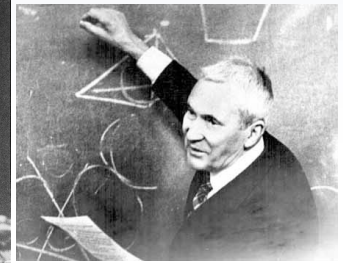
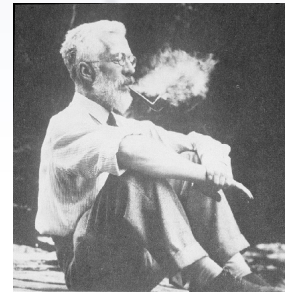
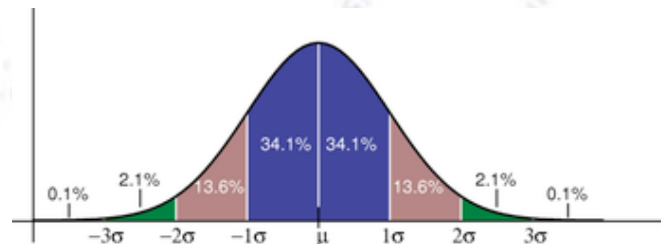


# Applied Statistics

Measuring the length of a Table...

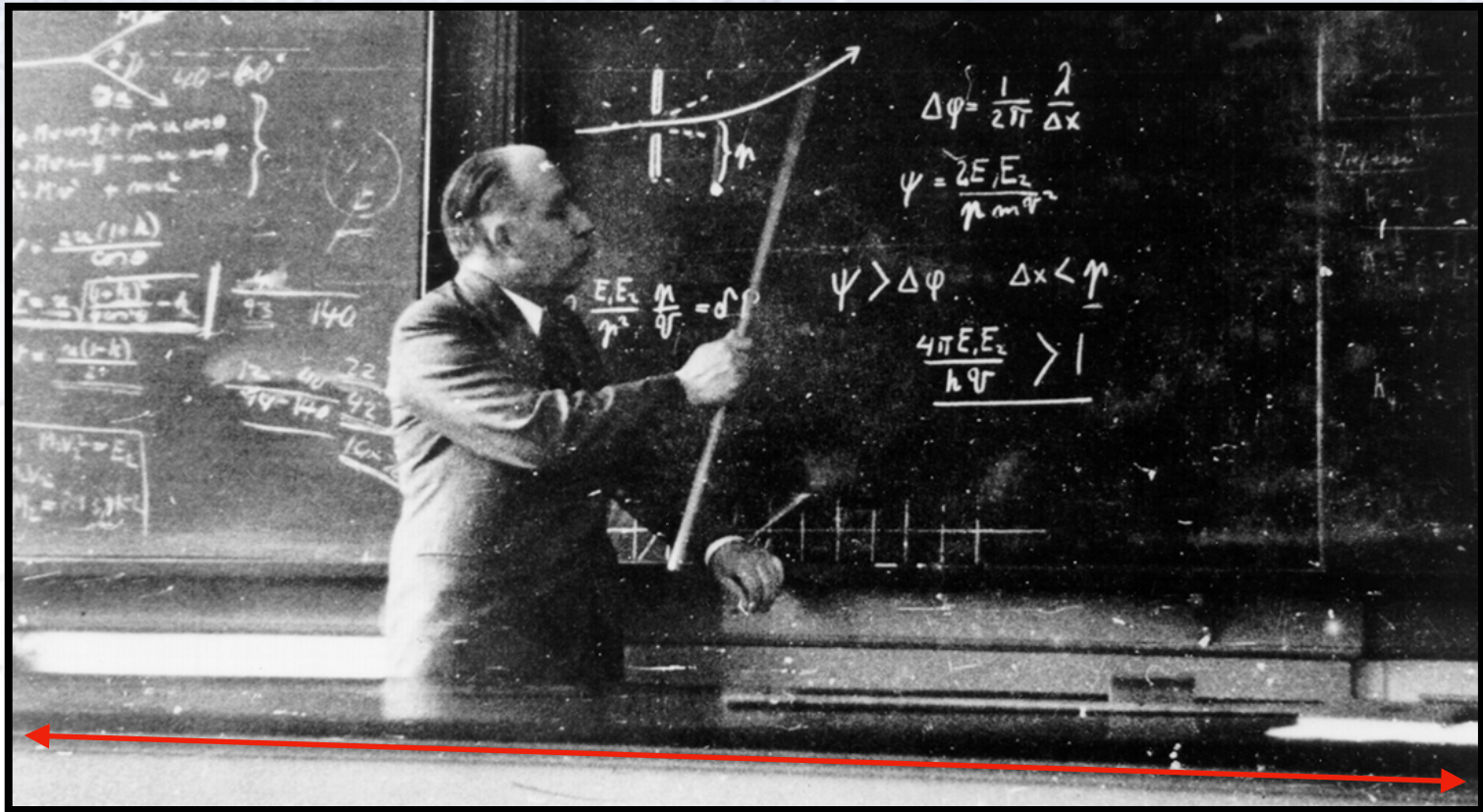


Troels C. Petersen (NBI)



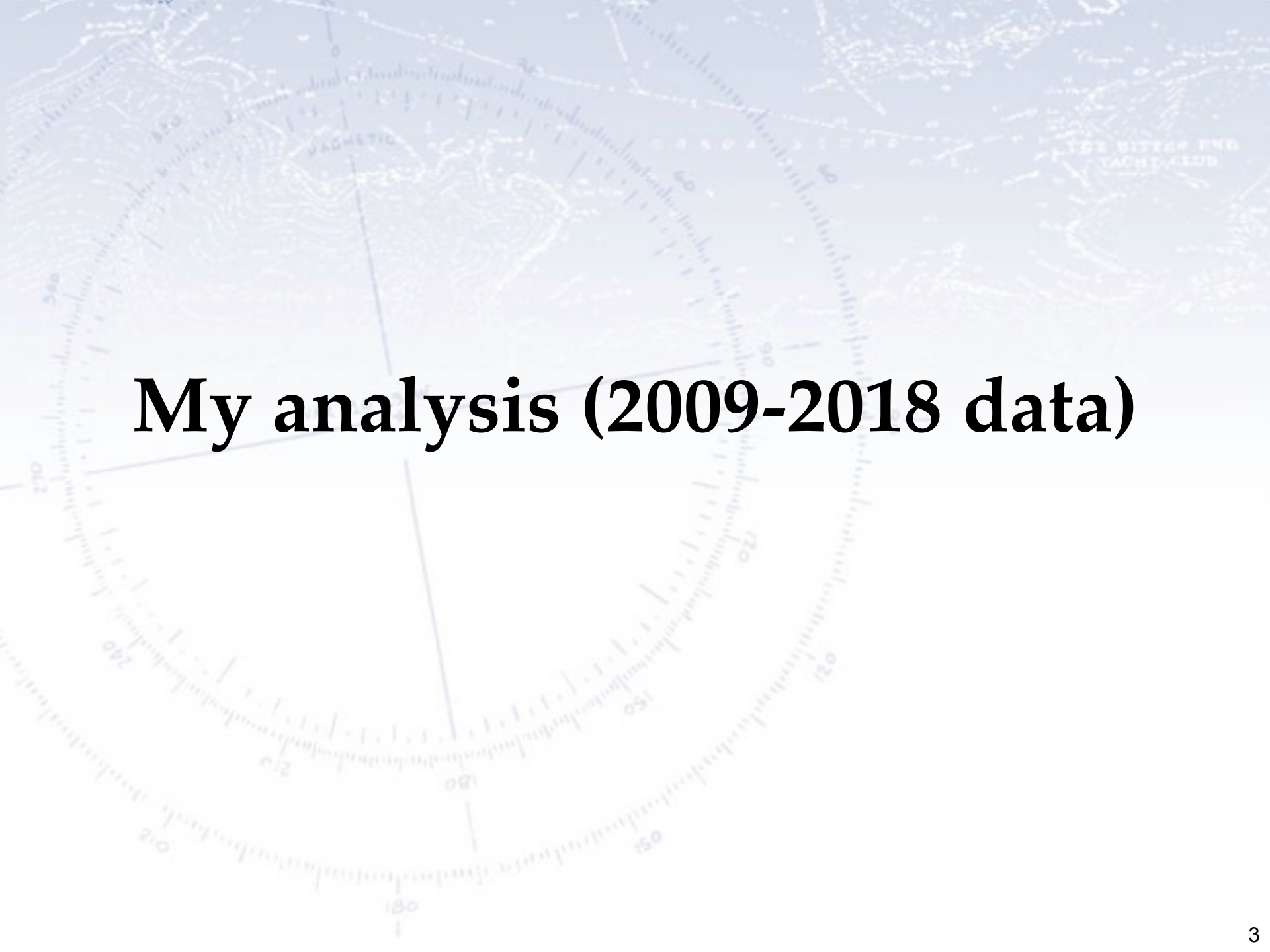
*"Statistics is merely a quantisation of common sense"*

# The table in auditorium A



*“Everything is vague to a degree you do not realise till you have tried to make it precise.”*

[Bertrand Russell, 1872-1970]

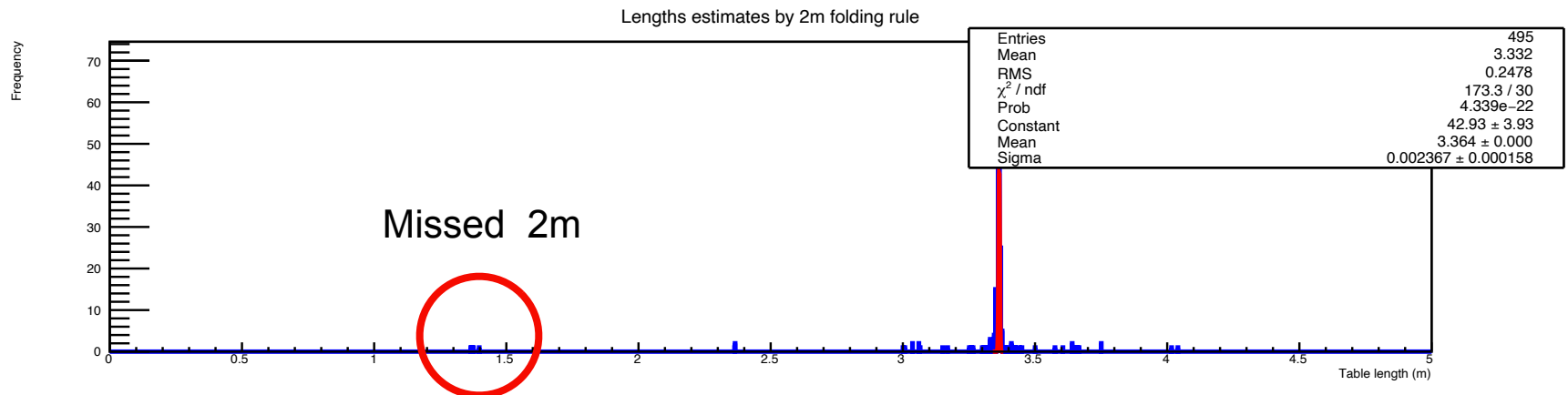
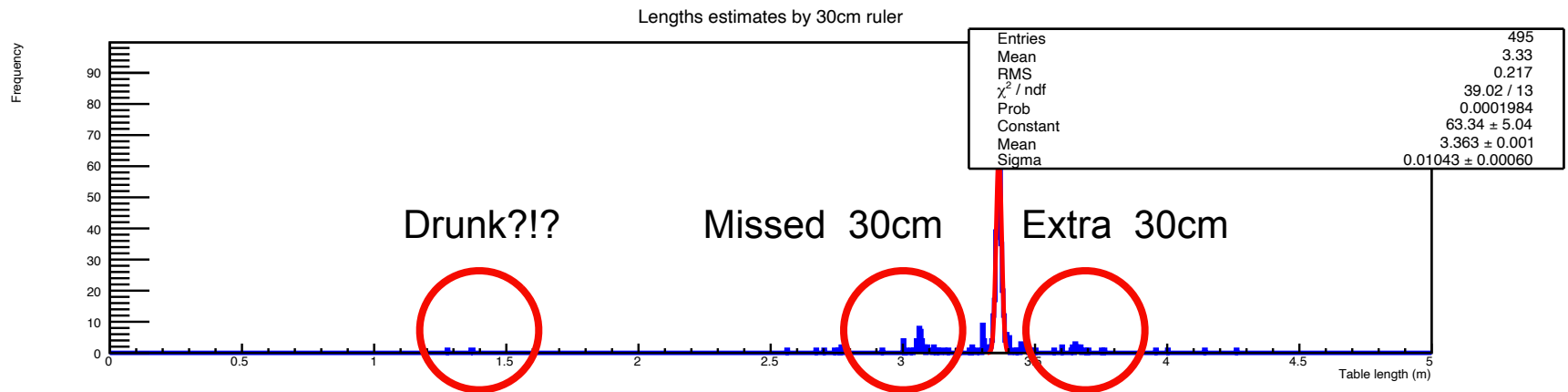


# My analysis (2009-2018 data)

# The table measurement data

The initial dataset contains (valid measurements):

- 30cm measurements: 495      Range: [0.0, 5.0] m
- 2m measurements: 493      Range: [0.0, 5.0] m





# Raw (“Naive”) results

30cm:

Mean =  $3.3290 \pm 0.0098$  m

RMS = 0.22 m (N = 495)

2m:

Mean =  $3.3320 \pm 0.0112$  m

RMS = 0.25 m (N = 493)

Correspondence between 30cm and 2m measurement:

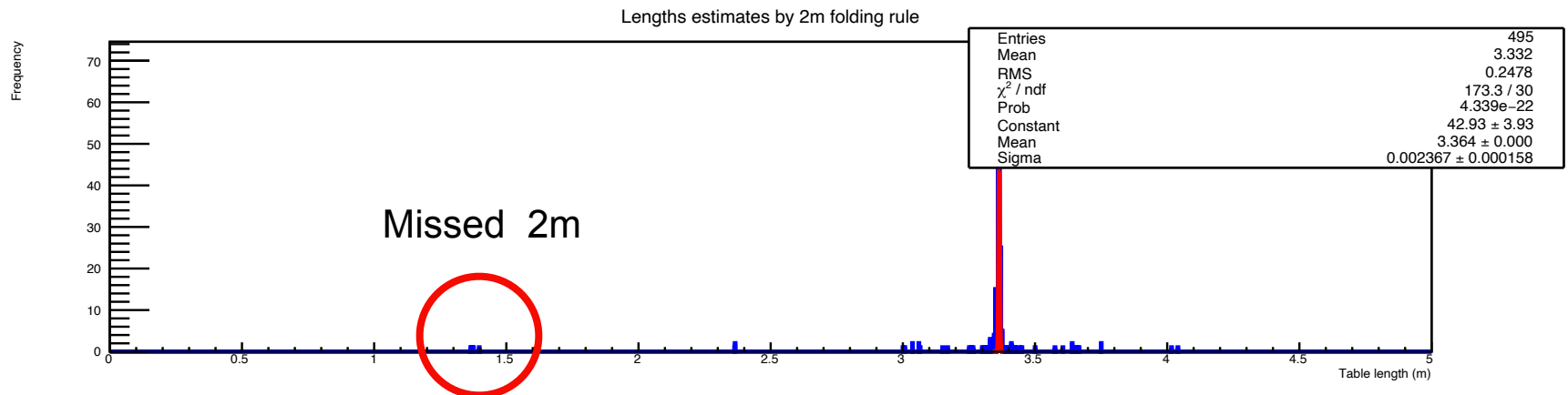
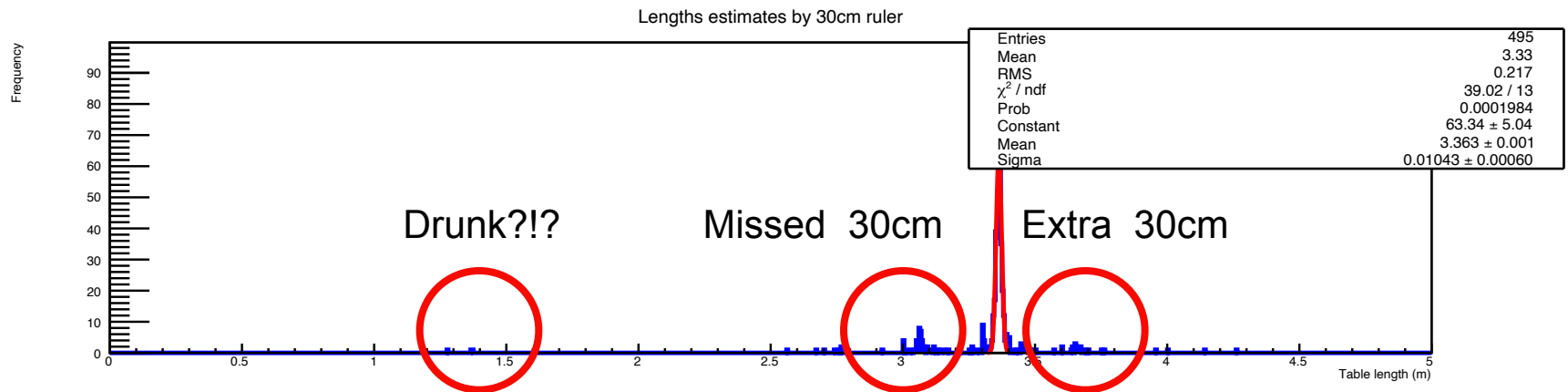
Diff =  $-0.0030 \pm 0.0148$  m (-0.20  $\sigma$ )

# Include offsets?

There are some clear and understandable mis-measurements.

