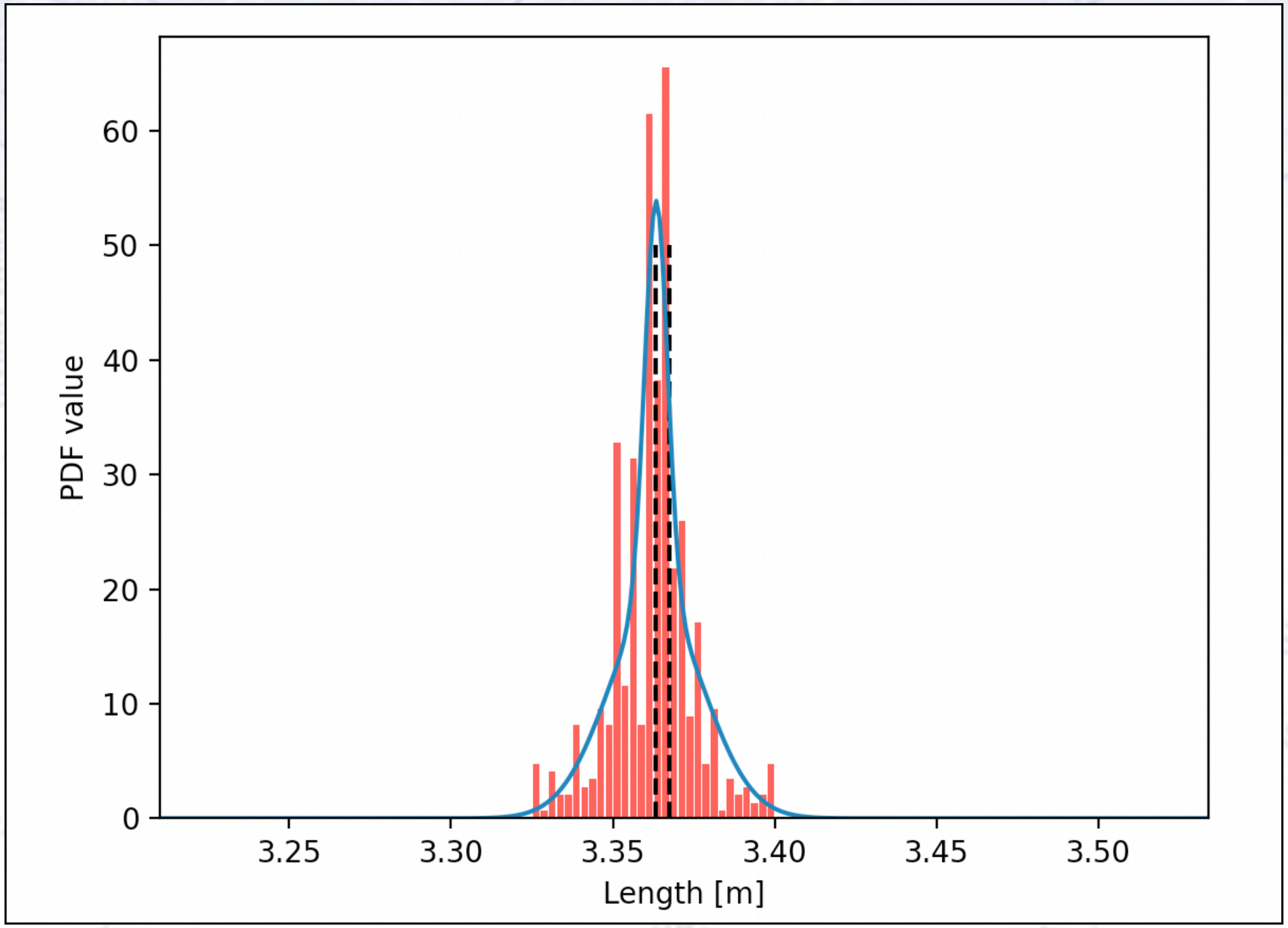
Weighted results

Considering the quoted uncertainties, we first need to evaluate their quality.

The plot to consider is a **PULL** plot, i.e. the distribution of: $z = \dfrac{x_i - \mu}{\sigma_i}$



Fitting the centered data with a double gaussian - unweighted case

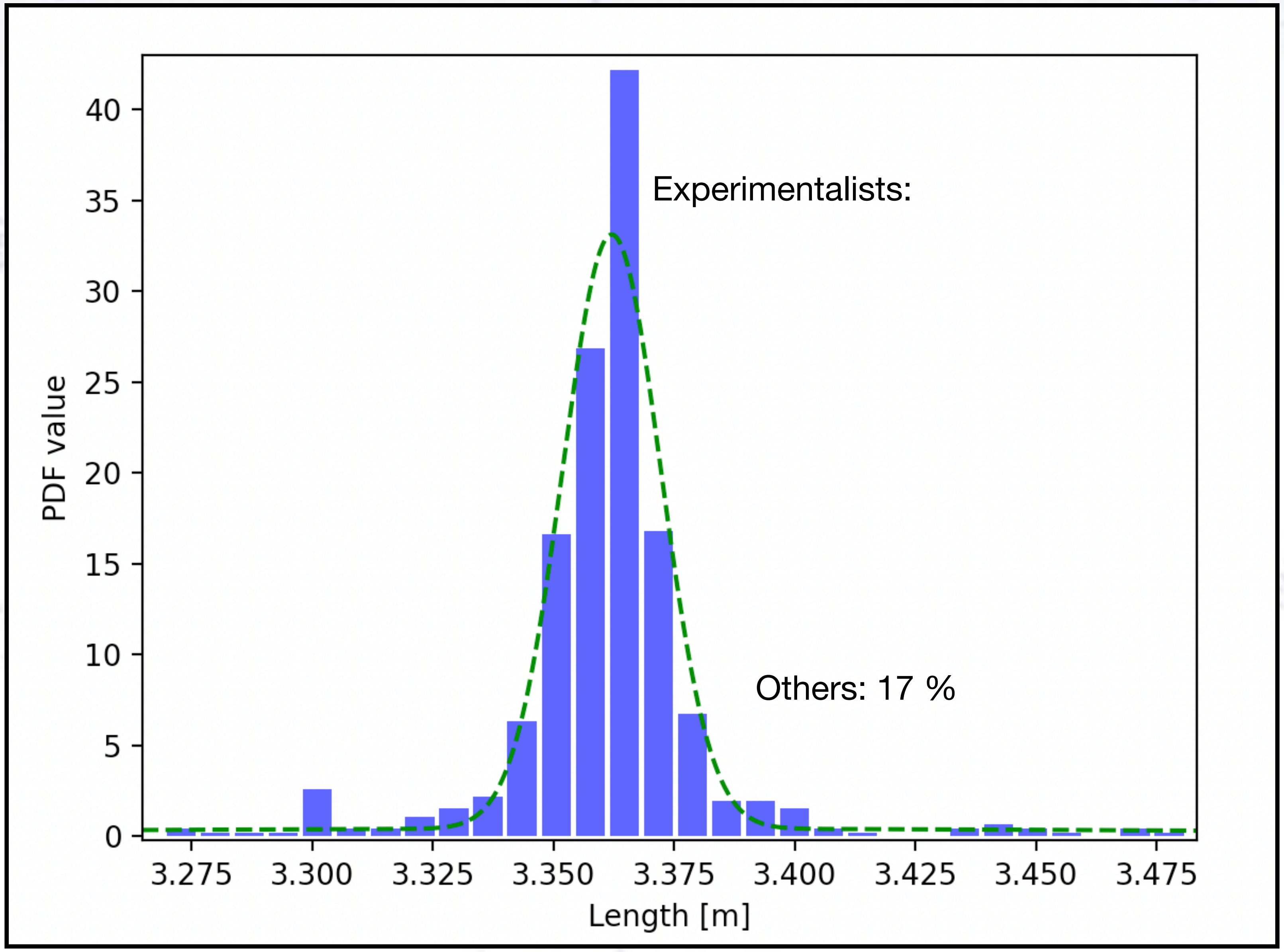


We realised that the center around the peak fitted OK with a double gaussian.

This also allows foran interpretation of the model: All measurers can be divided into two groups - the precise ("experimentalists") and the sloppy ("Others").

