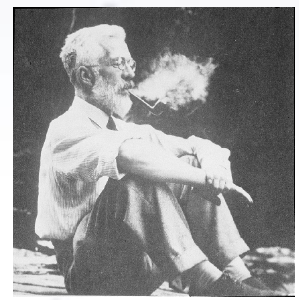
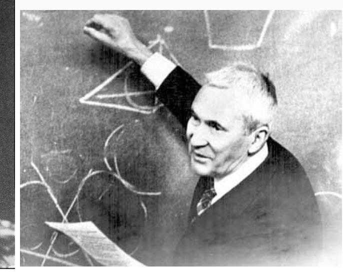
# Applied Statistics

## Two comments on ChiSquare





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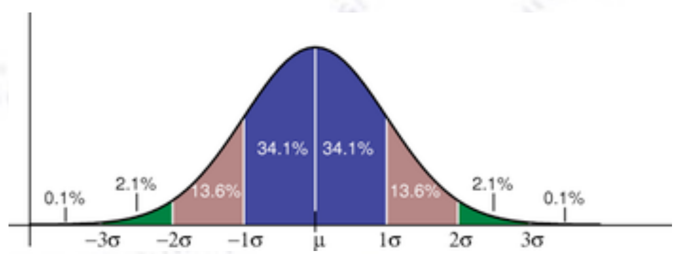


*"Statistics is merely a quantisation of common sense"*

# Defining the Chi-Square - for a fit

<u>Problem Statement:</u> Given N data points (x,y), adjust the parameter(s) θ of a model, such that it fits data best.

The best way to do this, given uncertainties $\sigma_i$ on $y_i$ is by minimising:

$$\chi^2(\theta) = \sum_i^N \frac{(y_i - f(x_i, \theta))^2}{\sigma_i^2}$$

**The power of this method is hard to overstate!**

Not only does it provide a simple, elegant and unique way of fitting data, but more importantly it provides a **goodness-of-fit measure**.

**This is the Chi-Square test!**

# Two ChiSquare definitions

Given a histogram, the number of Observed ($O_i$) events is compared to the number of Expected ($E_i$) events in terms of the uncertainty for each bin in the Pearson Chi-square:

$$\chi^2 = \sum_{\mathrm{i} \in \mathrm{bins}} \frac{(O_i - E_i)^2}{E_i}$$

Here, the uncertainty on the number of events is based on the **expectation**, and we thus divide by $E_i$ (which is the Poisson uncertainty squared). However, one can also use the uncertainty (squared) on the number of observed events $O_i$:

$$\chi^2 = \sum_{\mathrm{i} \in \mathrm{bins}} \frac{(O_i - E_i)^2}{O_i}$$

The advantage of the first, is that empty bins are accepted, but the disadvantage is that it is not clear how many bins to include. It is reverse for the second expression.

# Chi-Square for two histograms

The Chi-Square is generally defined as:

$$\chi^2 = \sum_{i \in bins} \frac{\text{Difference}^2}{\text{Error on Difference}^2}$$

Comparing two histograms,
the difference is simple to define:

$$\text{Difference} = O1_i - O2_i$$

The uncertainty on the count in a bin
is (assumed) Poisson distributed:

$$\text{Error on } O1_i = \sigma(O1_i) = \sqrt{O1_i}$$

Now we can calculate the denominator:

$$\text{Error on Difference} = \sqrt{\sigma(O1_i)^2 + \sigma(O2_i)^2} = \sqrt{O1_i + O2_i}$$

Inserting all of this yields the final result:

$$\chi^2 = \sum_{i \in bins} \frac{(O1_i - O2_i)^2}{O1_i + O2_i}$$