Should one correct and include these?

Depends on resulting improvement, but decide without seeing the final result.

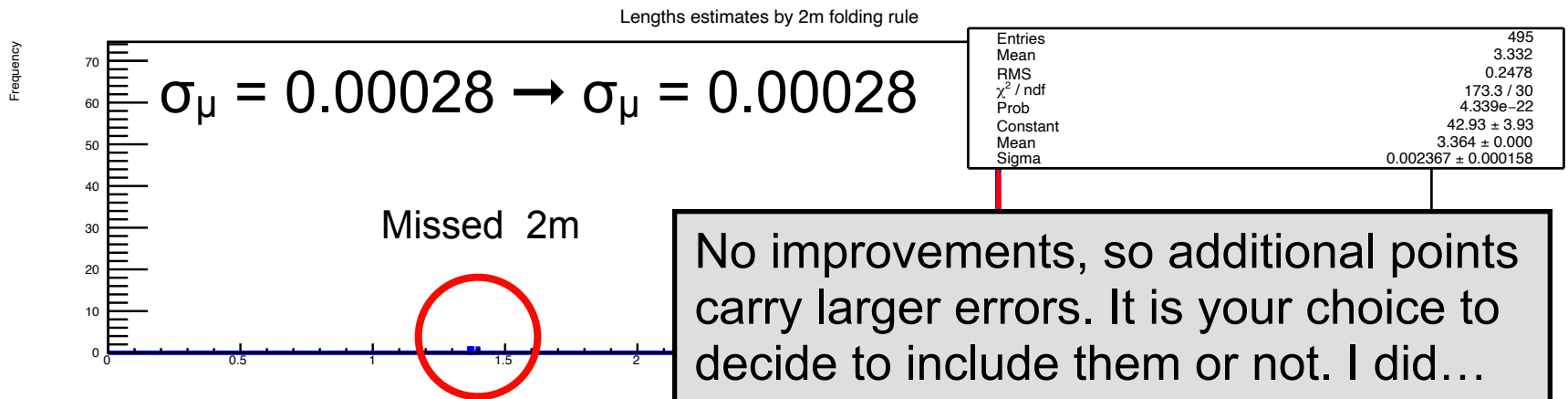
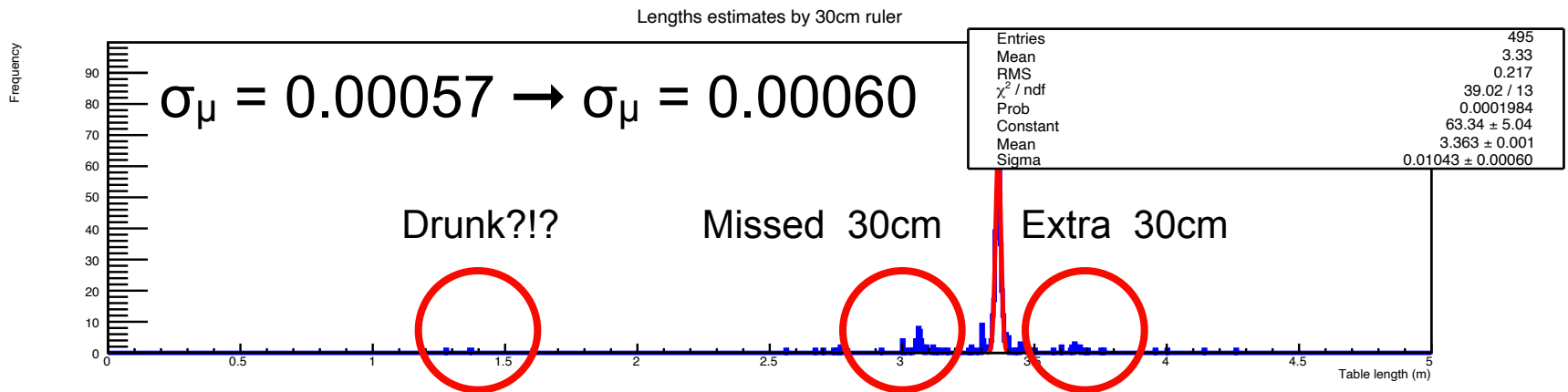


# Include offsets?

There are some clear and understandable mis-measurements.

Should one correct and include these?

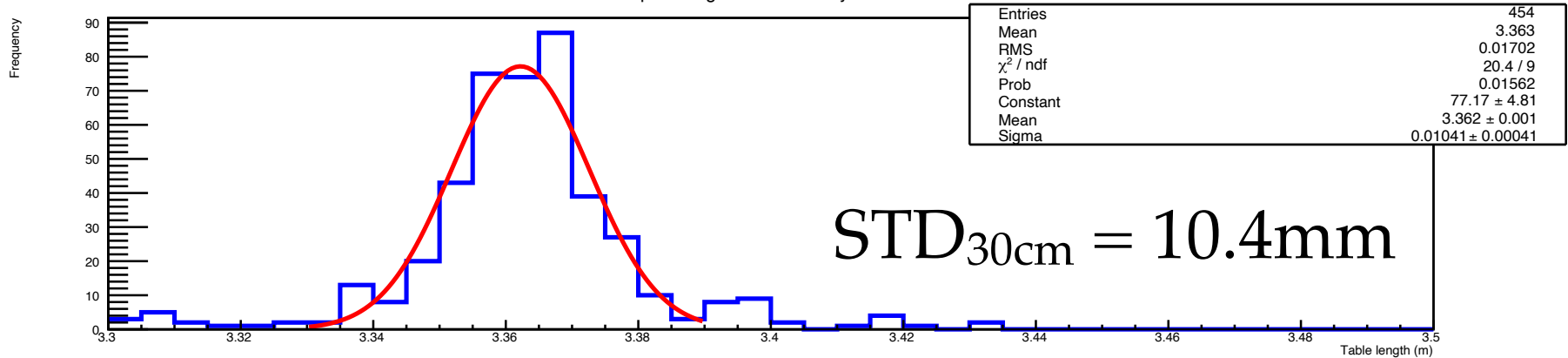
Depends on resulting improvement, but decide without seeing the final result.



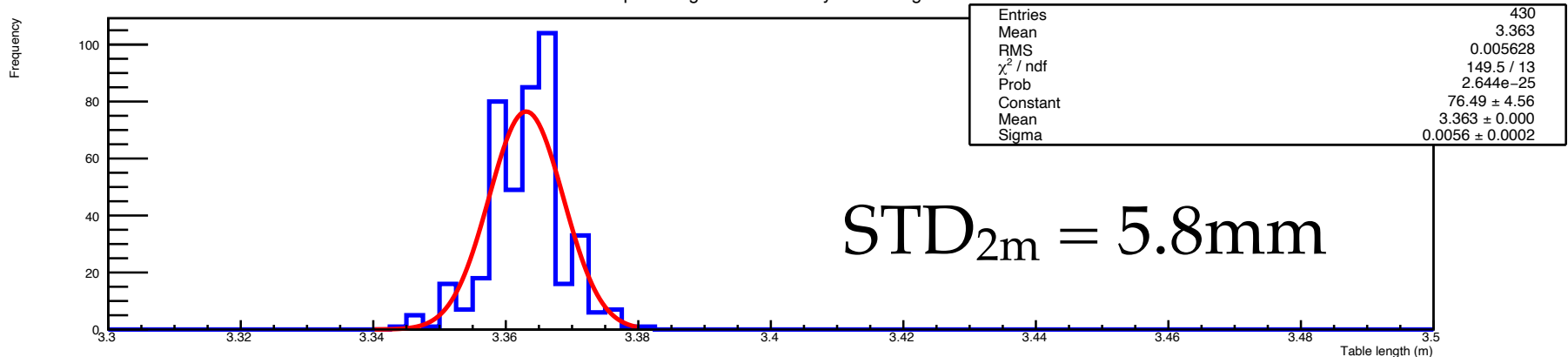
# Inspecting the data

The 30cm peak seems Gaussian ( $p=2.4\%$ ) with binning 0.005 (smaller gives peaks).  
The 2m peak does not seem Gaussian with any binning (here 0.0025), yet “collected”.

Accepted lengths estimates by 30cm ruler



Accepted lengths estimates by 2m folding rule

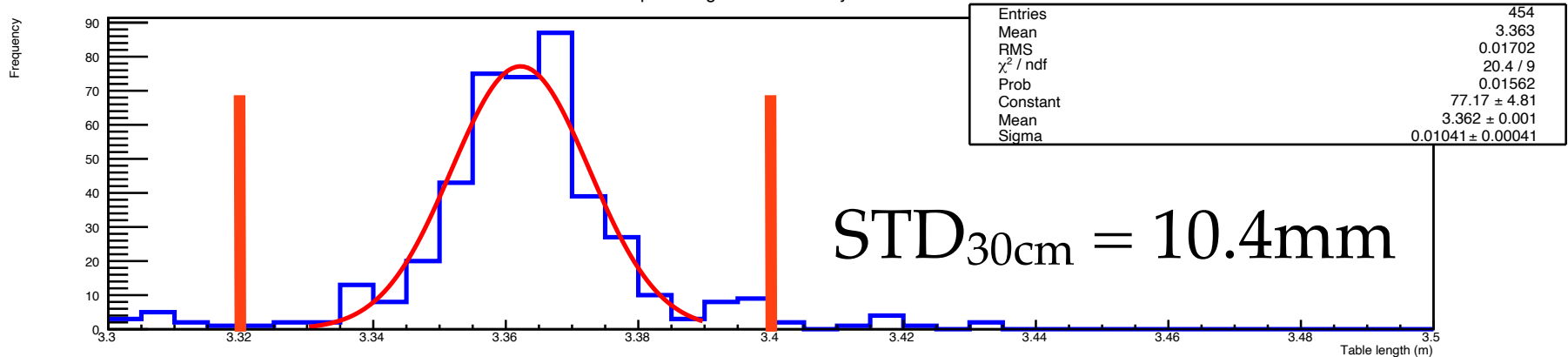




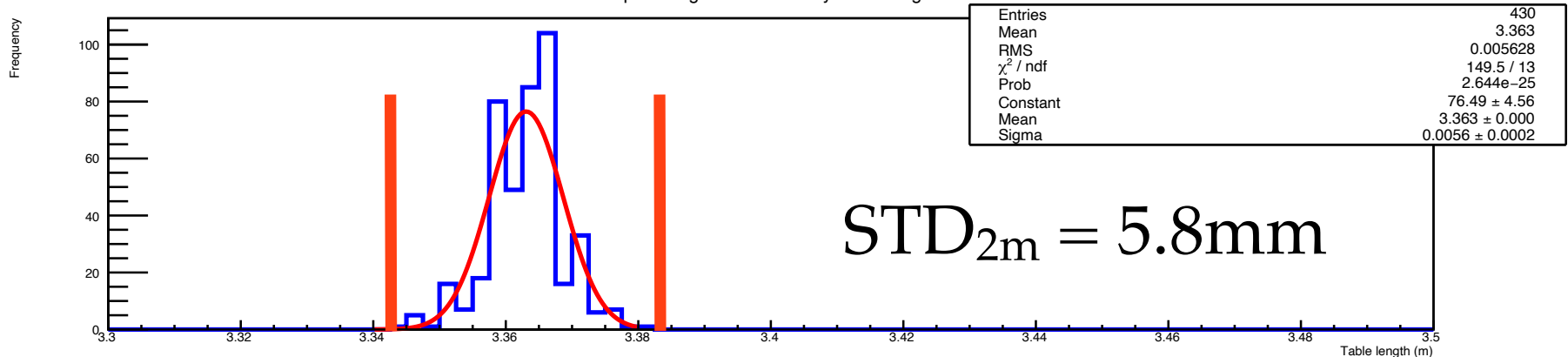
# Inspecting the data

There are clearly some **mis-measurements**, which we would like to **exclude**. Using the fitted width, and accepting that this only includes the best measurements, I could **decide** to include measurements within  $4 \times \text{STD}$ :

Accepted lengths estimates by 30cm ruler

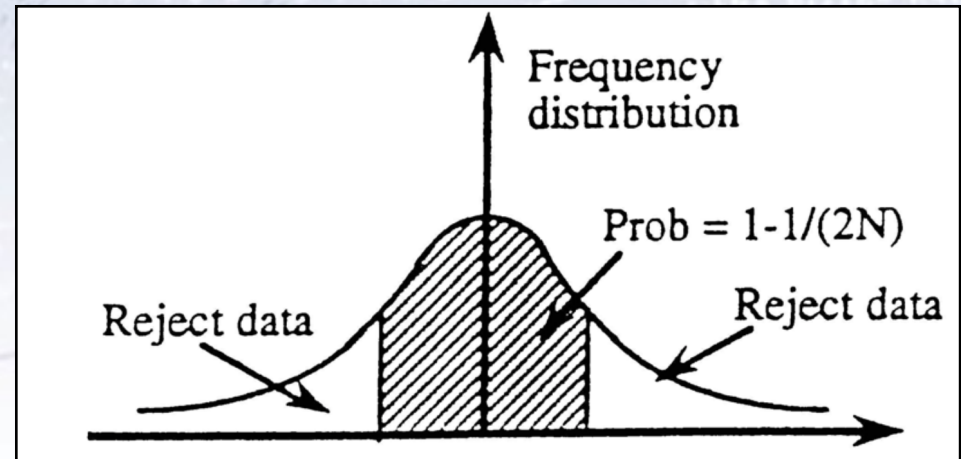


Accepted lengths estimates by 2m folding rule



# Removing data points

Removing improbable data points is formalised in **Chauvenet's Criterion**, though many other methods exist (see Peirce, Grubbs, etc.)