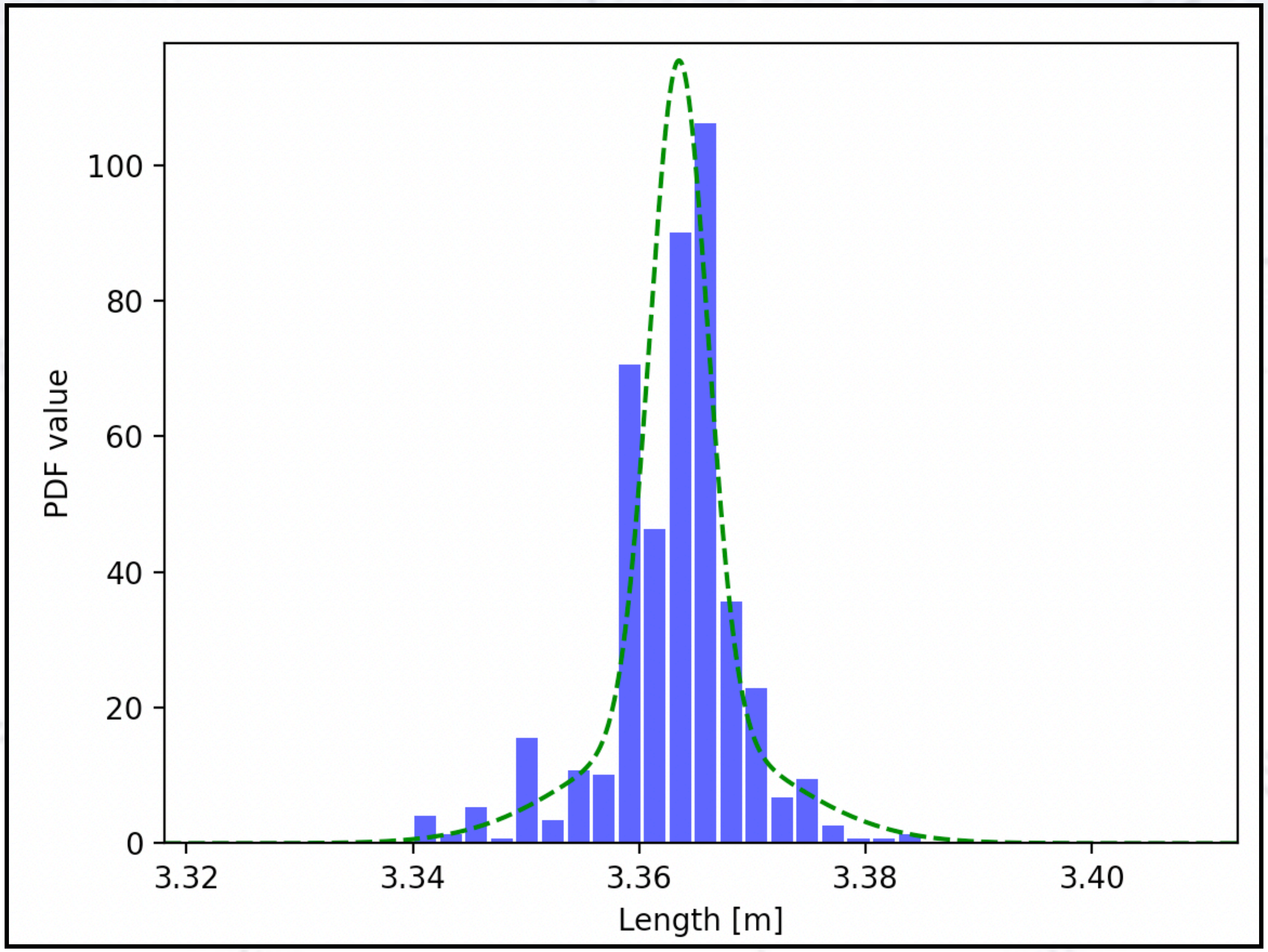
Weighted results with double gauss fit - 30cm

$$f(x|\mu,\sigma_1,\sigma_2,A) = \frac{A}{\sqrt{2\pi}\sigma_1}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma_1}\right)^2} + \frac{1-A}{\sqrt{2\pi}\sigma_2}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma_2}\right)^2}$$



$$\mu = 3.36207 \pm 0.00047$$
$$\sigma_1 = 0.159 \pm 0.012$$
$$\sigma_2 = 0.01015 \pm 0.00046$$
$$A = 0.1676 \pm 0.0177$$
$$N = 648$$

Weighted results with double gauss fit - 2m



$$\mu = 3.363519 \pm 0.00015$$
$$\sigma_1 = 0.00919 \pm 0.00057$$
$$\sigma_2 = 0.0026 \pm 0.00017$$
$$A = 0.362 \pm 0.042$$
$$N = 661$$

This is within 3 sigma away from the 30 cm measurements!

Fitting for a result

A completely different approach is to fit the RAW data, hence describing all data points instead of excluding some.

This approach is philosophically more clean, but certainly not easy!

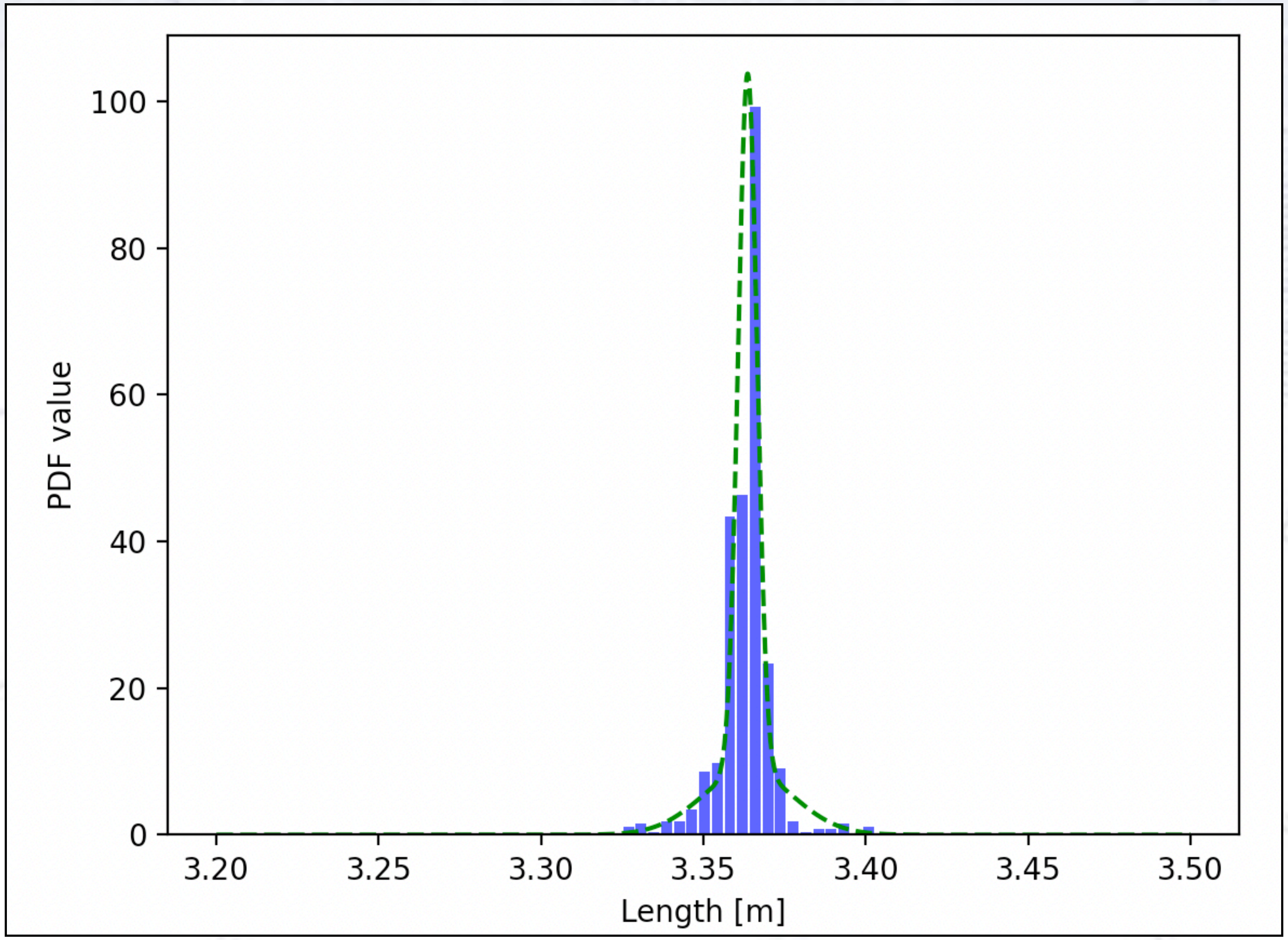
Challenges:

- Measurements has many different resolutions.
- There are several peaks in the data (30cm case).
- Some measurements are clearly rounded.

While all of these can be accommodated, it is still a challenge, at the following "fitting around" took me half and hour!

