

Solution for the Error Propagation exercise

The length and width of the table was measured to:

$$L = 3.5 \pm 0.4 \text{ m}$$

$$W = 0.8 \pm 0.2 \text{ m}$$

The Circumference, Area and Diagonal and their errors are calculated as following (assuming no correlation):

$$\begin{aligned} C &= 2L + 2W & A &= L \cdot W \\ \sigma_C &= \sqrt{(2\sigma_L)^2 + (2\sigma_W)^2} & \sigma_A &= \sqrt{(W\sigma_L)^2 + (L\sigma_W)^2} \\ C &= 8.6 \pm 0.9 \text{ m} & A &= 2.8 \pm 0.8 \text{ m} \end{aligned}$$

$$\begin{aligned} \sigma_D &= \sqrt{\left(\frac{L\sigma_L}{\sqrt{L^2 + W^2}}\right)^2 + \left(\frac{W\sigma_W}{\sqrt{L^2 + W^2}}\right)^2} \\ D &= 3.6 \pm 0.4 \text{ m} \end{aligned}$$

Now with correlation:

$$\begin{aligned} C &= 2L + 2W & A &= L \cdot W \\ \sigma_C &= \sqrt{(2\sigma_L)^2 + (2\sigma_W)^2 + (4\sigma_{LW})^2} & \sigma_A &= \sqrt{(W\sigma_L)^2 + (L\sigma_W)^2 + 2LW\sigma_{LW}^2} \end{aligned}$$

$$\sigma_D = \sqrt{\left(\frac{L\sigma_L}{\sqrt{L^2 + W^2}}\right)^2 + \left(\frac{W\sigma_W}{\sqrt{L^2 + W^2}}\right)^2 + \frac{LW}{L^2 + W^2}\sigma_{LW}^2}$$

Recall that $\sigma_{LW}^2 = V_{LW}$ and $\rho_{LW} = \frac{V_{LW}}{\sigma_L\sigma_W}$.

By knowing $\rho_{LW} = 0.5$, we can calculate V_{LW} and thereby find the errors with correlation:

$$C = 8.6 \pm 1.2 \text{ m}$$

$$A = 2.8 \pm 0.8 \text{ m}$$

$$D = 3.6 \pm 0.4 \text{ m}$$