The idea is to assume that the distribution is Gaussian, and ask what the probability of the farthest point is. If it is below some value, which is to be determined ahead of applying the criterion, then the point is removed, and the criterion is reapplied until no more points should be removed.

I choose to say, that if the outermost point in the Gaussian case has **less than 5% chance of being this far out** (taking the total number of points into account), then I reject it.

However, **ALWAYS keep a record of your original data**, as it may contain more effects than you originally thought.

# ...a fair hearing?

Rejected 30cm measurements:

304: L=1.275 dL=2.071 Nsig=10.89	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=495	mean=3.3460	rms=0.1901	-> Rejected
368: L=1.365 dL=1.985 Nsig=11.97	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=494	mean=3.3501	rms=0.1659	-> Rejected
42: L=1.370 dL=1.984 Nsig=14.19	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=493	mean=3.3542	rms=0.1398	-> Rejected
44: L=4.260 dL=0.902 Nsig= 8.39	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=492	mean=3.3582	rms=0.1075	-> Rejected
146: L=2.563 dL=0.793 Nsig= 7.97	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=491	mean=3.3564	rms=0.0995	-> Rejected
158: L=4.140 dL=0.782 Nsig= 8.41	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=490	mean=3.3580	rms=0.0929	-> Rejected
143: L=2.670 dL=0.686 Nsig= 7.98	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=489	mean=3.3564	rms=0.0860	-> Rejected
477: L=2.700 dL=0.658 Nsig= 8.19	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=488	mean=3.3578	rms=0.0803	-> Rejected
308: L=4.004 dL=0.645 Nsig= 8.64	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=487	mean=3.3591	rms=0.0746	-> Rejected
427: L=2.744 dL=0.614 Nsig= 8.93	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=486	mean=3.3578	rms=0.0687	-> Rejected
407: L=2.759 dL=0.600 Nsig= 9.55	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=485	mean=3.3591	rms=0.0628	-> Rejected
313: L=3.955 dL=0.595 Nsig=10.49	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=484	mean=3.3603	rms=0.0567	-> Rejected
269: L=2.768 dL=0.591 Nsig=11.86	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=483	mean=3.3591	rms=0.0498	-> Rejected
380: L=2.769 dL=0.591 Nsig=14.10	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=482	mean=3.3603	rms=0.0419	-> Rejected
133: L=3.180 dL=0.182 Nsig= 5.65	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=481	mean=3.3616	rms=0.0321	-> Rejected
71: L=3.200 dL=0.162 Nsig= 5.21	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=480	mean=3.3619	rms=0.0311	-> Rejected
320: L=3.205 dL=0.157 Nsig= 5.20	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=479	mean=3.3623	rms=0.0302	-> Rejected
319: L=3.215 dL=0.148 Nsig= 5.02	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=478	mean=3.3626	rms=0.0294	-> Rejected
194: L=3.221 dL=0.142 Nsig= 4.96	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=477	mean=3.3629	rms=0.0286	-> Rejected
1: L=3.250 dL=0.113 Nsig= 4.06	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=476	mean=3.3632	rms=0.0279	-> Rejected
175: L=3.471 dL=0.108 Nsig= 3.92	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=475	mean=3.3634	rms=0.0274	-> Rejected
133: L=3.470 dL=0.107 Nsig= 3.95	p1=0.00000000, p_all=0.00000000 >? pmin=0.050	N=474	mean=3.3632	rms=0.0270	-> Rejected
13: L=3.470 dL=0.107 Nsig= 4.02	p1=0.00002883, p_all=0.01354566 >? pmin=0.050	N=473	mean=3.3630	rms=0.0266	-> Rejected
154: L=3.467 dL=0.104 Nsig= 3.98	p1=0.00003406, p_all=0.01594736 >? pmin=0.050	N=472	mean=3.3628	rms=0.0262	-> Rejected
130: L=3.260 dL=0.103 Nsig= 3.98	p1=0.00003423, p_all=0.01599227 >? pmin=0.050	N=471	mean=3.3625	rms=0.0258	-> Rejected
230: L=3.260 dL=0.103 Nsig= 4.05	p1=0.00002507, p_all=0.01171573 >? pmin=0.050	N=470	mean=3.3628	rms=0.0253	-> Rejected
444: L=3.461 dL=0.098 Nsig= 3.93	p1=0.00004185, p_all=0.01943772 >? pmin=0.050	N=469	mean=3.3630	rms=0.0249	-> Rejected
79: L=3.460 dL=0.097 Nsig= 3.96	p1=0.00003687, p_all=0.01710909 >? pmin=0.050	N=468	mean=3.3628	rms=0.0245	-> Rejected
43: L=3.455 dL=0.092 Nsig= 3.83	p1=0.00006418, p_all=0.02952774 >? pmin=0.050	N=467	mean=3.3626	rms=0.0241	-> Rejected
113: L=3.455 dL=0.093 Nsig= 3.90	p1=0.00004898, p_all=0.02256478 >? pmin=0.050	N=466	mean=3.3624	rms=0.0238	-> Rejected
433: L=3.272 dL=0.090 Nsig= 3.85	p1=0.00005881, p_all=0.02697886 >? pmin=0.050	N=465	mean=3.3622	rms=0.0234	-> Rejected
81: L=3.450 dL=0.088 Nsig= 3.80	p1=0.00007216, p_all=0.03292944 >? pmin=0.050	N=464	mean=3.3624	rms=0.0231	-> Rejected
345: L=3.450 dL=0.088 Nsig= 3.87	p1=0.00005540, p_all=0.02532365 >? pmin=0.050	N=463	mean=3.3622	rms=0.0227	-> Rejected
351: L=3.450 dL=0.088 Nsig= 3.93	p1=0.00004175, p_all=0.01910567 >? pmin=0.050	N=462	mean=3.3620	rms=0.0224	-> Rejected
333: L=3.450 dL=0.088 Nsig= 4.01	p1=0.00003083, p_all=0.01411413 >? pmin=0.050	N=461	mean=3.3618	rms=0.0220	-> Rejected
324: L=3.448 dL=0.086 Nsig= 3.99	p1=0.00003299, p_all=0.01506122 >? pmin=0.050	N=460	mean=3.3616	rms=0.0217	-> Rejected
167: L=3.447 dL=0.086 Nsig= 4.02	p1=0.00002922, p_all=0.01332302 >? pmin=0.050	N=459	mean=3.3614	rms=0.0213	-> Rejected
126: L=3.445 dL=0.084 Nsig= 4.00	p1=0.00003155, p_all=0.01434519 >? pmin=0.050	N=458	mean=3.3612	rms=0.0209	-> Rejected
162: L=3.445 dL=0.084 Nsig= 4.08	p1=0.00002276, p_all=0.01034822 >? pmin=0.050	N=457	mean=3.3610	rms=0.0206	-> Rejected
340: L=3.280 dL=0.081 Nsig= 4.00	p1=0.00003223, p_all=0.01458856 >? pmin=0.050	N=456	mean=3.3609	rms=0.0202	-> Rejected
393: L=3.285 dL=0.076 Nsig= 3.82	p1=0.00006640, p_all=0.02975990 >? pmin=0.050	N=455	mean=3.3610	rms=0.0199	-> Rejected
121: L=3.433 dL=0.072 Nsig= 3.66	p1=0.00012405, p_all=0.05476548 >? pmin=0.050	N=454	mean=3.3612	rms=0.0196	-> Accepted

I rejected:  
41 data points from the 30cm sample,  
63 data points from the 2m sample.  
And I inspected each and every one!

# Unweighted results

30cm:

Mean =  $3.36120 \pm 0.00092$  m

RMS = 0.020 m (N = 454)

2m:

Mean =  $3.36295 \pm 0.00026$  m

RMS = 0.005 m (N = 430)

Correspondence between 30cm and 2m measurement:

Diff =  $-0.00175 \pm 0.00130$  m (-1.35  $\sigma$ )

With corrections and Chavenet's ( $p=0.05$ )

Improvement in error from naive to selected measurement 30cm: 10.6

Improvement in error from naive to selected measurement 2m: 42.7

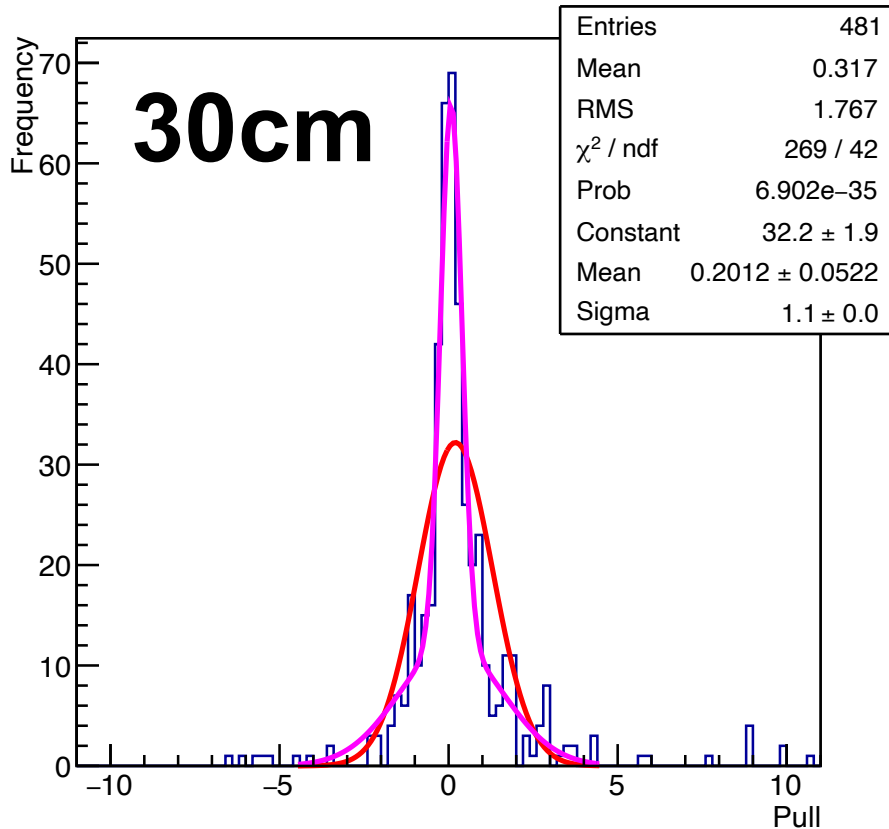


# Weighted analysis

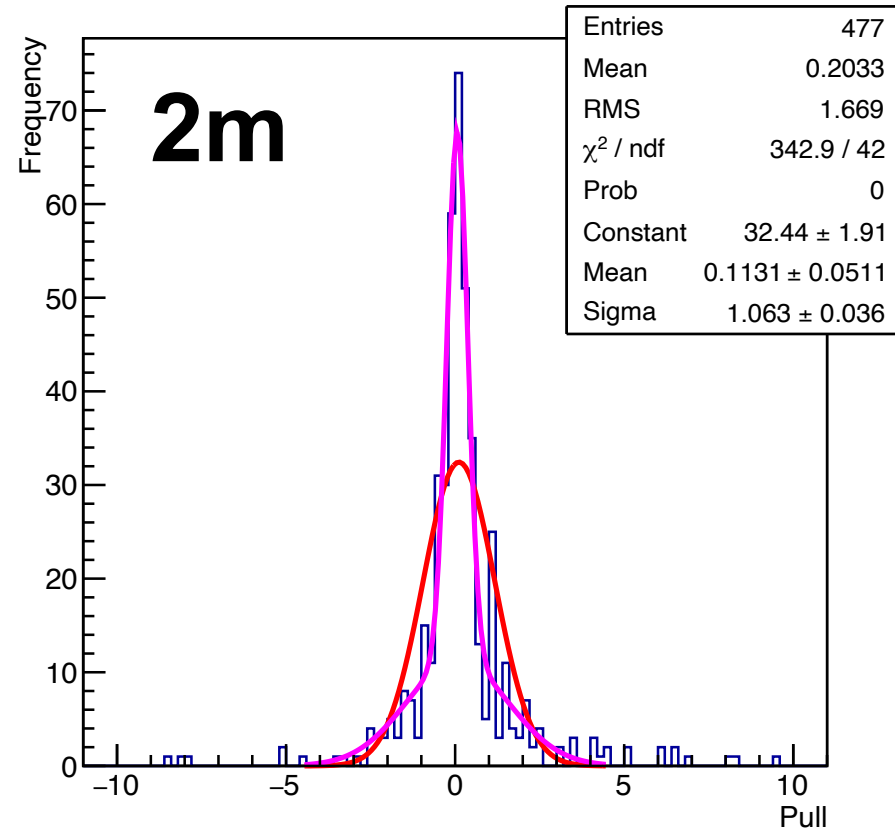
Considering the quoted uncertainties, we first need to evaluate their quality.

The plot to consider is a **PULL** plot, i.e. the distribution of: 
$$\mathcal{Z} = \frac{x_i - \mu}{\sigma_i}$$

Pull distribution - 30cm



Pull distribution - 2m



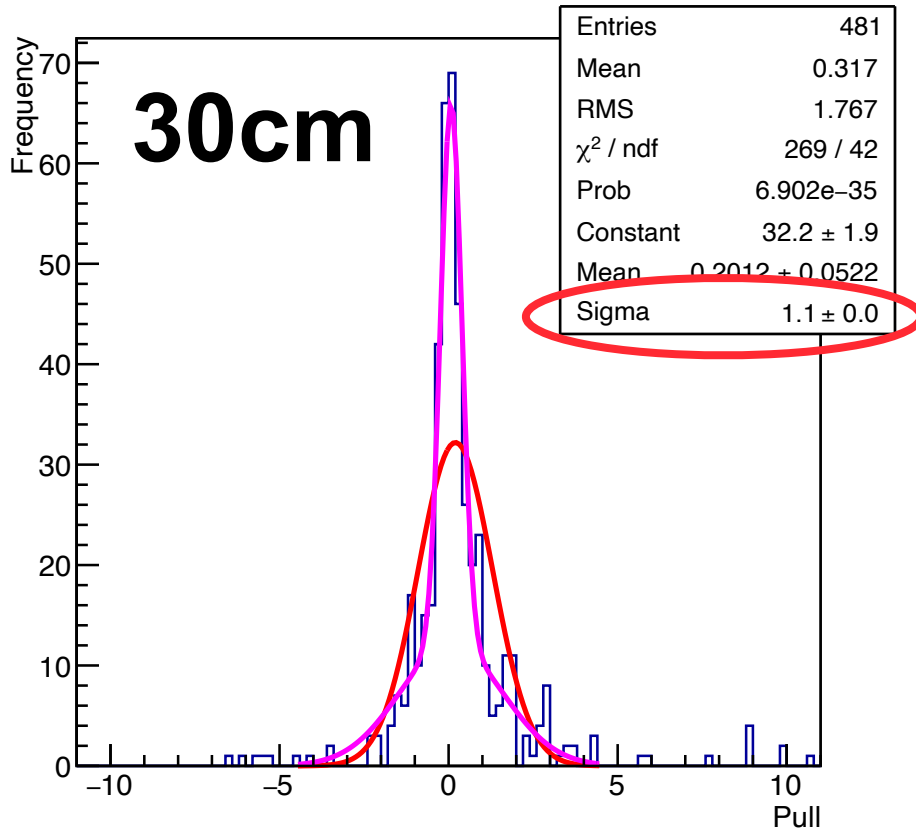
The pulls should be unit Gaussian. I **decide** to exclude measurements beyond  $4.5\sigma$ .