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.
John von Neumann

Fitting with double gaussian - unweighted 2 m case



$\mu = 3.36299 \pm 0.00022$

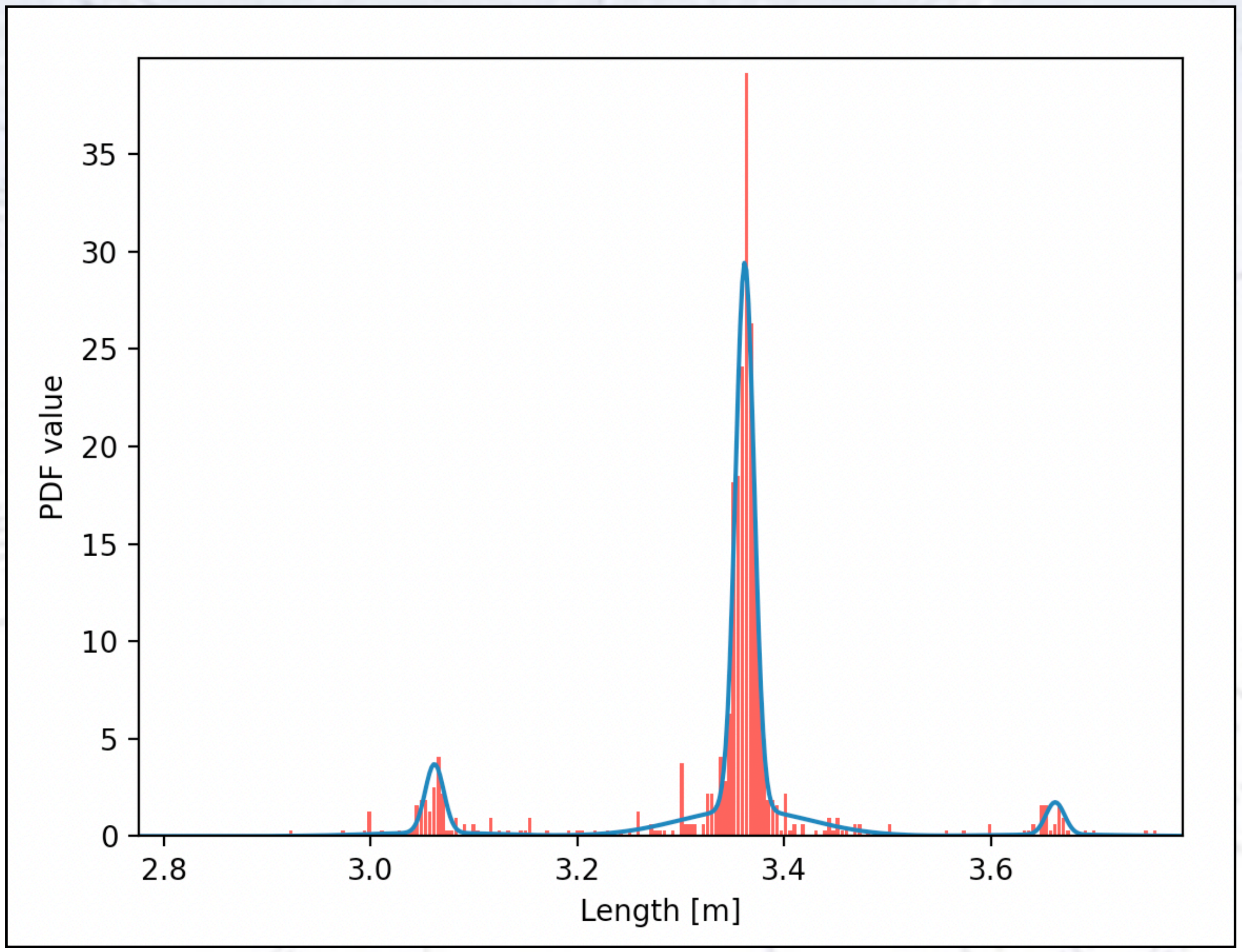
$\sigma_1 = 0.01404671374004074 \pm 0.00087$

$\sigma_2 = 0.00292 \pm 0.00018$

$A = 0.303 \pm 0.032$

$N = 681$

Fitting all three peaks - with a double gaussian



$$\mu = 3.36182 \pm 0.00043$$
$$\sigma_1 = 0.00898 \pm 0.00042$$
$$\sigma_2 = 0.0629 \pm 0.0045$$
$$A = 0.749 \pm 0.023$$
$$C_1 = 0.0439 \pm 0.0075$$
$$C_2 = 0.107 \pm 0.011$$

With this function, we have 6 free parameters and included "all" datapoints (except the outliers) so N = 761.

Note the uncertainty on the length of the table is 430 micrometer!

FCN = -1403 EDM = 0.000112 (Goal: 0.0001)		Ncalls = 208 (208 total) up = 0.5		
Valid Minimum	Valid Parameters	No Parameters at limit		
Below EDM threshold (goal x 10)		Below call limit		
Hesse ok	Has Covariance	Accurate	Pos. def.	Not forced

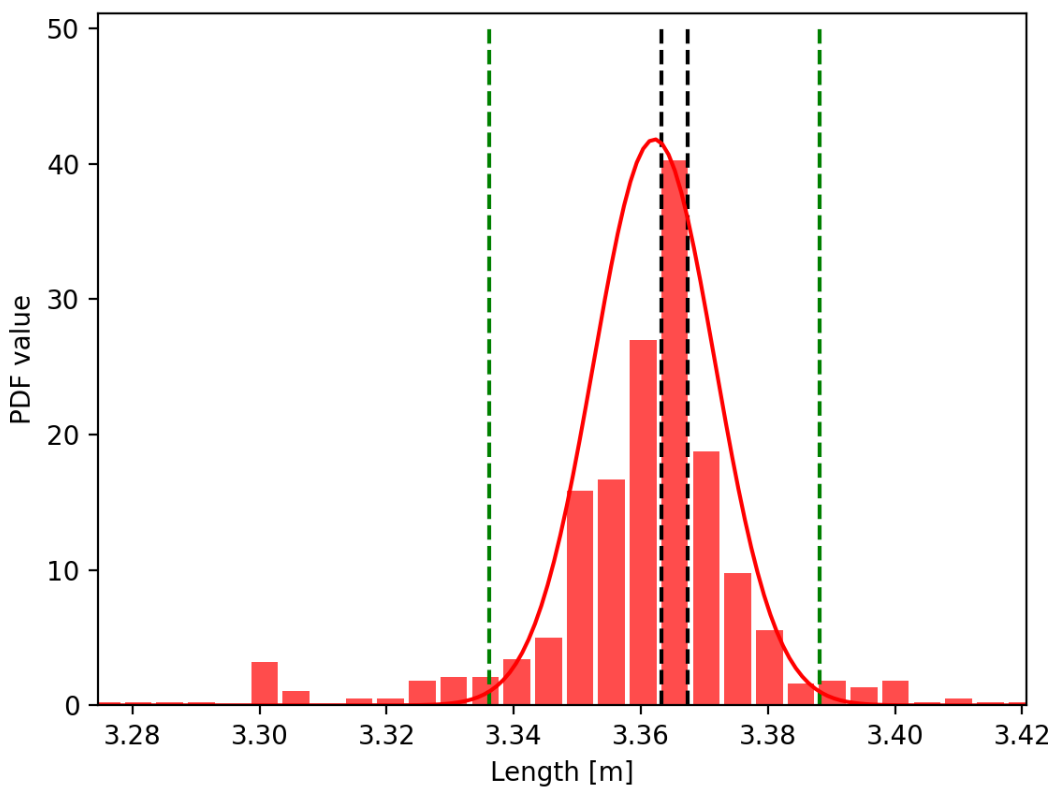
This is also within 3 sigma away from the 2 m measurements!

Does it have to be gaussian?

The Central limit Theorem, tells us that data is typically gaussian. This is a strong argument, that is typically good enough for our main hypotheses to be approximatively right.

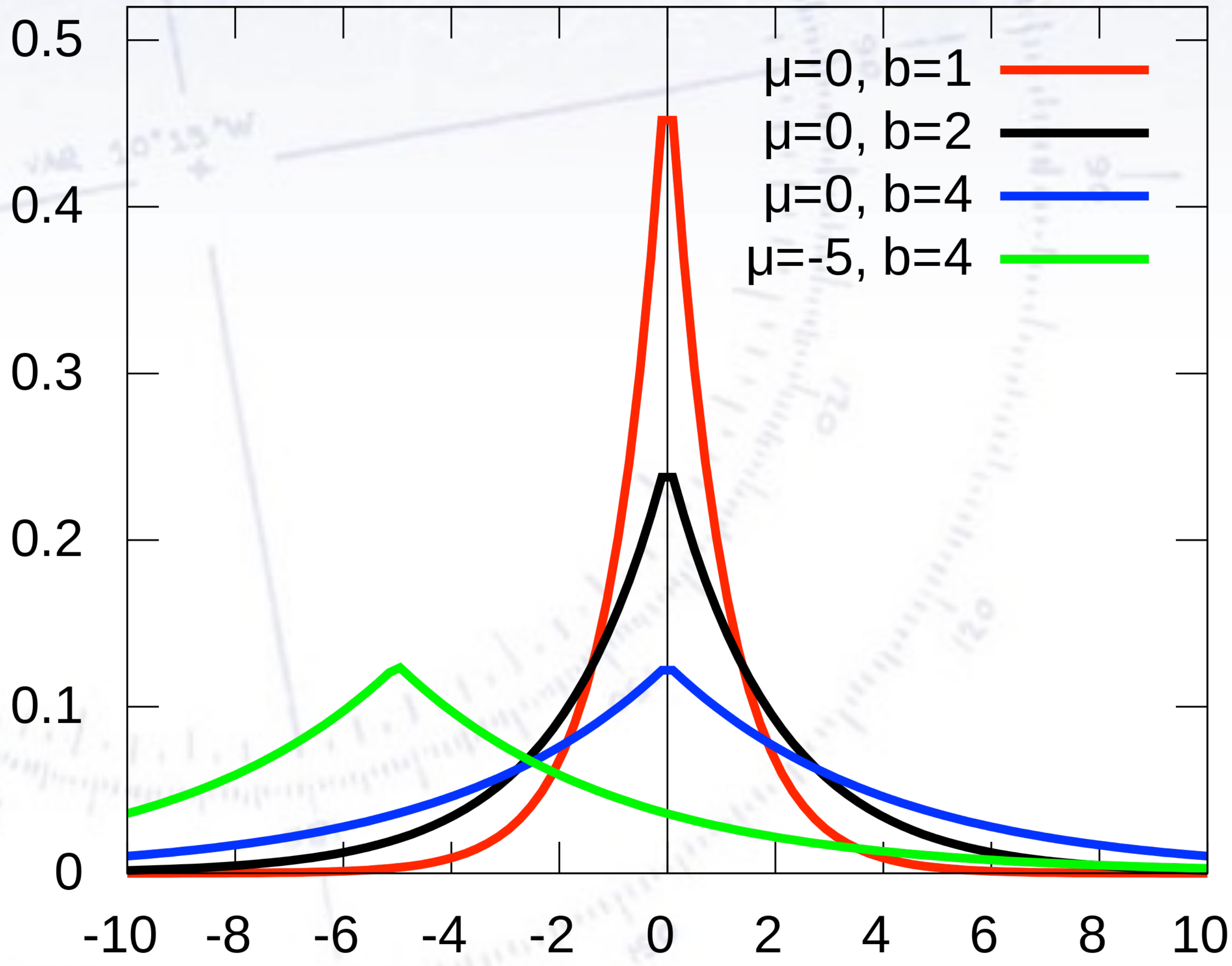
However having looked at the data, it is clearly not gaussian… The double gaussian worked better because the tails were not punished as much…

ince the gaussian falls like exp(-x^2), then maybe something decaying more slowly - as exp(-x)…

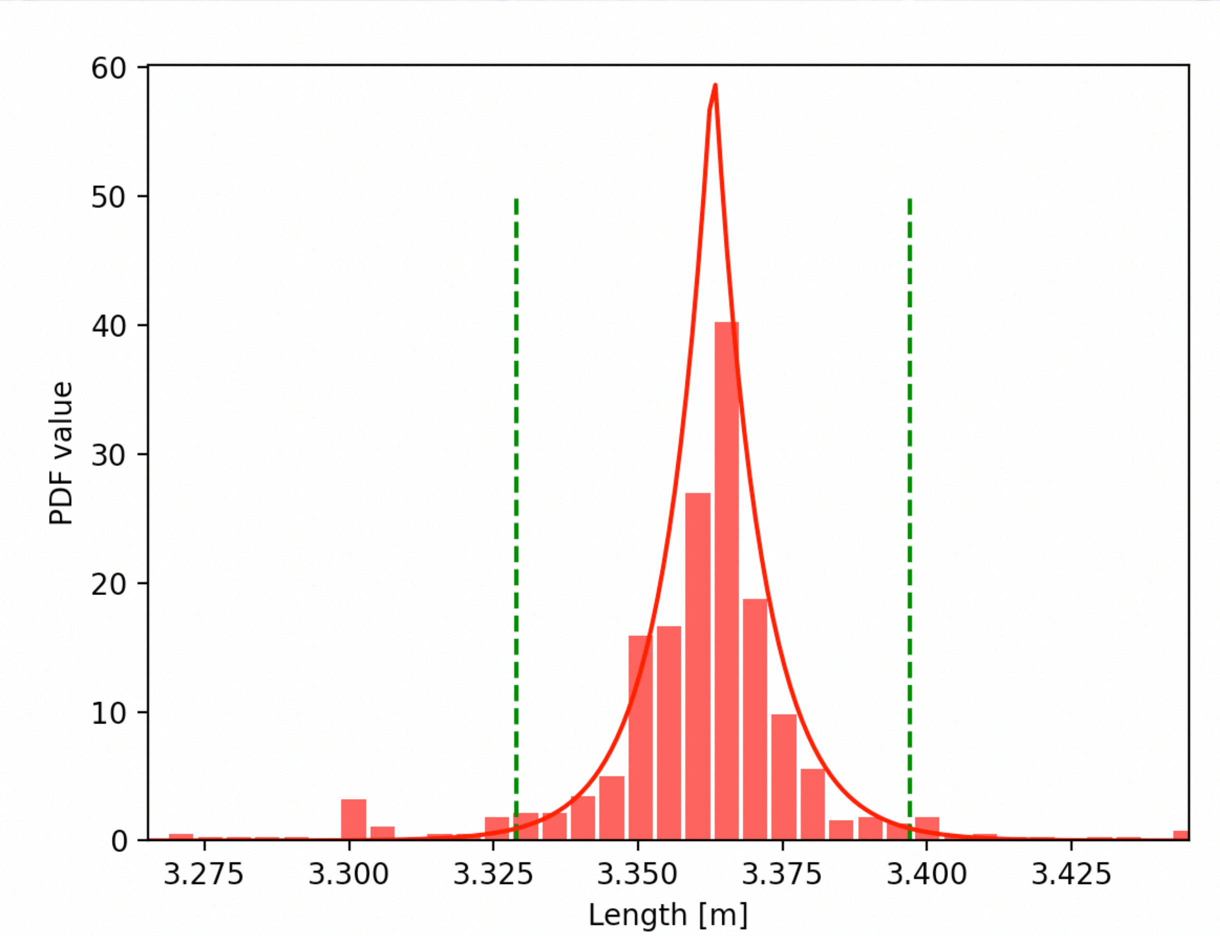


The Laplace distribution

$$f(x \mid \mu, b) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b} \right)$$



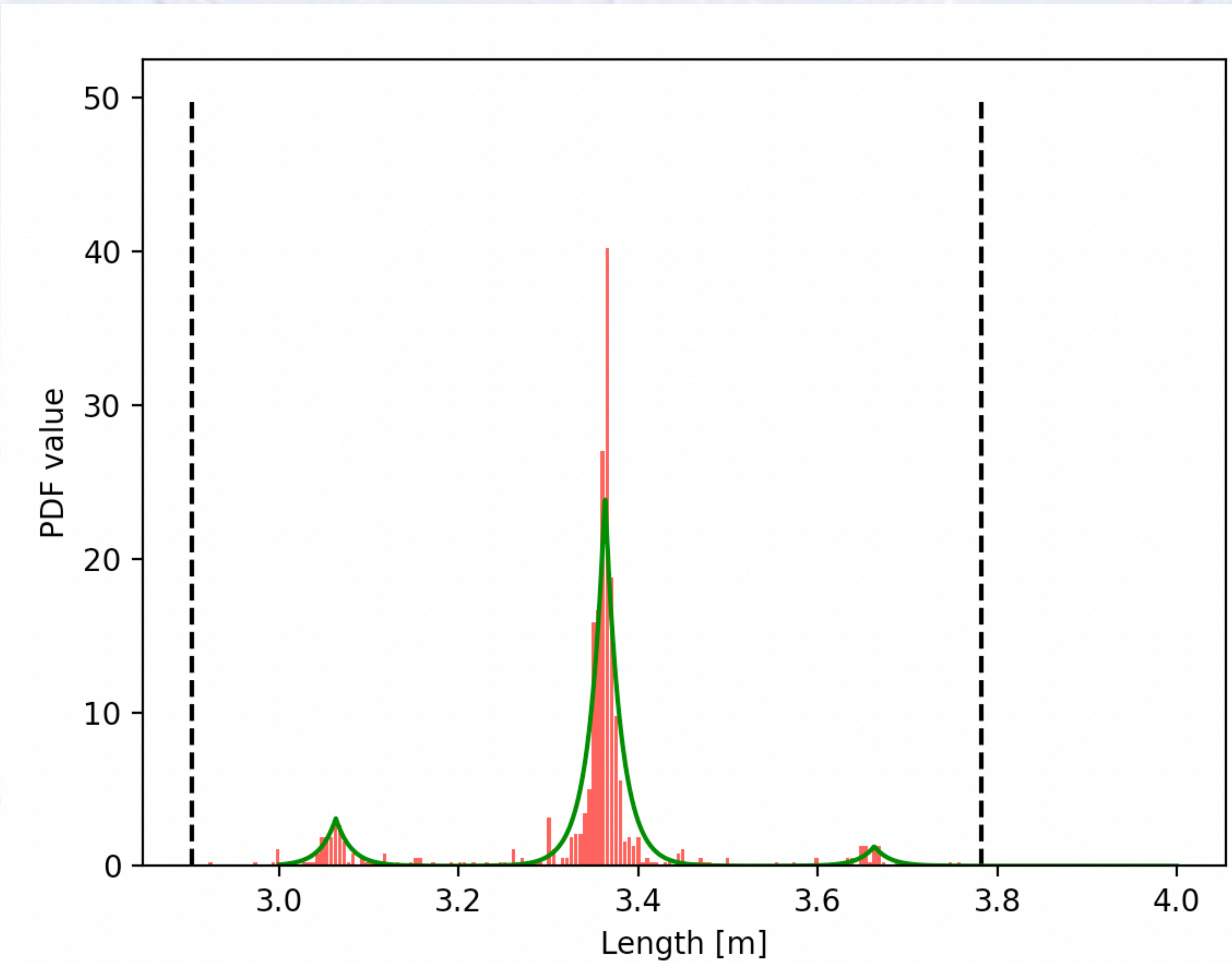
Fitting with the Laplace distribution



$$\mu = 3.363000 \pm 0.000019$$
$$b = 0.00815 \pm 0.00034$$
$$N = 571.0$$

Finding the region with the lowest likelihood gives a range with 571 datapoints included. Here we can use Minuit to find the two parameters and their uncertainty