# Weighted analysis

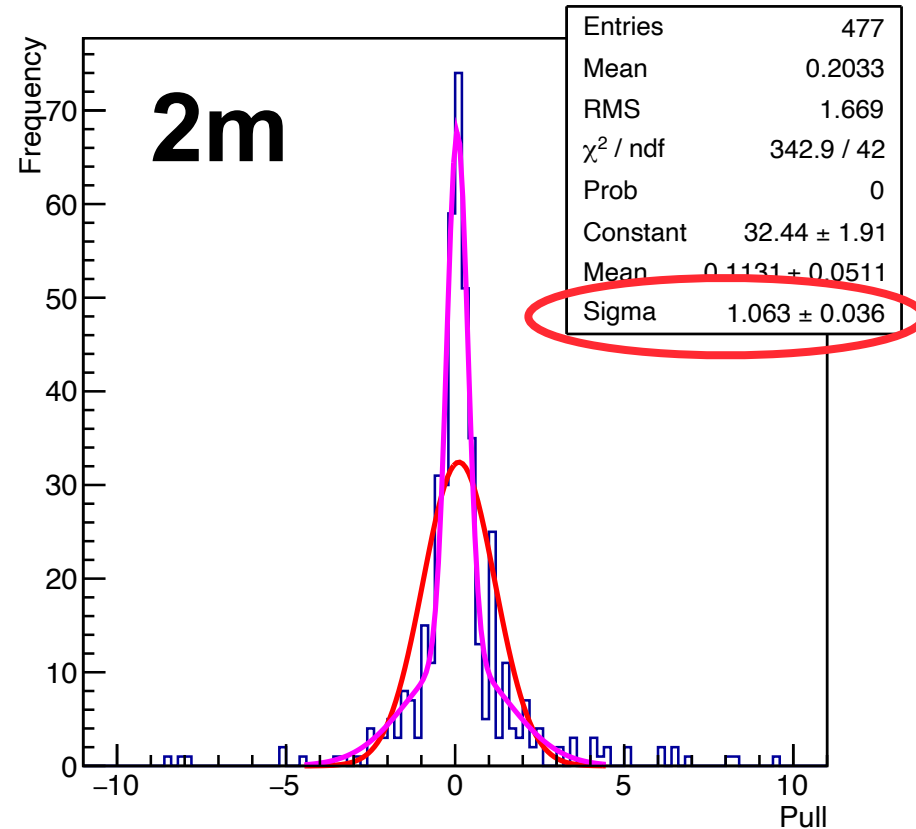
Considering the quoted uncertainties, we first need to evaluate their quality.

The plot to consider is a **PULL** plot, i.e. the distribution of: 
$$\mathcal{Z} = \frac{x_i - \mu}{\sigma_i}$$

Pull distribution - 30cm



Pull distribution - 2m



The pulls should be unit Gaussian. I **decide** to exclude measurements beyond  $4.5\sigma$ .

# Excluded data due to bad pull

30cm: Warning! Large pull: L = 3.4700 +- 0.0080	pull = 13.60	2m: Warning! Large pull: L = 3.0600 +- 0.0050	pull = -60.59
30cm: Warning! Large pull: L = 3.3700 +- 0.0010	pull = 8.80	2m: Warning! Large pull: L = 3.7500 +- 0.0200	pull = 19.35
30cm: Warning! Large pull: L = 1.3700 +- 0.0100	pull = -199.12	2m: Warning! Large pull: L = 3.3680 +- 0.0010	pull = 5.05
30cm: Warning! Large pull: L = 4.2600 +- 0.1000	pull = 8.99	2m: Warning! Large pull: L = 3.6500 +- 0.0300	pull = 9.57
30cm: Warning! Large pull: L = 3.3000 +- 0.0100	pull = -6.12	2m: Warning! Large pull: L = 3.3200 +- 0.0050	pull = -8.59
30cm: Warning! Large pull: L = 2.6700 +- 0.0100	pull = -69.12	2m: Warning! Large pull: L = 3.6660 +- 0.0500	pull = 6.06
30cm: Warning! Large pull: L = 2.5630 +- 0.0200	pull = -39.91	2m: Warning! Large pull: L = 3.5750 +- 0.0020	pull = 106.03
30cm: Warning! Large pull: L = 4.1400 +- 0.0400	pull = 19.47	2m: Warning! Large pull: L = 3.0360 +- 0.0400	pull = -8.17
30cm: Warning! Large pull: L = 3.4670 +- 0.0120	pull = 8.82	2m: Warning! Large pull: L = 3.3500 +- 0.0010	pull = -12.95
30cm: Warning! Large pull: L = 3.4710 +- 0.0050	pull = 21.96	2m: Warning! Large pull: L = 3.0360 +- 0.0050	pull = -65.39
30cm: Warning! Large pull: L = 3.4470 +- 0.0050	pull = 17.16	2m: Warning! Large pull: L = 2.3630 +- 0.0050	pull = -199.99
30cm: Warning! Large pull: L = 3.2210 +- 0.0310	pull = -4.52	2m: Warning! Large pull: L = 3.3150 +- 0.0006	pull = -79.92
30cm: Warning! Large pull: L = 3.3470 +- 0.0010	pull = -14.20	2m: Warning! Large pull: L = 3.4170 +- 0.0120	pull = 4.50
30cm: Warning! Large pull: L = 3.3650 +- 0.0005	pull = 7.60	2m: Warning! Large pull: L = 3.6400 +- 0.0100	pull = 27.71
30cm: Warning! Large pull: L = 3.3670 +- 0.0010	pull = 5.80	2m: Warning! Large pull: L = 4.0400 +- 0.0200	pull = 33.85
30cm: Warning! Large pull: L = 3.3290 +- 0.0010	pull = 6.47	2m: Warning! Large pull: L = 4.0160 +- 0.1010	pull = 6.47
30cm: Warning! Large pull: L = 3.2600 +- 0.0010	pull = -59.79	2m: Warning! Large pull: L = 3.0640 +- 0.0050	pull = -59.79
30cm: Warning! Large pull: L = 3.4240 +- 0.0010	pull = 24.75	2m: Warning! Large pull: L = 3.6600 +- 0.0120	pull = 24.75
30cm: Warning! Large pull: L = 3.3050 +- 0.0010	pull = -7.95	2m: Warning! Large pull: L = 3.3550 +- 0.0010	pull = -7.95
30cm: Warning! Large pull: L = 3.3000 +- 0.0010	pull = 77.41	2m: Warning! Large pull: L = 3.7500 +- 0.0050	pull = 77.41
30cm: Warning! Large pull: L = 2.7680 +- 0.0010	pull = 6.05	2m: Warning! Large pull: L = 3.3690 +- 0.0010	pull = 6.05
30cm: Warning! Large pull: L = 3.4200 +- 0.0010	pull = -44.49	2m: Warning! Large pull: L = 3.0070 +- 0.0080	pull = -44.49
30cm: Warning! Large pull: L = 3.3900 +- 0.0010	pull = -38.99	2m: Warning! Large pull: L = 3.1680 +- 0.0050	pull = -38.99
30cm: Warning! Large pull: L = 1.2750 +- 0.0010	pull = -60.59	2m: Warning! Large pull: L = 3.0600 +- 0.0050	pull = -60.59
30cm: Warning! Large pull: L = 4.0040 +- 0.0300	pull = 21.43	2m: Warning! Large pull: L = 3.3720 +- 0.0020	pull = 4.53
30cm: Warning! Large pull: L = 3.9550 +- 0.0550	pull = 10.80	2m: Warning! Large pull: L = 3.1500 +- 0.0100	pull = -21.29
30cm: Warning! Large pull: L = 3.2150 +- 0.0110	pull = -13.29	2m: Warning! Large pull: L = 2.3640 +- 0.0300	pull = -33.30
30cm: Warning! Large pull: L = 1.3650 +- 0.0050	pull = -399.24	2m: Warning! Large pull: L = 3.4200 +- 0.0050	pull = 11.41
30cm: Warning! Large pull: L = 3.4110 +- 0.0050	pull = 9.96	2m: Warning! Large pull: L = 3.3720 +- 0.0011	pull = 8.23
30cm: Warning! Large pull: L = 3.4500 +- 0.0050	pull = 17.76	2m: Warning! Large pull: L = 3.3970 +- 0.0030	pull = 11.35
30cm: Warning! Large pull: L = 2.7690 +- 0.0050	pull = -118.44	2m: Warning! Large pull: L = 3.6050 +- 0.0150	pull = 16.14
30cm: Warning! Large pull: L = 3.3080 +- 0.0100	pull = -5.32	2m: Warning! Large pull: L = 3.3070 +- 0.0050	pull = -11.19
30cm: Warning! Large pull: L = 3.3700 +- 0.0010	pull = 8.80	2m: Warning! Large pull: L = 3.3730 +- 0.0020	pull = 5.03
30cm: Warning! Large pull: L = 2.7590 +- 0.0020	pull = -301.10	2m: Warning! Large pull: L = 3.3760 +- 0.0020	pull = 6.53
30cm: Warning! Large pull: L = 2.7440 +- 0.0500	pull = -12.34	2m: Warning! Large pull: L = 2.3650 +- 0.0050	pull = -199.59
30cm: Warning! Large pull: L = 3.3350 +- 0.0010	pull = -26.20	2m: Warning! Large pull: L = 2.3650 +- 0.0040	pull = -249.49
30cm: Warning! Large pull: L = 3.4610 +- 0.0100	pull = 9.98	2m: Warning! Large pull: L = 3.0020 +- 0.0700	pull = -5.16
30cm: Warning! Large pull: L = 2.7000 +- 0.0300	pull = -22.04	2m: Warning! Large pull: L = 3.2600 +- 0.0200	pull = -5.15
		2m: Warning! Large pull: L = 3.2650 +- 0.0010	pull = -97.95
		2m: Warning! Large pull: L = 3.5000 +- 0.0200	pull = 6.85
		2m: Warning! Large pull: L = 3.1695 +- 0.0020	pull = -96.72
		2m: Warning! Large pull: L = 3.3660 +- 0.0003	pull = 12.20
		2m: Warning! Large pull: L = 3.6540 +- 0.0200	pull = 14.55
		2m: Warning! Large pull: L = 3.3710 +- 0.0010	pull = 8.05
		2m: Warning! Large pull: L = 3.0650 +- 0.0050	pull = -59.59

I rejected:  
38 data points from the 30cm sample,  
45 data points from the 2m sample.  
And I inspected each and every one!



# Excluded data due to bad pull

30cm: Warning! Large pull: L = 3.4700 +- 0.0080	pull = 13.60	2m: Warning! Large pull: L = 3.0600 +- 0.0050	pull = -60.59
30cm: Warning! Large pull: L = 3.3700 +- 0.0010	pull = 8.80	2m: Warning! Large pull: L = 3.7500 +- 0.0200	pull = 19.35
30cm: Warning! Large pull: L = 1.3700 +- 0.0100	pull = -199.12	2m: Warning! Large pull: L = 3.3680 +- 0.0010	pull = 5.05
30cm: Warning! Large pull: L = 4.2600 +- 0.1000	pull = 8.99	2m: Warning! Large pull: L = 3.6500 +- 0.0300	pull = 9.57
30cm: Warning! Large pull: L = 3.3000 +- 0.0100	pull = -6.12	2m: Warning! Large pull: L = 3.3200 +- 0.0050	pull = -8.59
30cm: Warning! Large pull: L = 2.6700 +- 0.0100	pull = -69.12	2m: Warning! Large pull: L = 3.6660 +- 0.0500	pull = 6.06
30cm: Warning! Large pull: L = 2.5630 +- 0.0200	pull = -39.91	2m: Warning! Large pull: L = 3.5750 +- 0.0020	pull = 106.03
30cm: Warning! Large pull: L = 4.1400 +- 0.0400	pull = 19.47	2m: Warning! Large pull: L = 3.0360 +- 0.0400	pull = -8.17
30cm: Warning! Large pull: L = 3.4670 +- 0.0120	pull = 8.82	2m: Warning! Large pull: L = 3.3500 +- 0.0010	pull = -12.95
30cm: Warning! Large pull: L = 3.4710 +- 0.0050	pull = 21.96	2m: Warning! Large pull: L = 3.0360 +- 0.0050	pull = -65.39
30cm: Warning! Large pull: L = 3.4470 +- 0.0050	pull = 17.16	2m: Warning! Large pull: L = 2.3630 +- 0.0050	pull = -199.99
30cm: Warning! Large pull: L = 3.2210 +- 0.0310	pull = -4.52	2m: Warning! Large pull: L = 3.3150 +- 0.0006	pull = -79.92
30cm: Warning! Large pull: L = 3.3470 +- 0.0010	pull = -14.20	2m: Warning! Large pull: L = 3.4170 +- 0.0120	pull = 4.50
30cm: Warning! Large pull: L = 3.3650 +- 0.0005	pull = 7.60	2m: Warning! Large pull: L = 3.6400 +- 0.0100	pull = 27.71
30cm: Warning! Large pull: L = 3.3670 +- 0.0010	pull = 5.80	2m: Warning! Large pull: L = 4.0400 +- 0.0200	pull = 33.85
30cm: Warning! Large pull: L = 3.3290 +- 0.0010	pull = 6.47	2m: Warning! Large pull: L = 4.0160 +- 0.1010	pull = 6.47
30cm: Warning! Large pull: L = 3.2600 +- 0.0010	pull = -59.79	2m: Warning! Large pull: L = 3.0640 +- 0.0050	pull = -59.79
30cm: Warning! Large pull: L = 3.4240 +- 0.0010	pull = 24.75	2m: Warning! Large pull: L = 3.6600 +- 0.0120	pull = 24.75
30cm: Warning! Large pull: L = 3.3050 +- 0.0010	pull = -7.95	2m: Warning! Large pull: L = 3.3550 +- 0.0010	pull = -7.95
30cm: Warning! Large pull: L = 3.3000 +- 0.0010	pull = 77.41	2m: Warning! Large pull: L = 3.7500 +- 0.0050	pull = 77.41
30cm: Warning! Large pull: L = 2.7680 +- 0.0010	pull = 6.05	2m: Warning! Large pull: L = 3.3690 +- 0.0010	pull = 6.05
30cm: Warning! Large pull: L = 3.4200 +- 0.0010	pull = -44.49	2m: Warning! Large pull: L = 3.0070 +- 0.0080	pull = -44.49
30cm: Warning! Large pull: L = 3.3900 +- 0.0010	pull = -38.99	2m: Warning! Large pull: L = 3.1680 +- 0.0050	pull = -38.99
30cm: Warning! Large pull: L = 1.2750 +- 0.0010	pull = -60.59	2m: Warning! Large pull: L = 3.0600 +- 0.0050	pull = -60.59
30cm: Warning! Large pull: L = 4.0040 +- 0.0300	pull = 21.43	2m: Warning! Large pull: L = 3.3720 +- 0.0020	pull = 4.53
30cm: Warning! Large pull: L = 3.9550 +- 0.0550	pull = 10.80	2m: Warning! Large pull: L = 3.1500 +- 0.0100	pull = -21.29
30cm: Warning! Large pull: L = 3.2150 +- 0.0110	pull = -13.29	2m: Warning! Large pull: L = 2.3640 +- 0.0300	pull = -33.30
30cm: Warning! Large pull: L = 1.3650 +- 0.0050	pull = -399.24	2m: Warning! Large pull: L = 3.4200 +- 0.0050	pull = 11.41
30cm: Warning! Large pull: L = 3.4110 +- 0.0050	pull = 9.96	2m: Warning! Large pull: L = 3.3720 +- 0.0011	pull = 8.23
30cm: Warning! Large pull: L = 3.4500 +- 0.0050	pull = 17.76	2m: Warning! Large pull: L = 3.3970 +- 0.0030	pull = 11.35
30cm: Warning! Large pull: L = 2.7690 +- 0.0050	pull = -118.44	2m: Warning! Large pull: L = 3.6050 +- 0.0150	pull = 16.14
30cm: Warning! Large pull: L = 3.3080 +- 0.0100	pull = -5.32	2m: Warning! Large pull: L = 3.3070 +- 0.0050	pull = -11.19
30cm: Warning! Large pull: L = 3.3700 +- 0.0010	pull = 8.80	2m: Warning! Large pull: L = 3.3730 +- 0.0020	pull = 5.03
30cm: Warning! Large pull: L = 2.7590 +- 0.0020	pull = -301.10	2m: Warning! Large pull: L = 3.3760 +- 0.0020	pull = 6.53
30cm: Warning! Large pull: L = 2.7440 +- 0.0500	pull = -12.34	2m: Warning! Large pull: L = 2.3650 +- 0.0050	pull = -199.59
30cm: Warning! Large pull: L = 3.3350 +- 0.0010	pull = -26.20	2m: Warning! Large pull: L = 2.3650 +- 0.0040	pull = -249.49
30cm: Warning! Large pull: L = 3.4610 +- 0.0100	pull = 9.98	2m: Warning! Large pull: L = 3.0020 +- 0.0700	pull = -5.16
30cm: Warning! Large pull: L = 2.7000 +- 0.0300	pull = -22.04	2m: Warning! Large pull: L = 3.2600 +- 0.0200	pull = -5.15
		2m: Warning! Large pull: L = 3.2650 +- 0.0010	pull = -97.95
		2m: Warning! Large pull: L = 3.5000 +- 0.0200	pull = 6.85
		2m: Warning! Large pull: L = 3.1885 +- 0.0020	pull = 86.72
		2m: Warning! Large pull: L = 3.3660 +- 0.0003	pull = 12.20
		2m: Warning! Large pull: L = 3.3540 +- 0.0200	pull = -14.55
		2m: Warning! Large pull: L = 3.3710 +- 0.0010	pull = 8.05
		2m: Warning! Large pull: L = 3.0650 +- 0.0050	pull = -59.59