Fitting "all" data with the Laplace distribution



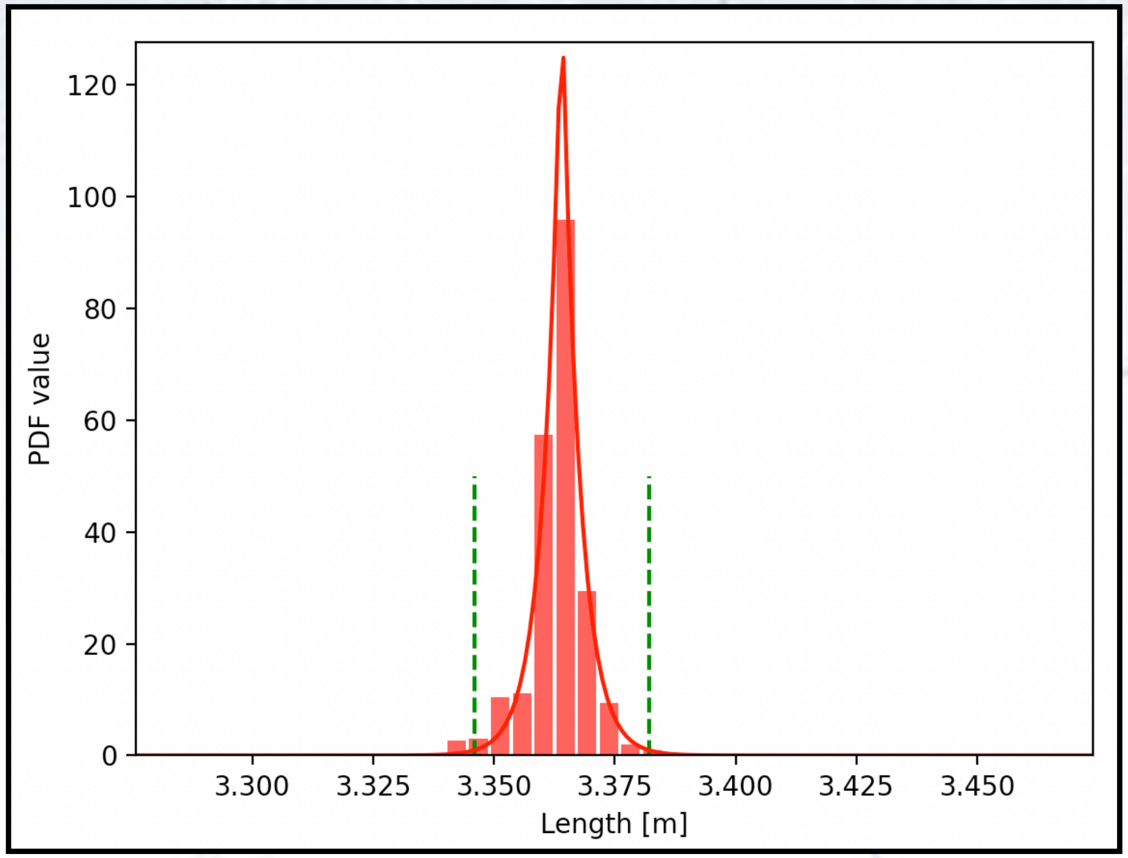
$$\mu = 3.3629999 \pm 0.0000162$$
$$b = 0.01740 \pm 0.00065$$
$$C_1 = 0.0445 \pm 0.0076$$
$$C_2 = 0.1080 \pm 0.0113$$
$$N = 765$$

FCN = -1403 EDM = 0.000112 (Goal: 0.0001)		Ncalls = 208 (208 total) up = 0.5		
Valid Minimum	Valid Parameters	No Parameters at limit		
Below EDM threshold (goal x 10)		Below call limit		
Hesse ok	Has Covariance	Accurate	Pos. def.	Not forced

With this function, we need only 4 free parameters and need only to reject the datapoints more than 4 sigma away from the original dataset!

Note the uncertainty on the length of the table is 16 micrometer!

Comparing with the 2m case



$$\mu = 3.3639999 \pm 0.0000025$$
$$b = 0.00362 \pm 0.00014$$
$$N = 663$$

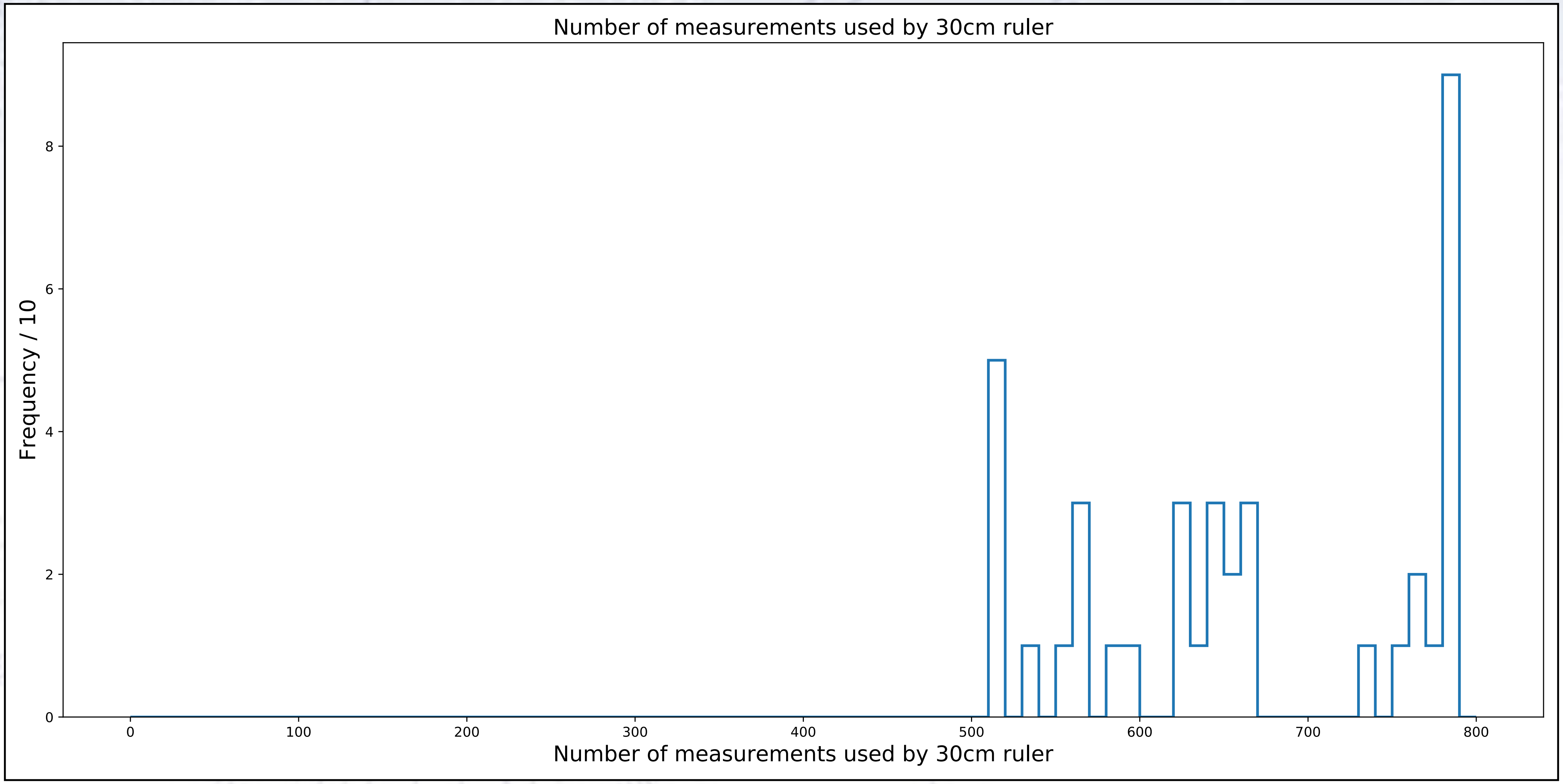
From the dataset of two meters, we can apply the same method, fitting the Laplace distribution, and here we get an uncertainty on the micrometer. Note however that the actual value is 1 millimetre longer than the 30 cm case…

Do we trust errors this small…? Hmm by using common sense we would expect an error on the mean to be approximately 2mm/sqrt(625) approx 80 um…

Your analyses

Your measurement results

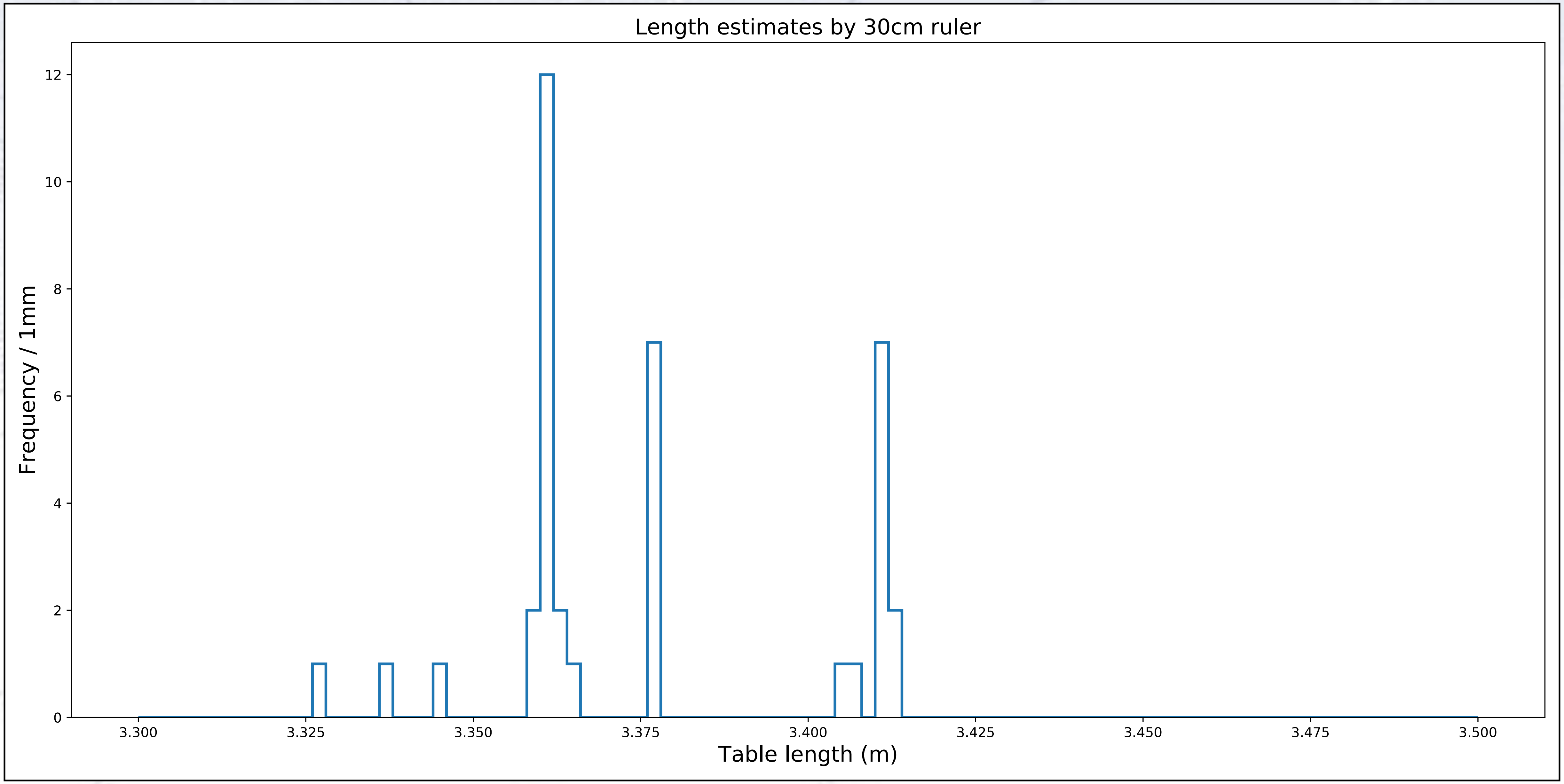
The number of measurements used varied quiet a bit.



But remember that the impact is only sqrt(N), and thus not that important!

Length results

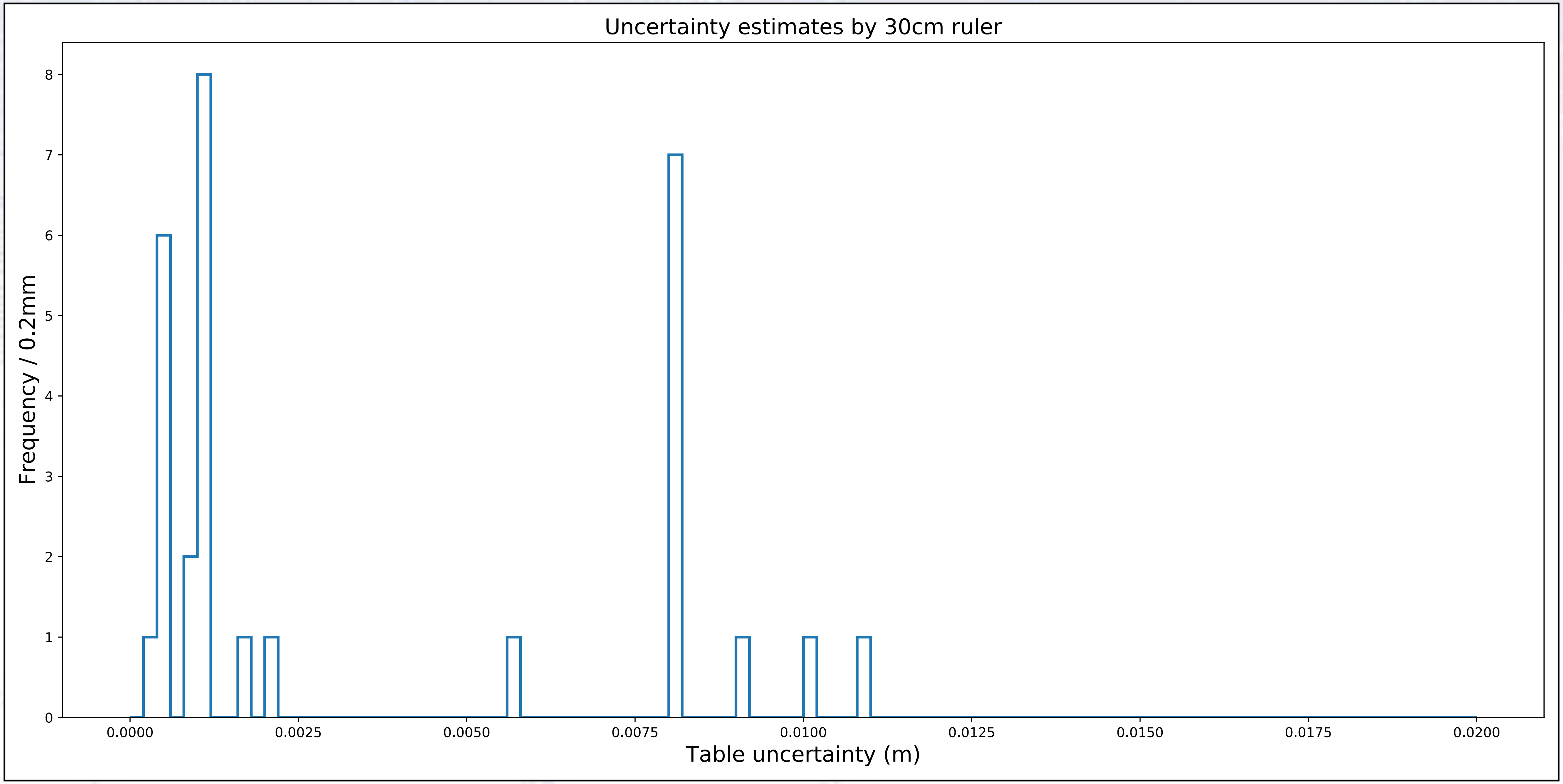
Results are relatively consistent… though some are above 3.4m, which is not correct!



I got: L(unweighted) = 3.36227 ± 0.00061 m, L(weighted) = 3.36371 ± 0.00035 m

Uncertainties

The uncertainties varied quite a bit - almost a factor 100! Think about that.



I got: L(unweighted) = 3.36227 ± 0.00061 m, L(weighted) = 3.36371 ± 0.00035 m

Conclusions

<u>Specifically on the analysis:</u>

- Greatest improvement came from simply removing mis-measurements!
- Weighted result was a further improvement, but required good uncertainties.
- The uncertainties are accepted as “reasonable”, as they have good pull distributions, and improve the result.
- The 30cm and 2m results match, giving credibility to the stated precision.

<u>More generally:</u>

- What appears to be a trivial task, turns out to require some thought anyhow.
  (Ask yourself how many fellow students would have been able to get a good result and error?)
- There were several choices to be made in the analysis:
  1. Which measurements to accept.
  2. Which uncertainties to accept.
  3. To correct or discard understood mis-measurements.
- All this can be solved with simple Python code.