I rejected:  
38 data points from the 30cm sample,  
45 data points from the 2m sample.  
And I inspected each and every one!

Notice, that close measurement with too small errors are rejected!



# Weighted results

30cm:

Mean =  $3.36376 \pm 0.00030$  m

RMS = undefined! (N = 443)

2m:

Mean =  $3.36385 \pm 0.00014$  m

RMS = undefined! (N = 432)

Correspondence between 30cm and 2m measurement:

Diff =  $-0.00009 \pm 0.00033$  m (-0.27  $\sigma$ )

30cm:  $\sigma_{\mu} = 0.00092 \rightarrow \sigma_{\mu} = 0.00030$  (factor 3.1)

2m:  $\sigma_{\mu} = 0.00026 \rightarrow \sigma_{\mu} = 0.00014$  (factor 1.8)

# Weighted results

30cm:

Mean =  $3.36376 \pm 0.00030$  m

Chi2 = 460.5, Ndof = 442, Prob = 0.26

2m:

Mean =  $3.36385 \pm 0.00014$  m

Chi2 = 451.1, Ndof = 431, Prob = 0.24

Correspondence between 30cm and 2m measurement:

Diff =  $-0.00009 \pm 0.00033$  m (-0.27  $\sigma$ )

30cm:  $\sigma_{\mu} = 0.00092 \rightarrow \sigma_{\mu} = 0.00030$  (factor 3.1)

2m:  $\sigma_{\mu} = 0.00026 \rightarrow \sigma_{\mu} = 0.00014$  (factor 1.8)

# A problem?

Correspondence: unweighted vs. weighted 30cm measurements:

$$\text{Diff} = 0.00257 \pm 0.00097 \quad (2.65 \text{ sigma})$$

Correspondence: unweighted vs. weighted 2m measurements:

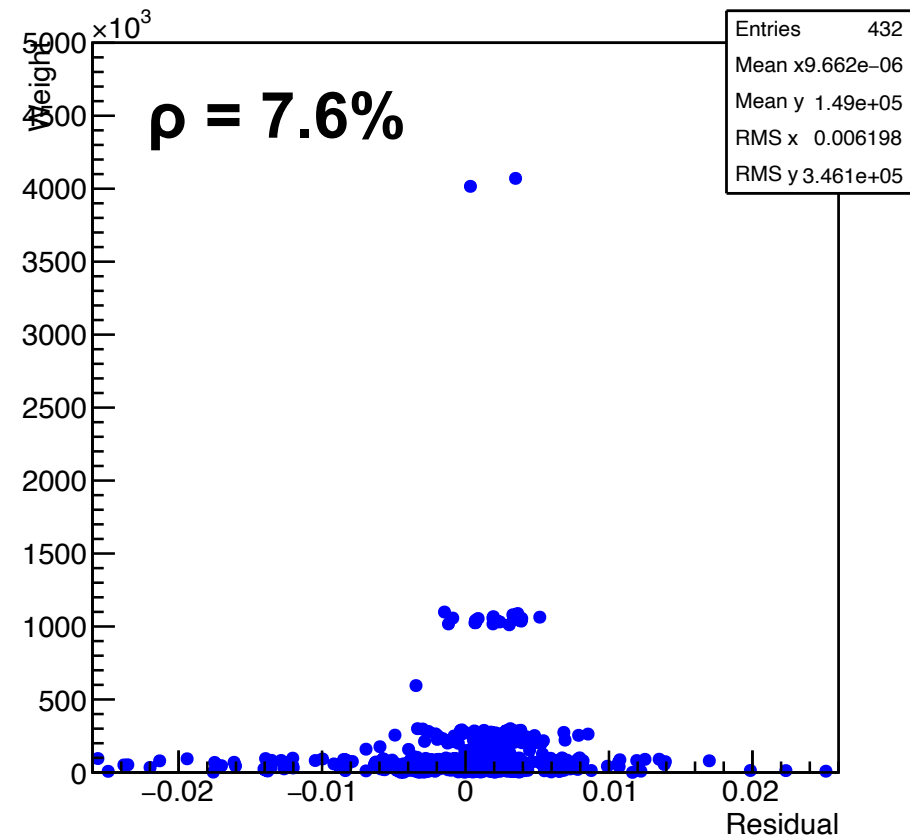
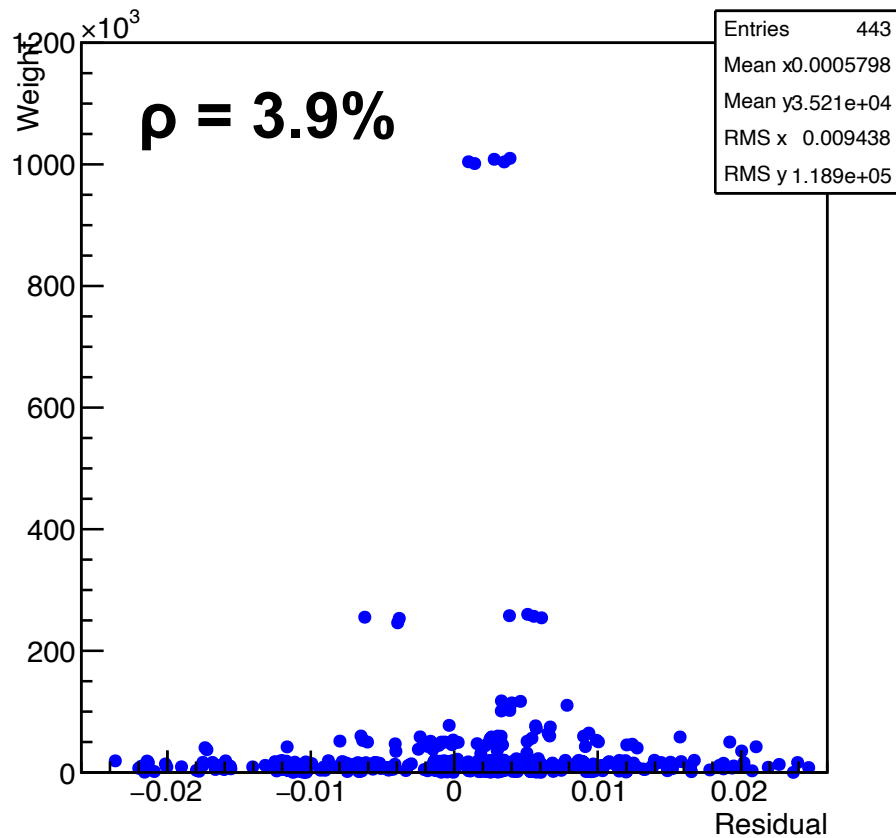
$$\text{Diff} = 0.00090 \pm 0.00030 \quad (3.04 \text{ sigma})$$





# A correlation?

Plotted is the **measurement residuals** (measurement - mean) as a function of the **weight of the measurement** (i.e.  $1/\text{uncertainty}^2$ ).

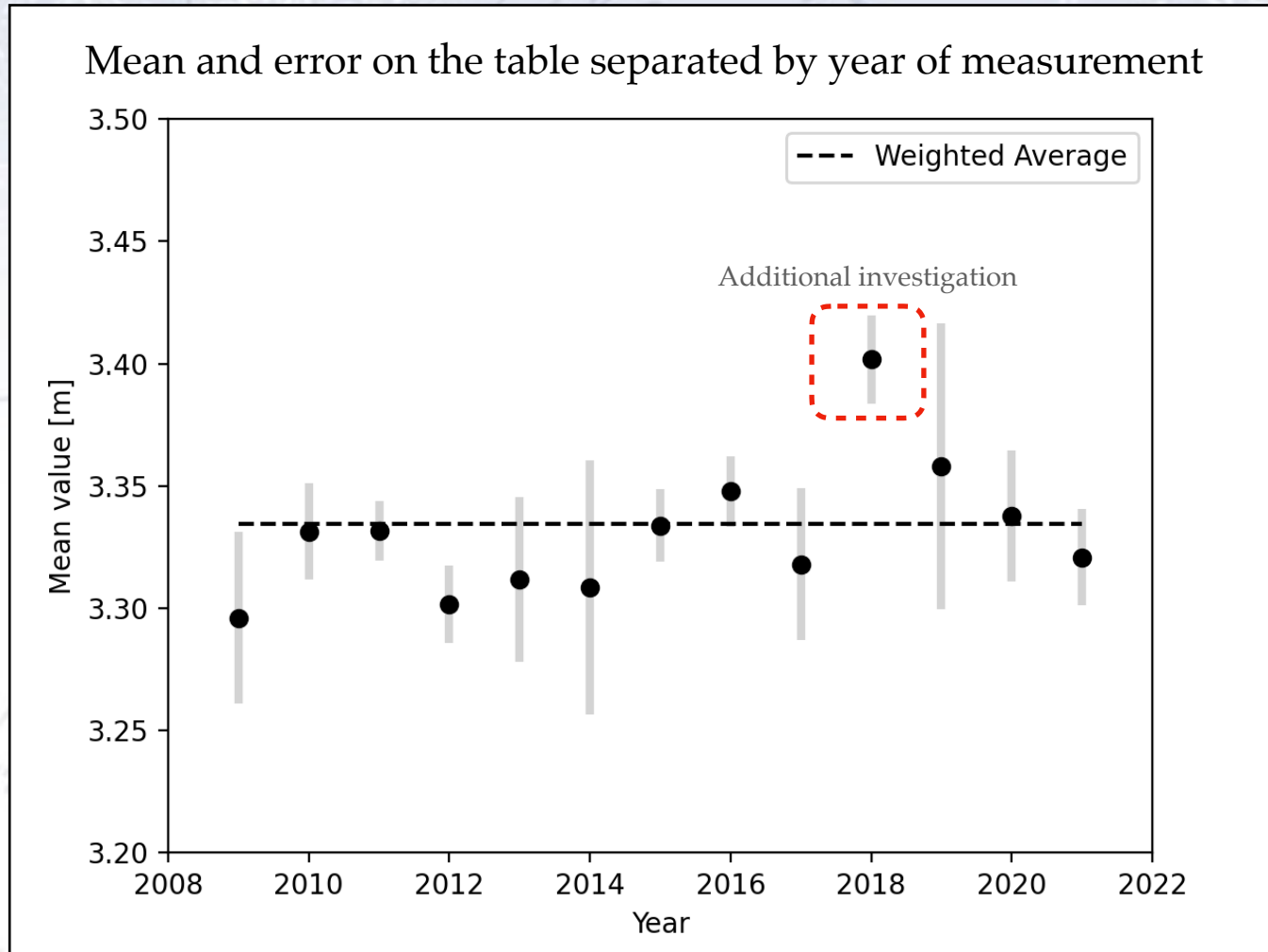


There is a slight correlation between the lengths and uncertainties quoted!

*"Longer measurements have smaller errors!"*. Why? I don't know. But data shows it!



# Crosschecking by year



These measurements are in OK agreement, except 2018!

# Fitting for a result

A completely different approach is to fit the RAW data, hence describing all data points instead of excluding some.

This approach is philosophically more clean, but certainly not easy!

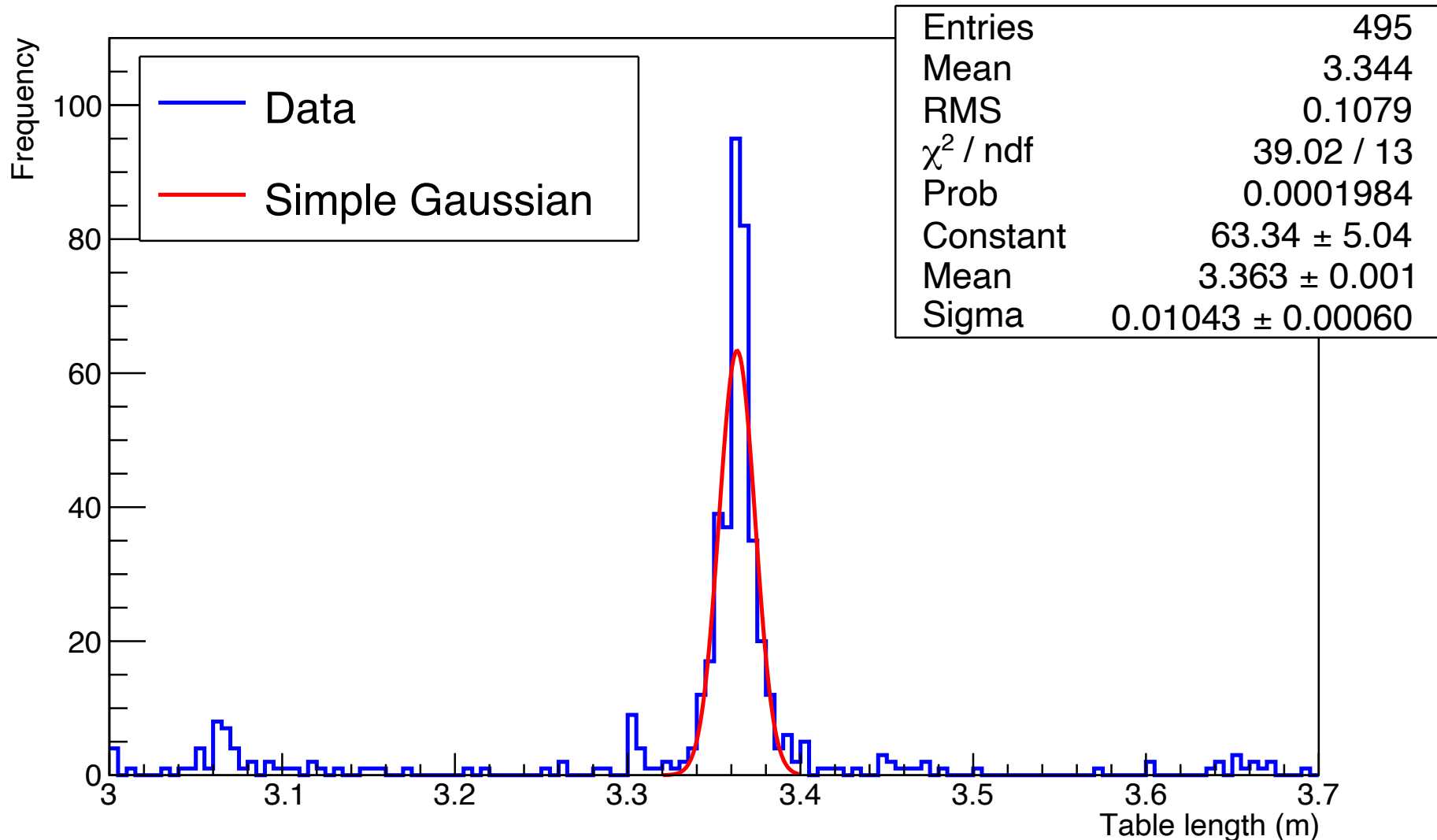
Challenges:

- Measurements has many different resolutions.
- There are several peaks in the data (30cm case).
- Some measurements are clearly rounded.

While all of these can be accommodated, it is still a challenge, at the following “fitting around” took me half and hour!