Bonus Slides

…a fair hearing?

```
Rejected 30cm measurements:
 304: L=1.275  dL=2.071  Nsig=10.89   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=495  mean=3.3460  rms=0.1901   -> Rejected
 368: L=1.365  dL=1.985  Nsig=11.97   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=494  mean=3.3501  rms=0.1659   -> Rejected
  42: L=1.370  dL=1.984  Nsig=14.19   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=493  mean=3.3542  rms=0.1398   -> Rejected
  44: L=4.260  dL=0.902  Nsig= 8.39   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=492  mean=3.3582  rms=0.1075   -> Rejected
 146: L=2.563  dL=0.793  Nsig= 7.97   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=491  mean=3.3564  rms=0.0995   -> Rejected
 158: L=4.140  dL=0.782  Nsig= 8.41   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=490  mean=3.3580  rms=0.0929   -> Rejected
 143: L=2.670  dL=0.686  Nsig= 7.98   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=489  mean=3.3564  rms=0.0860   -> Rejected
 477: L=2.700  dL=0.658  Nsig= 8.19   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=488  mean=3.3578  rms=0.0803   -> Rejected
 308: L=4.004  dL=0.645  Nsig= 8.64   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=487  mean=3.3591  rms=0.0746   -> Rejected
 427: L=2.744  dL=0.614  Nsig= 8.93   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=486  mean=3.3578  rms=0.0687   -> Rejected
 407: L=2.759  dL=0.600  Nsig= 9.55   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=485  mean=3.3591  rms=0.0628   -> Rejected
 313: L=3.955  dL=0.595  Nsig=10.49   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=484  mean=3.3603  rms=0.0567   -> Rejected
 269: L=2.768  dL=0.591  Nsig=11.86   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=483  mean=3.3591  rms=0.0498   -> Rejected
 380: L=2.769  dL=0.591  Nsig=14.10   p1=0.00000000, p_all=0.00000000 >? pmin=0.050   N=482  mean=3.3603  rms=0.0419   -> Rejected
 133: L=3.180  dL=0.182  Nsig= 5.65   p1 … an=3.3616  rms=0.0321   -> Rejected
  71: L=3.200  dL=0.162  Nsig= 5.21   p1 … an=3.3619  rms=0.0311   -> Rejected
 320: L=3.205  dL=0.157  Nsig= 5.20   p1 … an=3.3623  rms=0.0302   -> Rejected
 319: L=3.215  dL=0.148  Nsig= 5.02   p1 … an=3.3626  rms=0.0294   -> Rejected
 194: L=3.221  dL=0.142  Nsig= 4.96   p1 … an=3.3629  rms=0.0286   -> Rejected
   1: L=3.250  dL=0.113  Nsig= 4.06   p1= … n=3.3632  rms=0.0279   -> Rejected
 175: L=3.471  dL=0.108  Nsig= 3.92   p1 … an=3.3634  rms=0.0274   -> Rejected
 133: L=3.470  dL=0.107  Nsig= 3.95   p1 … an=3.3632  rms=0.0270   -> Rejected
  13: L=3.470  dL=0.107  Nsig= 4.02   p1=0.00002883, p_all=0.01354566 >? pmin=0.050   N=473  mean=3.3630  rms=0.0266   -> Rejected
 154: L=3.467  dL=0.104  Nsig= 3.98   p1=0.00003406, p_all=0.01594736 >? pmin=0.050   N=472  mean=3.3628  rms=0.0262   -> Rejected
 130: L=3.260  dL=0.103  Nsig= 3.98   p1=0.00003423, p_all=0.01599227 >? pmin=0.050   N=471  mean=3.3625  rms=0.0258   -> Rejected
 230: L=3.260  dL=0.103  Nsig= 4.05   p1=0.00002507, p_all=0.01171573 >? pmin=0.050   N=470  mean=3.3628  rms=0.0253   -> Rejected
 444: L=3.461  dL=0.098  Nsig= 3.93   p1=0.00004185, p_all=0.01943772 >? pmin=0.050   N=469  mean=3.3630  rms=0.0249   -> Rejected
  79: L=3.460  dL=0.097  Nsig= 3.96   p1=0.00003687, p_all=0.01710909 >? pmin=0.050   N=468  mean=3.3628  rms=0.0245   -> Rejected
  43: L=3.455  dL=0.092  Nsig= 3.83   p1=0.00006418, p_all=0.02952774 >? pmin=0.050   N=467  mean=3.3626  rms=0.0241   -> Rejected
 113: L=3.455  dL=0.093  Nsig= 3.90   p1=0.00004898, p_all=0.02256478 >? pmin=0.050   N=466  mean=3.3624  rms=0.0238   -> Rejected
 433: L=3.272  dL=0.090  Nsig= 3.85   p1=0.00005881, p_all=0.02697886 >? pmin=0.050   N=465  mean=3.3622  rms=0.0234   -> Rejected
  81: L=3.450  dL=0.088  Nsig= 3.80   p1=0.00007216, p_all=0.03292944 >? pmin=0.050   N=464  mean=3.3624  rms=0.0231   -> Rejected
 345: L=3.450  dL=0.088  Nsig= 3.87   p1=0.00005540, p_all=0.02532365 >? pmin=0.050   N=463  mean=3.3622  rms=0.0227   -> Rejected
 351: L=3.450  dL=0.088  Nsig= 3.93   p1=0.00004175, p_all=0.01910567 >? pmin=0.050   N=462  mean=3.3620  rms=0.0224   -> Rejected
 333: L=3.450  dL=0.088  Nsig= 4.01   p1=0.00003083, p_all=0.01411413 >? pmin=0.050   N=461  mean=3.3618  rms=0.0220   -> Rejected
 324: L=3.448  dL=0.086  Nsig= 3.99   p1=0.00003299, p_all=0.01506122 >? pmin=0.050   N=460  mean=3.3616  rms=0.0217   -> Rejected
 167: L=3.447  dL=0.086  Nsig= 4.02   p1=0.00002922, p_all=0.01332302 >? pmin=0.050   N=459  mean=3.3614  rms=0.0213   -> Rejected
 126: L=3.445  dL=0.084  Nsig= 4.00   p1=0.00003155, p_all=0.01434519 >? pmin=0.050   N=458  mean=3.3612  rms=0.0209   -> Rejected
 162: L=3.445  dL=0.084  Nsig= 4.08   p1=0.00002276, p_all=0.01034822 >? pmin=0.050   N=457  mean=3.3610  rms=0.0206   -> Rejected
 340: L=3.280  dL=0.081  Nsig= 4.00   p1=0.00003223, p_all=0.01458856 >? pmin=0.050   N=456  mean=3.3609  rms=0.0202   -> Rejected
 393: L=3.285  dL=0.076  Nsig= 3.82   p1=0.00006640, p_all=0.02975990 >? pmin=0.050   N=455  mean=3.3610  rms=0.0199   -> Rejected
 121: L=3.433  dL=0.072  Nsig= 3.66   p1=0.00012405, p_all=0.05476548 >? pmin=0.050   N=454  mean=3.3612  rms=0.0196   -> Accepted
```

Rejected:
41 data points from the 30cm sample,
63 data points from the 2m sample.
Each and every one was inspected!

Excluded data due to bad pull

30cm: Warning! Large pull: L = 3.4700 +- 0.0080 pull = 13.60
30cm: Warning! Large pull: L = 3.3700 +- 0.0010 pull = 8.80
30cm: Warning! Large pull: L = 1.3700 +- 0.0100 pull = -199.12
30cm: Warning! Large pull: L = 4.2600 +- 0.1000 pull = 8.99
30cm: Warning! Large pull: L = 3.3000 +- 0.0100 pull = -6.12
30cm: Warning! Large pull: L = 2.6700 +- 0.0100 pull = -69.12
30cm: Warning! Large pull: L = 2.5630 +- 0.0200 pull = -39.91
30cm: Warning! Large pull: L = 4.1400 +- 0.0400 pull = 19.47
30cm: Warning! Large pull: L = 3.4670 +- 0.0120 pull = 8.82
30cm: Warning! Large pull: L = 3.4710 +- 0.0050 pull = 21.96
30cm: Warning! Large pull: L = 3.4470 +- 0.0050 pull = 17.16
30cm: Warning! Large pull: L = 3.2210 +- 0.0310 pull = -4.52
30cm: Warning! Large pull: L = 3.3470 +- 0.0010 pull = -14.20
30cm: Warning! Large pull: L = 3.3650 +- 0.0005 pull = 7.60
30cm: Warning! Large pull: L = 3.3670 +- 0.0010 pull = 5.80
30cm: Warning! Large pull: L = 3.3290 +- 0.005
30cm: Warning! Large pull: L = 3.2600 +- 0.002
30cm: Warning! Large pull: L = 3.4240 +- 0.005
30cm: Warning! Large pull: L = 3.3050 +- 0.010
30cm: Warning! Large pull: L = 3.3000 +- 0.011
30cm: Warning! Large pull: L = 2.7680 +- 0.020
30cm: Warning! Large pull: L = 3.4200 +- 0.005
30cm: Warning! Large pull: L = 3.3900 +- 0.005
30cm: Warning! Large pull: L = 1.2750 +- 0.003
30cm: Warning! Large pull: L = 4.0040 +- 0.0300 pull = 21.43
30cm: Warning! Large pull: L = 3.9550 +- 0.0550 pull = 10.80
30cm: Warning! Large pull: L = 3.2150 +- 0.0110 pull = -13.29
30cm: Warning! Large pull: L = 1.3650 +- 0.0050 pull = -399.24
30cm: Warning! Large pull: L = 3.4110 +- 0.0050 pull = 9.96
30cm: Warning! Large pull: L = 3.4500 +- 0.0050 pull = 17.76
30cm: Warning! Large pull: L = 2.7690 +- 0.0050 pull = -118.44
30cm: Warning! Large pull: L = 3.3080 +- 0.0100 pull = -5.32
30cm: Warning! Large pull: L = 3.3700 +- 0.0010 pull = 8.80
30cm: Warning! Large pull: L = 2.7590 +- 0.0020 pull = -301.10
30cm: Warning! Large pull: L = 2.7440 +- 0.0500 pull = -12.34
30cm: Warning! Large pull: L = 3.3350 +- 0.0010 pull = -26.20
30cm: Warning! Large pull: L = 3.4610 +- 0.0100 pull = 9.98
30cm: Warning! Large pull: L = 2.7000 +- 0.0300 pull = -22.04