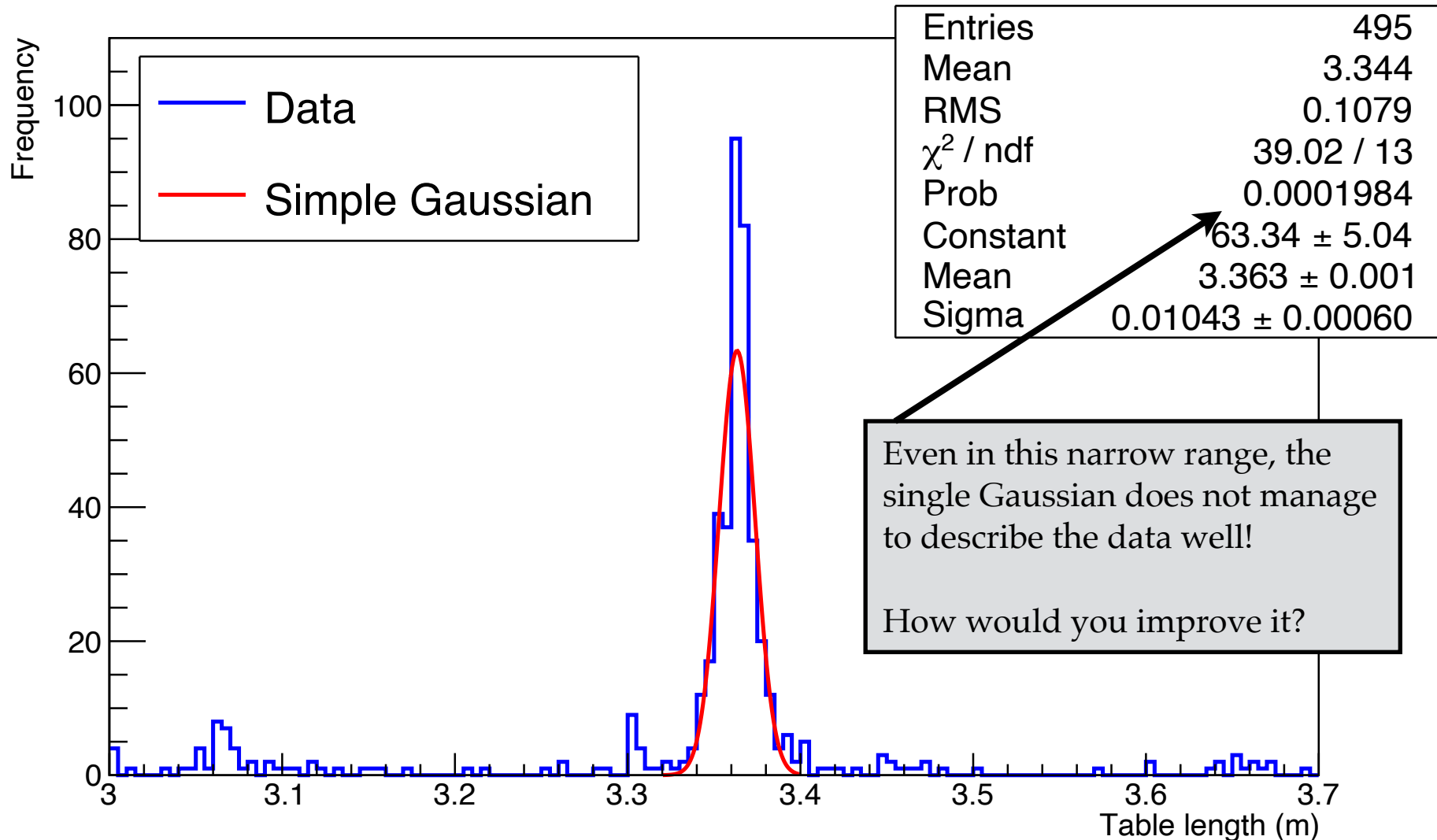
# Fitting for a result

Lengths estimates by 30cm ruler



# Fitting for a result

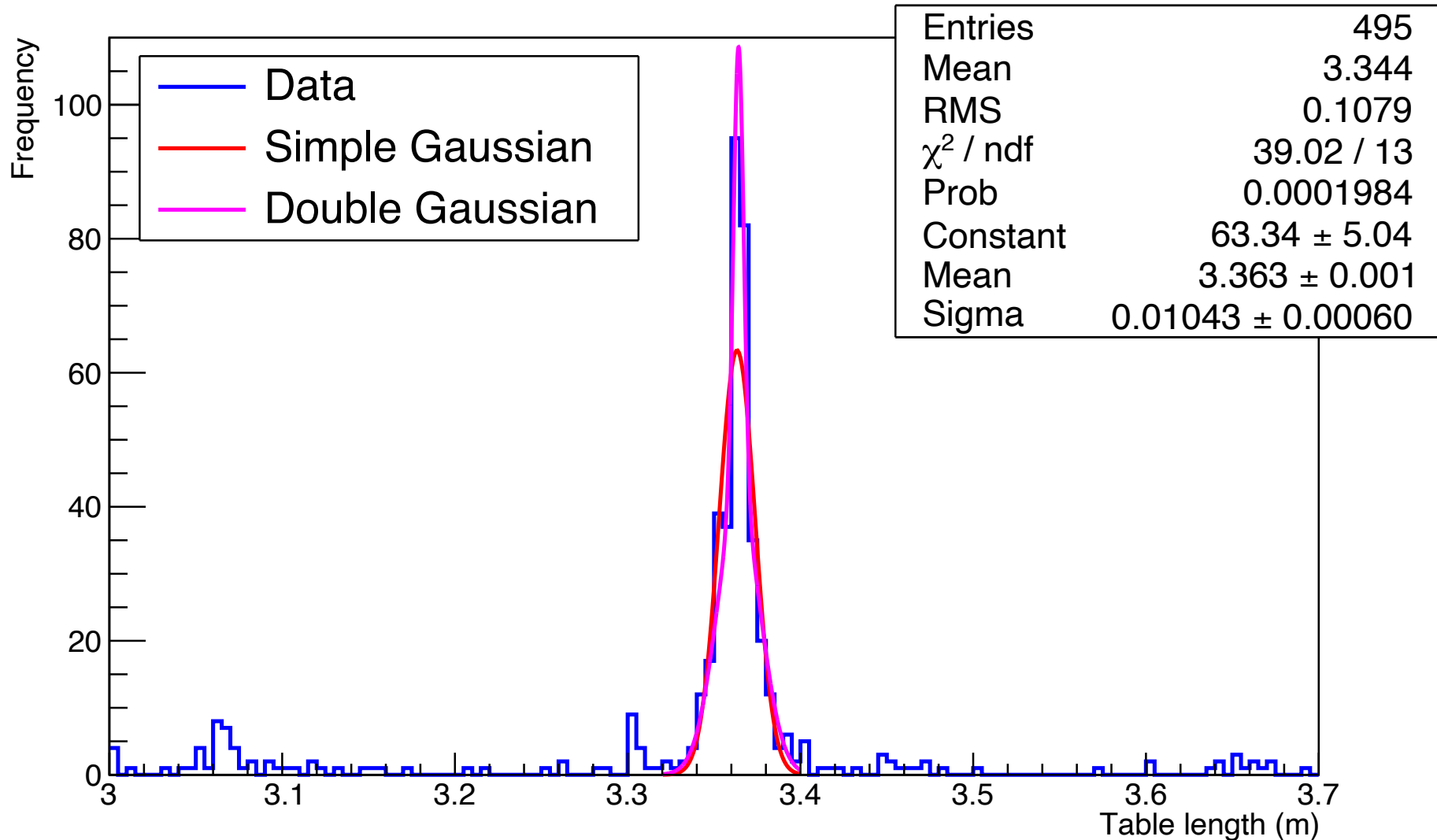
Lengths estimates by 30cm ruler





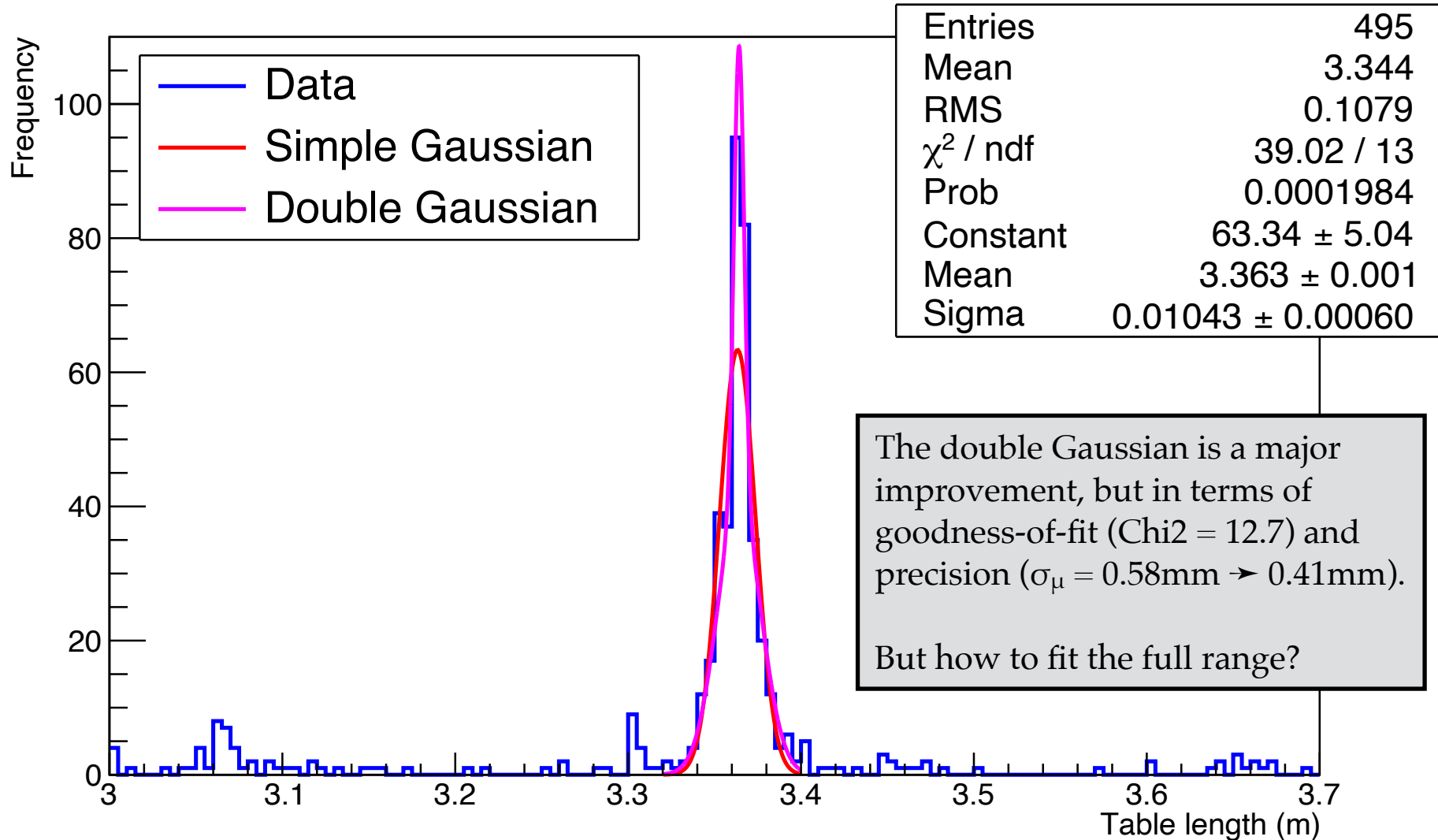
# Fitting for a result

Lengths estimates by 30cm ruler



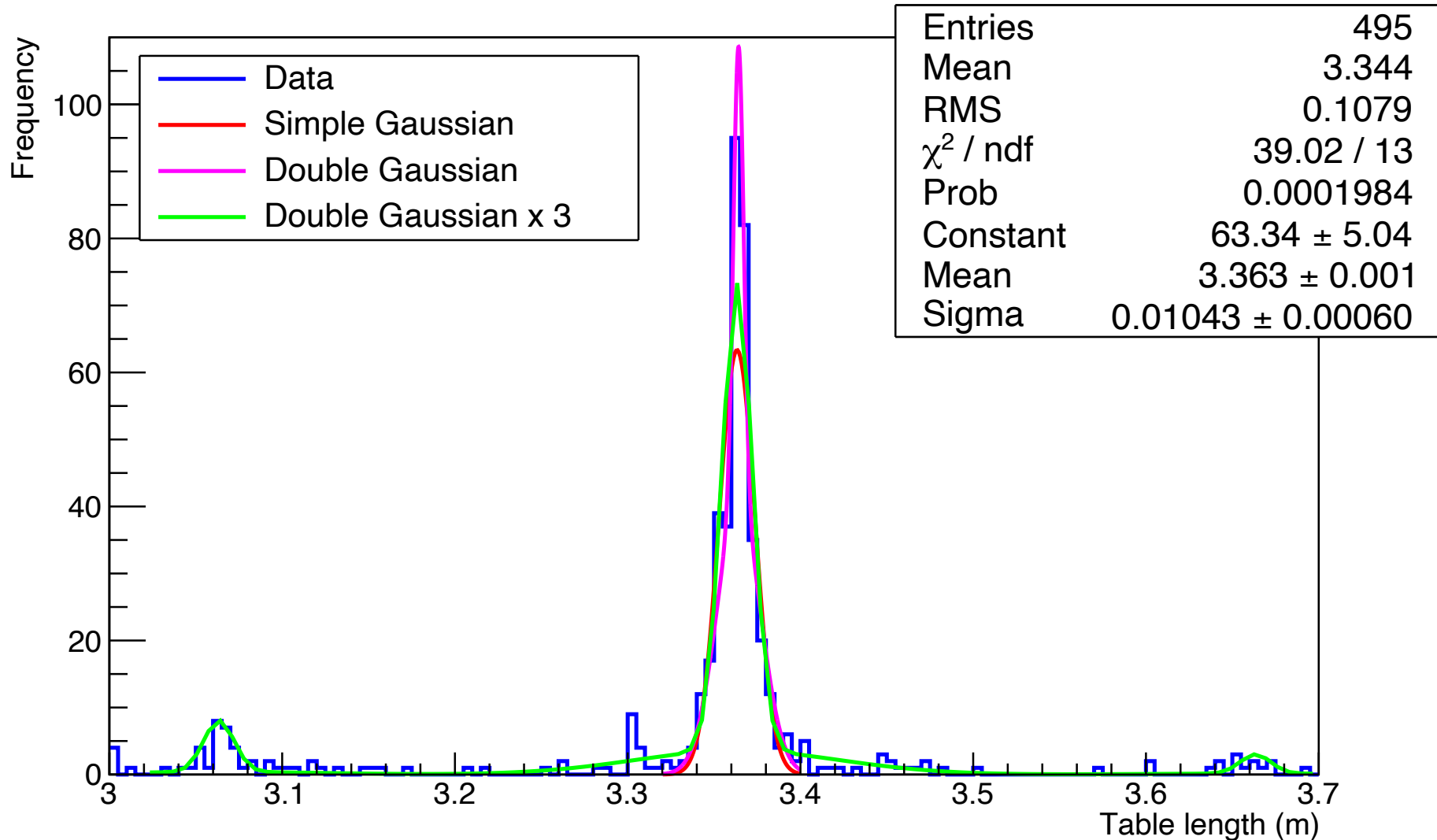
# Fitting for a result

Lengths estimates by 30cm ruler



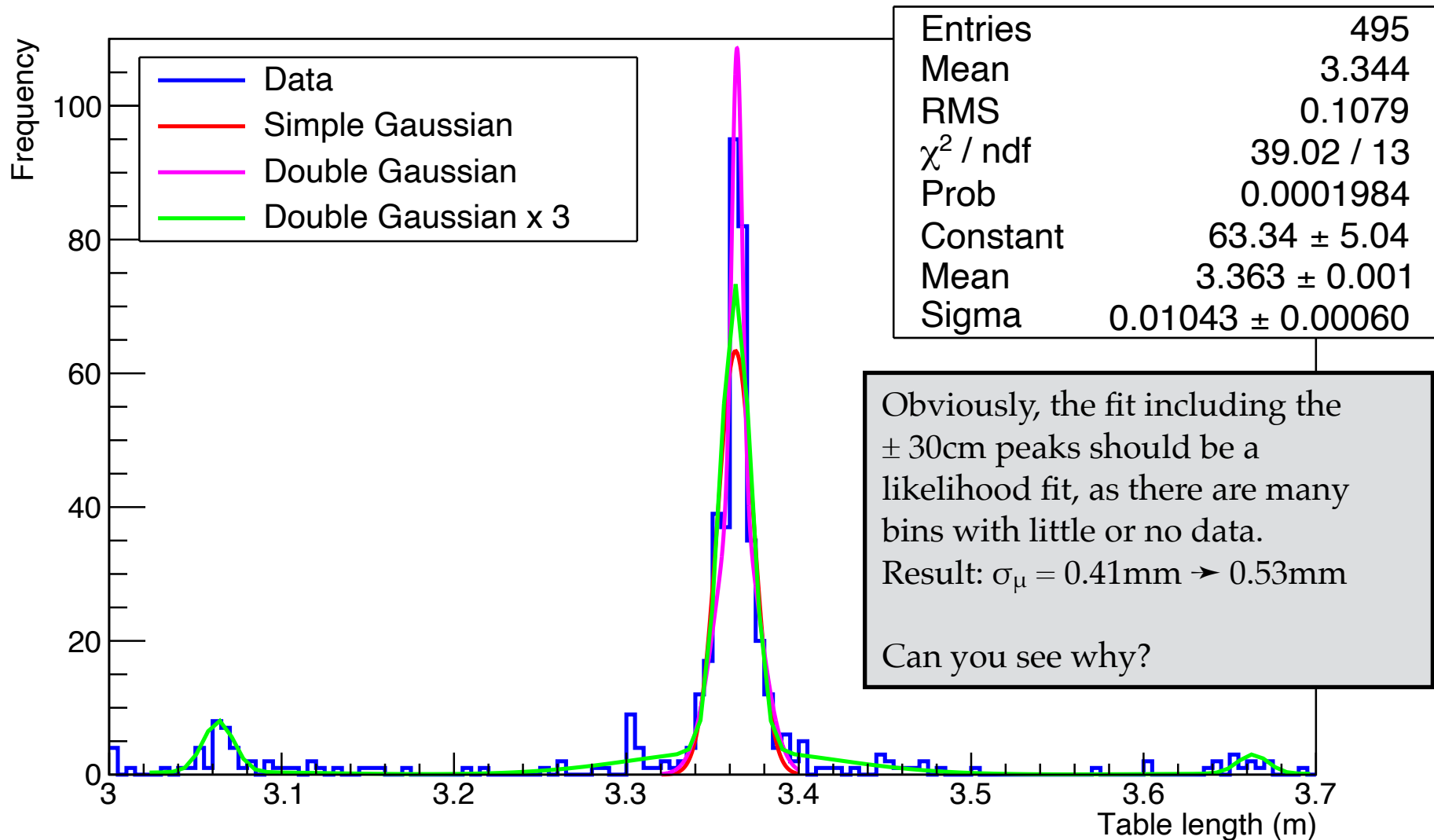
# Fitting for a result

Lengths estimates by 30cm ruler



# Fitting for a result

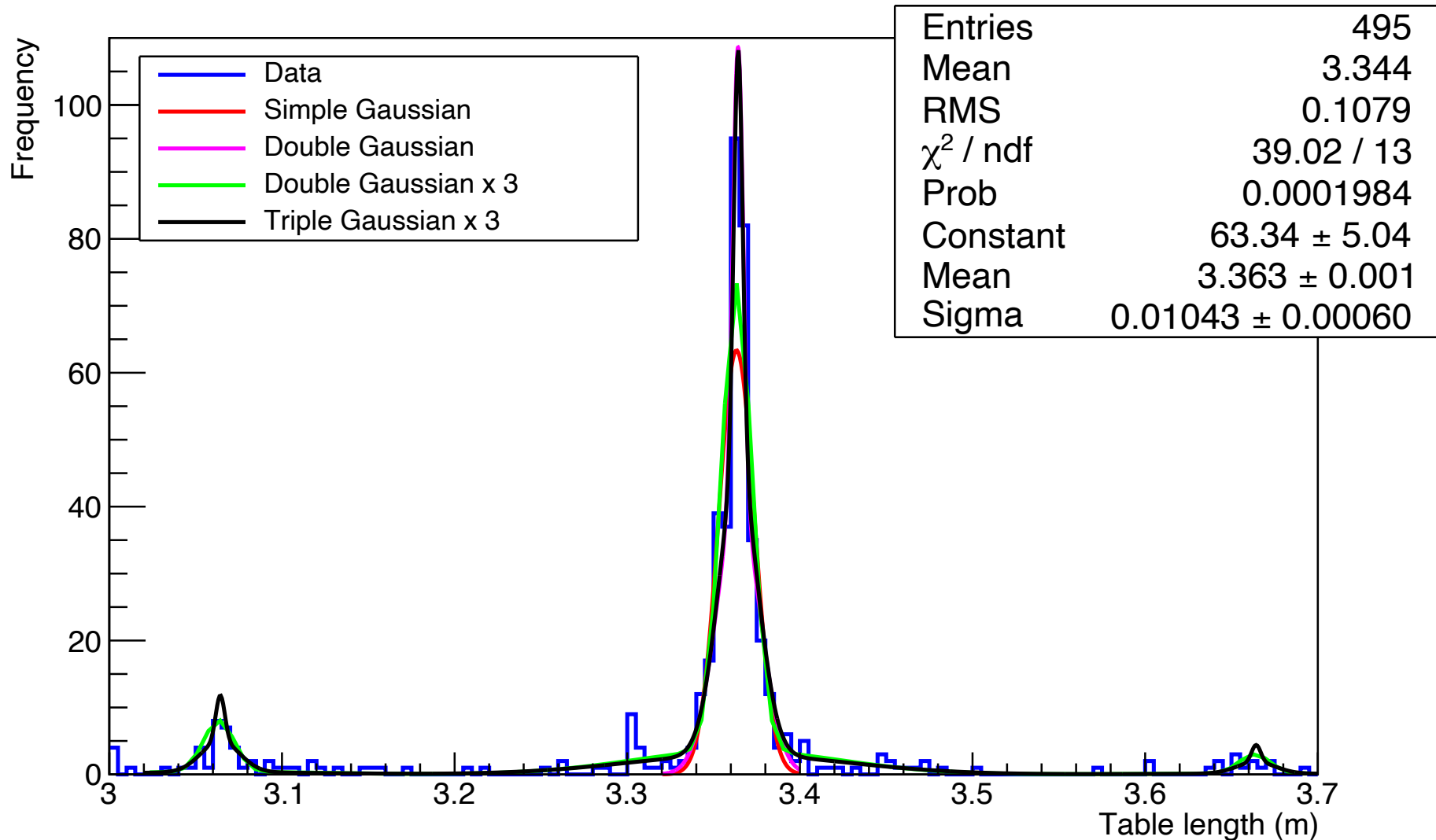
Lengths estimates by 30cm ruler





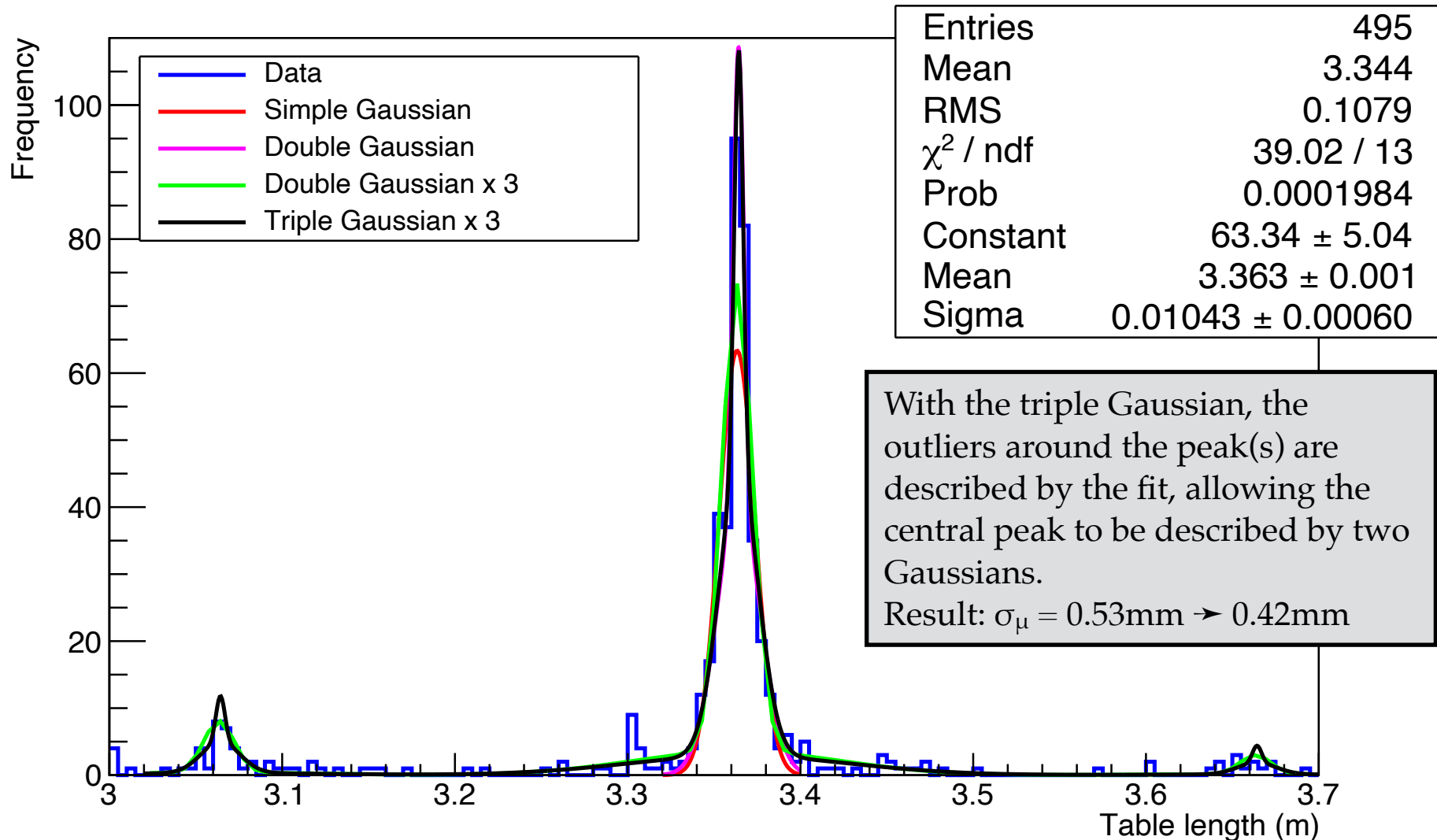
# Fitting for a result

Lengths estimates by 30cm ruler



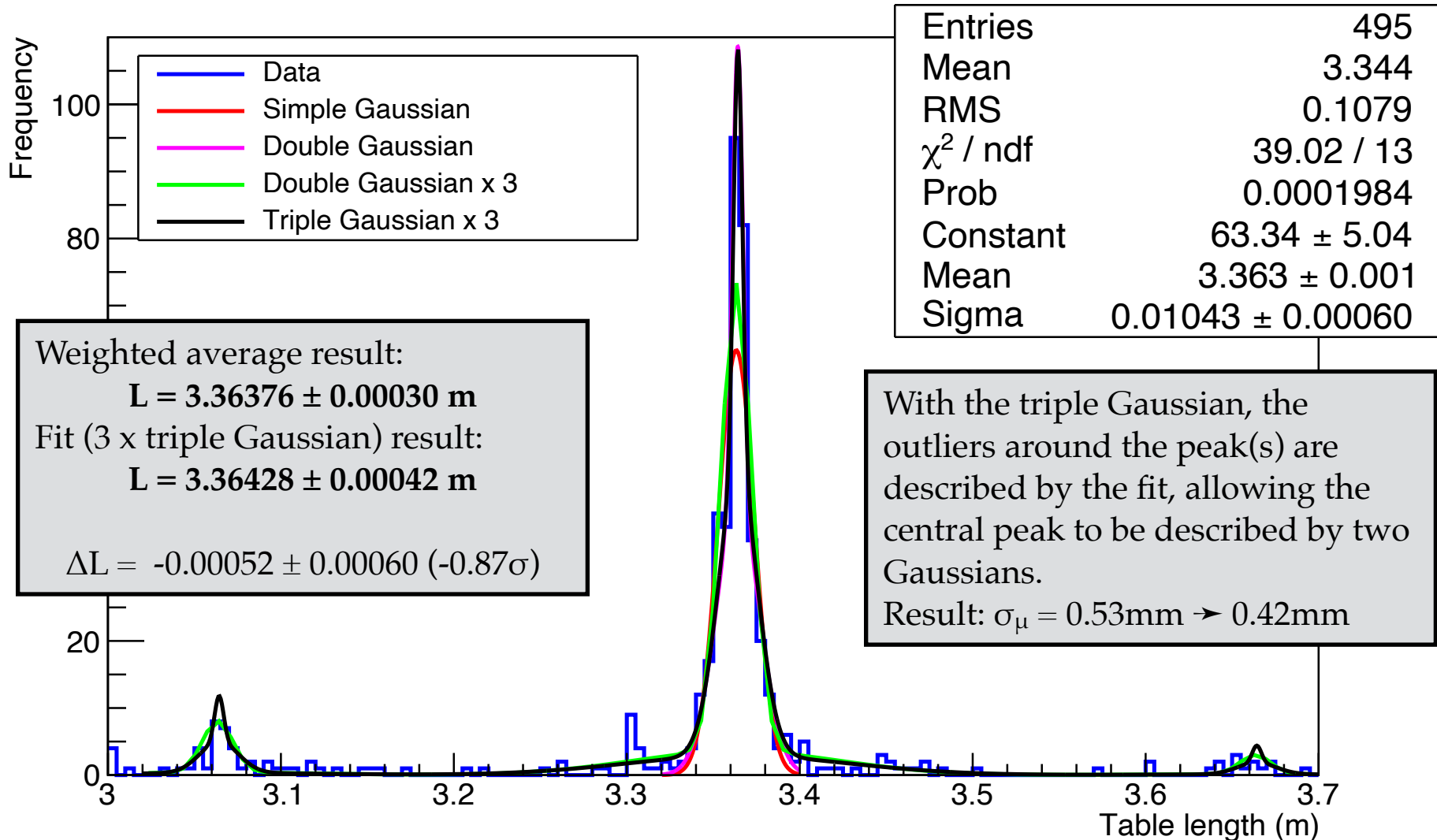
# Fitting for a result

Lengths estimates by 30cm ruler



# Fitting for a result

Lengths estimates by 30cm ruler





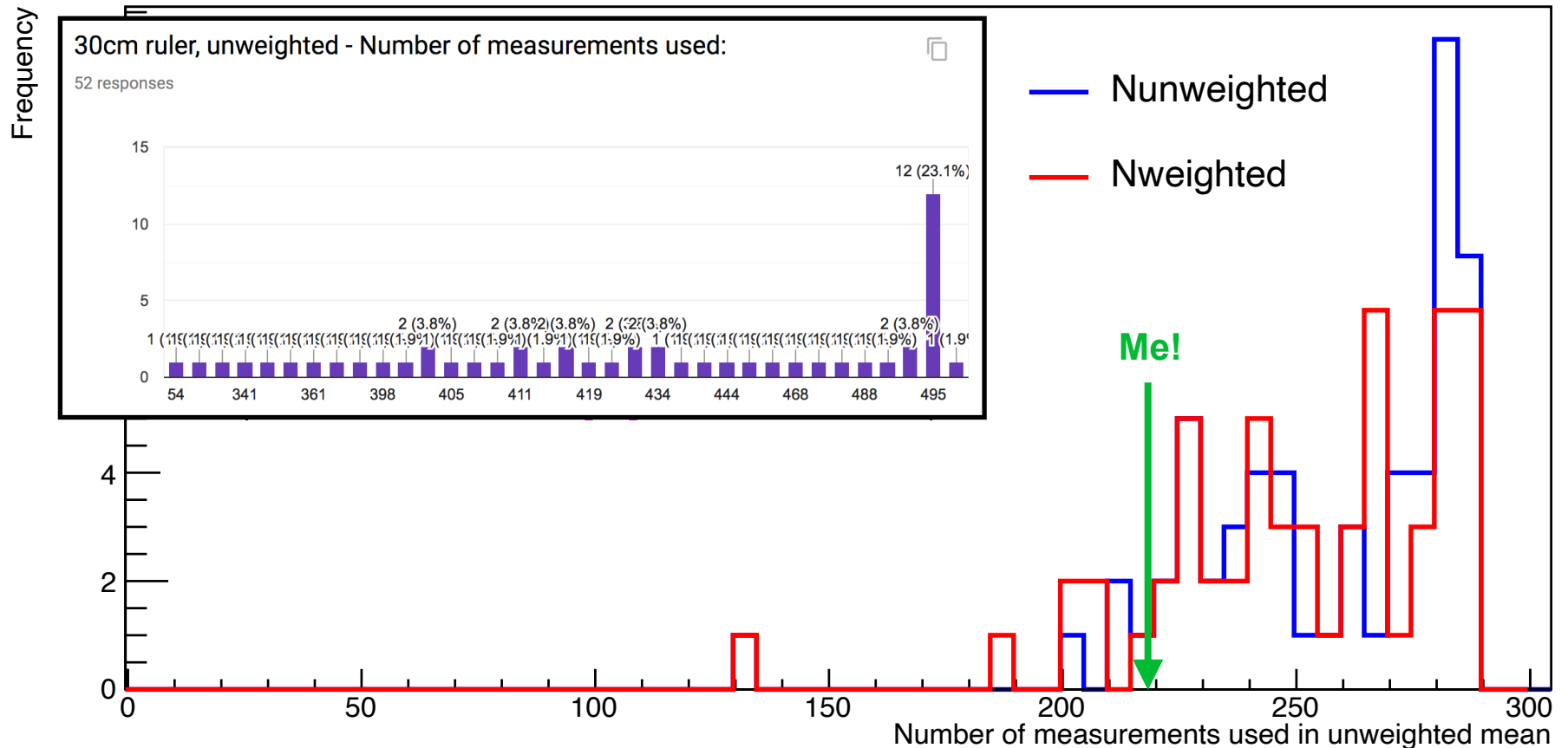
The background image is a nautical chart featuring a compass rose with magnetic isogonic lines. The chart includes labels such as 'MAGNETIC' and 'THE BITTER END YACHT CLUB'. The compass rose shows magnetic variation lines, and the chart includes various navigational markings and text.

# Student analyses comparison



# Your measurement results

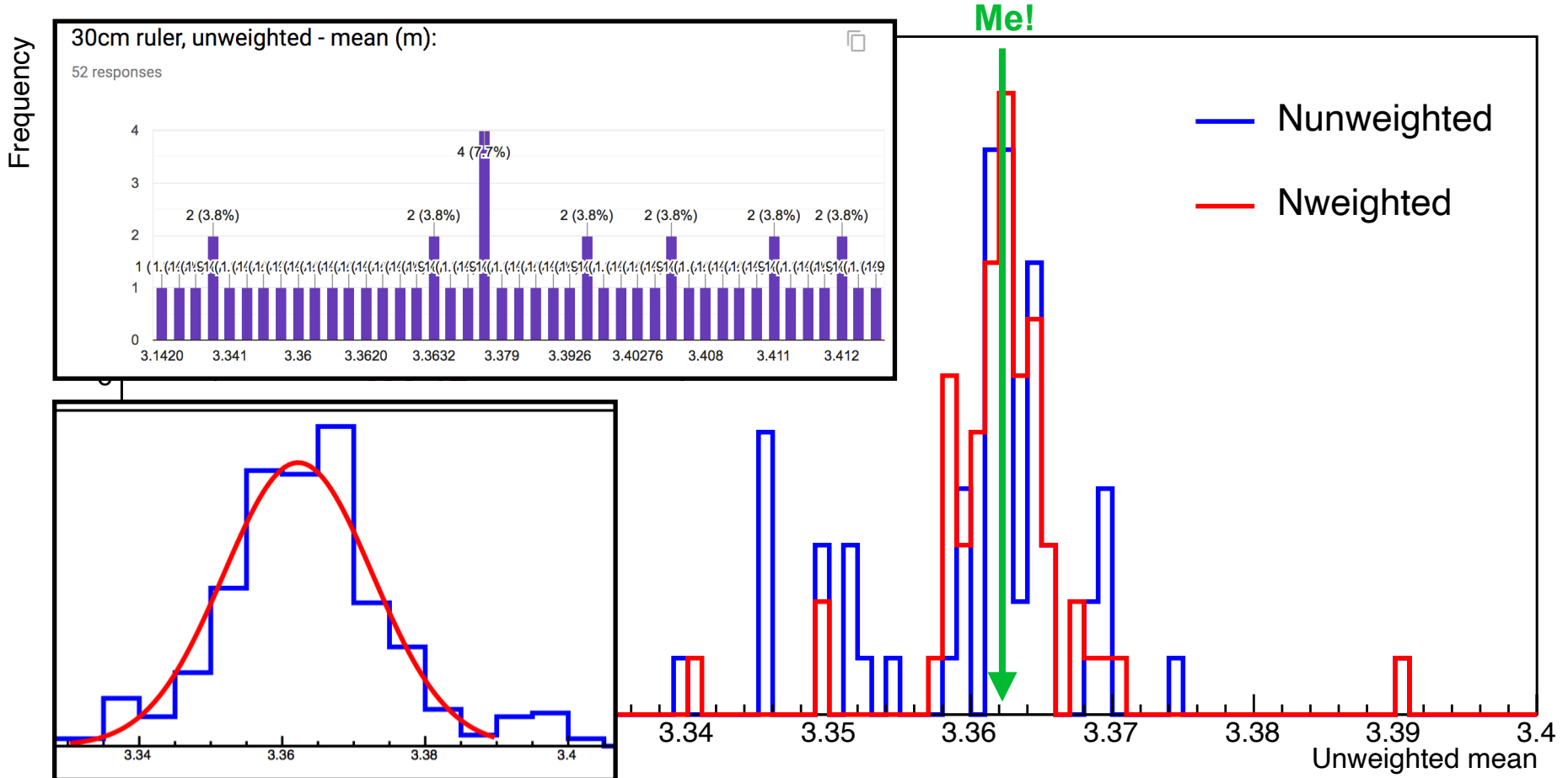
The number of measurements used varied quiet a bit.