2m: Warning! Large pull: L = 3.0600 +- 0.0050 pull = -60.59
2m: Warning! Large pull: L = 3.7500 +- 0.0200 pull = 19.35
2m: Warning! Large pull: L = 3.3680 +- 0.0010 pull = 5.05
2m: Warning! Large pull: L = 3.6500 +- 0.0300 pull = 9.57
2m: Warning! Large pull: L = 3.3200 +- 0.0050 pull = -8.59
2m: Warning! Large pull: L = 3.6660 +- 0.0500 pull = 6.06
2m: Warning! Large pull: L = 3.5750 +- 0.0020 pull = 106.03
2m: Warning! Large pull: L = 3.0360 +- 0.0400 pull = -8.17
2m: Warning! Large pull: L = 3.3500 +- 0.0010 pull = -12.95
2m: Warning! Large pull: L = 3.0360 +- 0.0050 pull = -65.39
2m: Warning! Large pull: L = 2.3630 +- 0.0050 pull = -199.99
2m: Warning! Large pull: L = 3.3150 +- 0.0006 pull = -79.92
2m: Warning! Large pull: L = 3.4170 +- 0.0120 pull = 4.50
2m: Warning! Large pull: L = 3.6400 +- 0.0100 pull = 27.71
2m: Warning! Large pull: L = 4.0400 +- 0.0200 pull = 33.85
L = 4.0160 +- 0.1010 pull = 6.47
L = 3.0640 +- 0.0050 pull = -59.79
L = 3.6600 +- 0.0120 pull = 24.75
L = 3.3550 +- 0.0010 pull = -7.95
L = 3.7500 +- 0.0050 pull = 77.41
L = 3.3690 +- 0.0010 pull = 6.05
L = 3.0070 +- 0.0080 pull = -44.49
L = 3.1680 +- 0.0050 pull = -38.99
L = 3.0600 +- 0.0050 pull = -60.59
2m: Warning! Large pull: L = 3.3720 +- 0.0020 pull = 4.53
2m: Warning! Large pull: L = 3.1500 +- 0.0100 pull = -21.29
2m: Warning! Large pull: L = 2.3640 +- 0.0300 pull = -33.30
2m: Warning! Large pull: L = 3.4200 +- 0.0050 pull = 11.41
2m: Warning! Large pull: L = 3.3720 +- 0.0011 pull = 8.23
2m: Warning! Large pull: L = 3.3970 +- 0.0030 pull = 11.35
2m: Warning! Large pull: L = 3.6050 +- 0.0150 pull = 16.14
2m: Warning! Large pull: L = 3.3070 +- 0.0050 pull = -11.19
2m: Warning! Large pull: L = 3.3730 +- 0.0020 pull = 5.03
2m: Warning! Large pull: L = 3.3760 +- 0.0020 pull = 6.53
2m: Warning! Large pull: L = 2.3650 +- 0.0050 pull = -199.59
2m: Warning! Large pull: L = 2.3650 +- 0.0040 pull = -249.49
2m: Warning! Large pull: L = 3.0020 +- 0.0700 pull = -5.16
2m: Warning! Large pull: L = 3.2600 +- 0.0200 pull = -5.15
2m: Warning! Large pull: L = 3.2650 +- 0.0010 pull = -97.95
2m: Warning! Large pull: L = 3.5000 +- 0.0200 pull = 6.85
2m: Warning! Large pull: L = 3.1695 +- 0.0020 pull = -96.72
2m: Warning! Large pull: L = 3.3660 +- 0.0003 pull = 12.20
2m: Warning! Large pull: L = 3.6540 +- 0.0200 pull = 14.55
2m: Warning! Large pull: L = 3.3710 +- 0.0010 pull = 8.05
2m: Warning! Large pull: L = 3.0650 +- 0.0050 pull = -59.59

Rejected:
38 data points from the 30cm sample,
45 data points from the 2m sample.
Each and every one was inspected!

Excluded data due to bad pull

30cm: Warning! Large pull: L = 3.4700 +- 0.0080   pull = 13.60
30cm: Warning! Large pull: L = 3.3700 +- 0.0010   pull =   8.80
30cm: Warning! Large pull: L = 1.3700 +- 0.0100   pull = -199.12
30cm: Warning! Large pull: L = 4.2600 +- 0.1000   pull =   8.99
30cm: Warning! Large pull: L = 3.3000 +- 0.0100   pull = -6.12
30cm: Warning! Large pull: L = 2.6700 +- 0.0100   pull = -69.12
30cm: Warning! Large pull: L = 2.5630 +- 0.0200   pull = -39.91
30cm: Warning! Large pull: L = 4.1400 +- 0.0400   pull = 19.47
30cm: Warning! Large pull: L = 3.4670 +- 0.0120   pull =   8.82
30cm: Warning! Large pull: L = 3.4710 +- 0.0050   pull = 21.96
30cm: Warning! Large pull: L = 3.4470 +- 0.0050   pull = 17.16
30cm: Warning! Large pull: L = 3.2210 +- 0.0310   pull = -4.52
30cm: Warning! Large pull: L = 3.3470 +- 0.0010   pull = -14.20
30cm: Warning! Large pull: L = 3.3650 +- 0.0005   pull =   7.60
30cm: Warning! Large pull: L = 3.3670 +- 0.0010   pull =   5.80
30cm: Warning! Large pull: L = 3.3290 +- 0.005
30cm: Warning! Large pull: L = 3.2600 +- 0.002
30cm: Warning! Large pull: L = 3.4240 +- 0.005
30cm: Warning! Large pull: L = 3.3050 +- 0.010
30cm: Warning! Large pull: L = 3.3000 +- 0.011
30cm: Warning! Large pull: L = 2.7680 +- 0.020
30cm: Warning! Large pull: L = 3.4200 +- 0.005
30cm: Warning! Large pull: L = 3.3900 +- 0.005
30cm: Warning! Large pull: L = 1.2750 +- 0.003
30cm: Warning! Large pull: L = 4.0040 +- 0.0300   pull = 21.43
30cm: Warning! Large pull: L = 3.9550 +- 0.0550   pull = 10.80
30cm: Warning! Large pull: L = 3.2150 +- 0.0110   pull = -13.29
30cm: Warning! Large pull: L = 1.3650 +- 0.0050   pull = -399.24
30cm: Warning! Large pull: L = 3.4110 +- 0.0050   pull =   9.96
30cm: Warning! Large pull: L = 3.4500 +- 0.0050   pull = 17.76
30cm: Warning! Large pull: L = 2.7690 +- 0.0050   pull = -118.44
30cm: Warning! Large pull: L = 3.3080 +- 0.0100   pull = -5.32
30cm: Warning! Large pull: L = 3.3700 +- 0.0010   pull =   8.80
30cm: Warning! Large pull: L = 2.7590 +- 0.0020   pull = -301.10
30cm: Warning! Large pull: L = 2.7440 +- 0.0500   pull = -12.34
30cm: Warning! Large pull: L = 3.3350 +- 0.0010   pull = -26.20
30cm: Warning! Large pull: L = 3.4610 +- 0.0100   pull =   9.98
30cm: Warning! Large pull: L = 2.7000 +- 0.0300   pull = -22.04

Rejected:
38 data points from the 30cm sample,
45 data points from the 2m sample.
Each and every one was inspected!

2m: Warning! Large pull: L = 3.0600 +- 0.0050   pull = -60.59
2m: Warning! Large pull: L = 3.7500 +- 0.0200   pull = 19.35
2m: Warning! Large pull: L = 3.3680 +- 0.0010   pull =   5.05
2m: Warning! Large pull: L = 3.6500 +- 0.0300   pull =   9.57
2m: Warning! Large pull: L = 3.3200 +- 0.0050   pull = -8.59
2m: Warning! Large pull: L = 3.6660 +- 0.0500   pull =   6.06
2m: Warning! Large pull: L = 3.5750 +- 0.0020   pull = 106.03
2m: Warning! Large pull: L = 3.0360 +- 0.0400   pull = -8.17
2m: Warning! Large pull: L = 3.3500 +- 0.0010   pull = -12.95
2m: Warning! Large pull: L = 3.0360 +- 0.0050   pull = -65.39
2m: Warning! Large pull: L = 2.3630 +- 0.0050   pull = -199.99
2m: Warning! Large pull: L = 3.3150 +- 0.0006   pull = -79.92
2m: Warning! Large pull: L = 3.4170 +- 0.0120   pull =   4.50
2m: Warning! Large pull: L = 3.6400 +- 0.0100   pull = 27.71
2m: Warning! Large pull: L = 4.0400 +- 0.0200   pull = 33.85
L = 4.0160 +- 0.1010   pull =   6.47
L = 3.0640 +- 0.0050   pull = -59.79
L = 3.6600 +- 0.0120   pull = 24.75
L = 3.3550 +- 0.0010   pull = -7.95
L = 3.7500 +- 0.0050   pull = 77.41
L = 3.3690 +- 0.0010   pull =   6.05
L = 3.0070 +- 0.0080   pull = -44.49
L = 3.1680 +- 0.0050   pull = -38.99
L = 3.0600 +- 0.0050   pull = -60.59
2m: Warning! Large pull: L = 3.3720 +- 0.0020   pull =   4.53
2m: Warning! Large pull: L = 3.1500 +- 0.0100   pull = -21.29
2m: Warning! Large pull: L = 2.3640 +- 0.0300   pull = -33.30
2m: Warning! Large pull: L = 3.4200 +- 0.0050   pull = 11.41
2m: Warning! Large pull: L = 3.3720 +- 0.0011   pull =   8.23
2m: Warning! Large pull: L = 3.3970 +- 0.0030   pull = 11.35
2m: Warning! Large pull: L = 3.6050 +- 0.0150   pull = 16.14
2m: Warning! Large pull: L = 3.3070 +- 0.0050   pull = -11.19
2m: Warning! Large pull: L = 3.3730 +- 0.0020   pull =   5.03
2m: Warning! Large pull: L = 3.3760 +- 0.0020   pull =   6.53
2m: Warning! Large pull: L = 2.3650 +- 0.0050   pull = -199.59
2m: Warning! Large pull: L = 2.3650 +- 0.0040   pull = -249.49
2m: Warning! Large pull: L = 3.0020 +- 0.0700   pull = -5.16
2m: Warning! Large pull: L = 3.2600 +- 0.0200   pull = -5.15
2m: Warning! Large pull: L = 3.2650 +- 0.0010   pull = -97.95
2m: Warning! Large pull: L = 3.5000 +- 0.0200   pull =   6.85
2m: Warning! Large pull: L = 3.1695 +- 0.0020   pull = -96.72
2m: Warning! Large pull: L = 3.3660 +- 0.0003   pull = 12.20
2m: Warning! Large pull: L = 3.6540 +- 0.0200   pull = 14.55
2m: Warning! Large pull: L = 3.3710 +- 0.0010   pull =   8.05
2m: Warning! Large pull: L = 3.0650 +- 0.0050   pull = -59.59

Notice, that close measurement
with too small errors are rejected!