But remember that the impact is only  $\sqrt{N}$ , and thus not that important!

# Length results

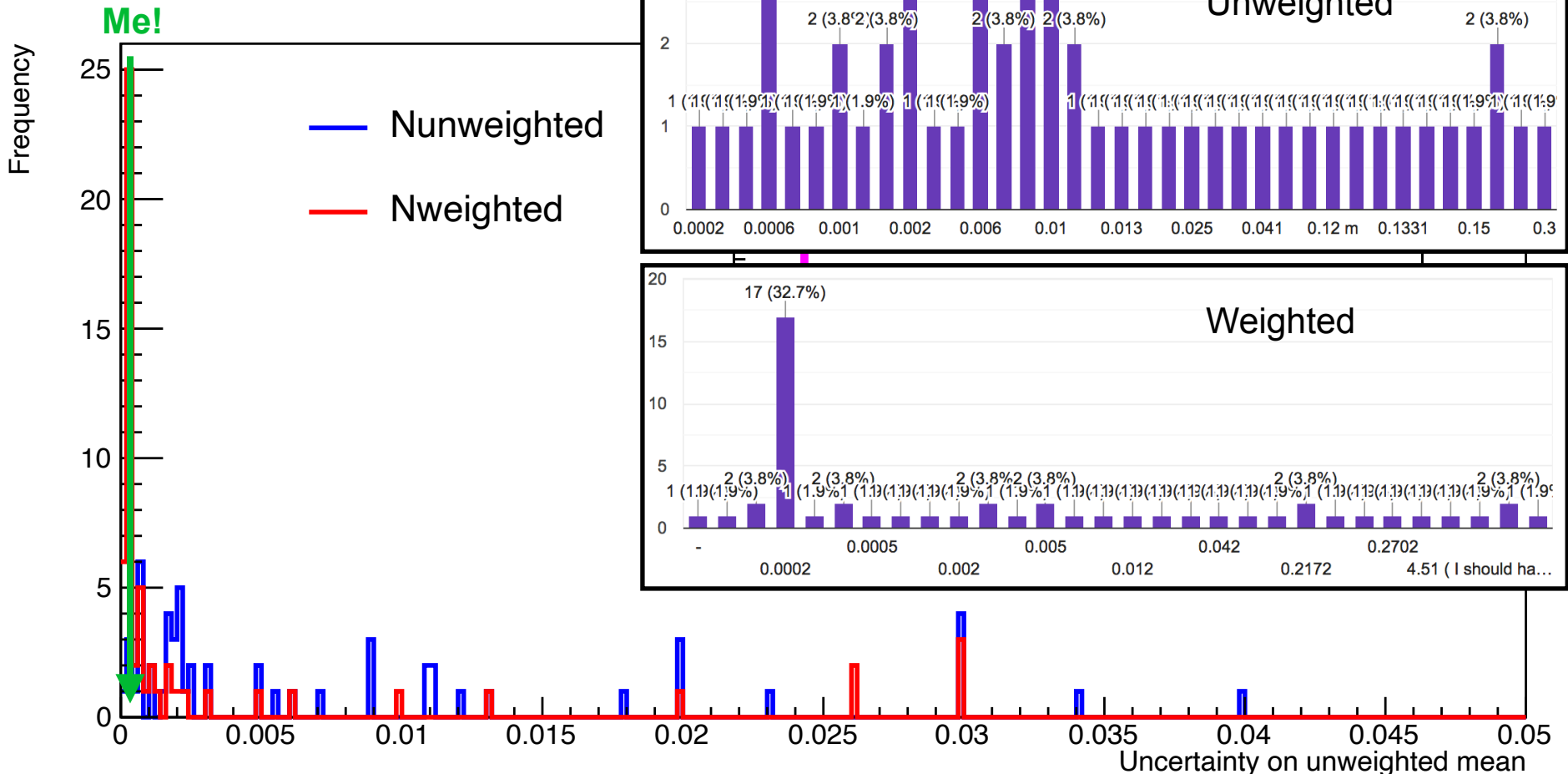
Results are relatively consistent... 80% of you get a value within 0.1% of "true"



I got:  $L(\text{unweighted}) = 3.36227 \pm 0.00061 \text{ m}$ ,  $L(\text{weighted}) = 3.36371 \pm 0.00035 \text{ m}$

# Uncertainties

The uncertainties varied quite a bit - by more than a factor 100! Think about that.



I got:  $L(\text{unweighted}) = 3.36227 \pm 0.00061 \text{ m}$ ,  $L(\text{weighted}) = 3.36371 \pm 0.00035 \text{ m}$

# Conclusions

## Specifically on the analysis:

- Greatest improvement came from simply removing mis-measurements!
- Weighted result was a further improvement, but required good uncertainties.
- The uncertainties are accepted as “reasonable”, as they have good pull distributions, and improve the result.
- The 30cm and 2m results match, giving credibility to the stated precision.

## More generally:

- What appears to be a trivial task, turns out to require some thought anyhow.  
(Ask yourself how many fellow students would have been able to get a good result and error?)
- There were several choices to be made in the analysis:
  1. Which measurements to accept.
  2. Which uncertainties to accept.
  3. To correct or discard understood mis-measurements.
- All this can be solved with simple Python code.

