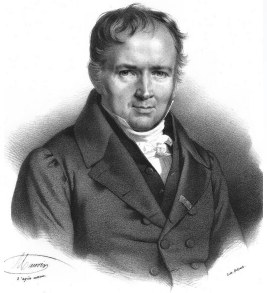
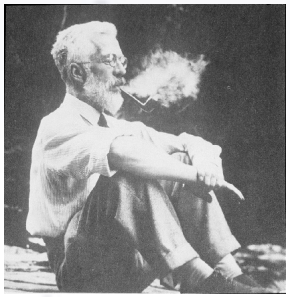
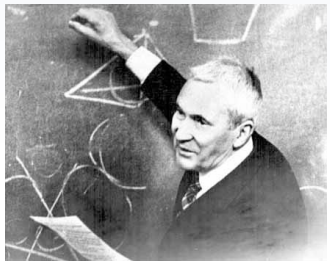
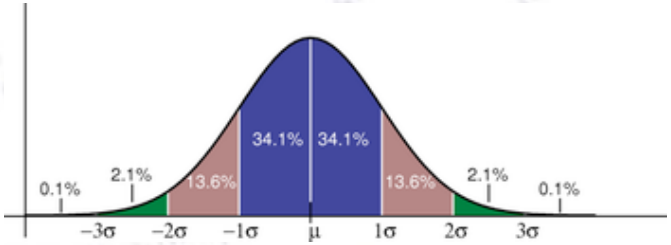
# Applied Statistics

## Mean and Width



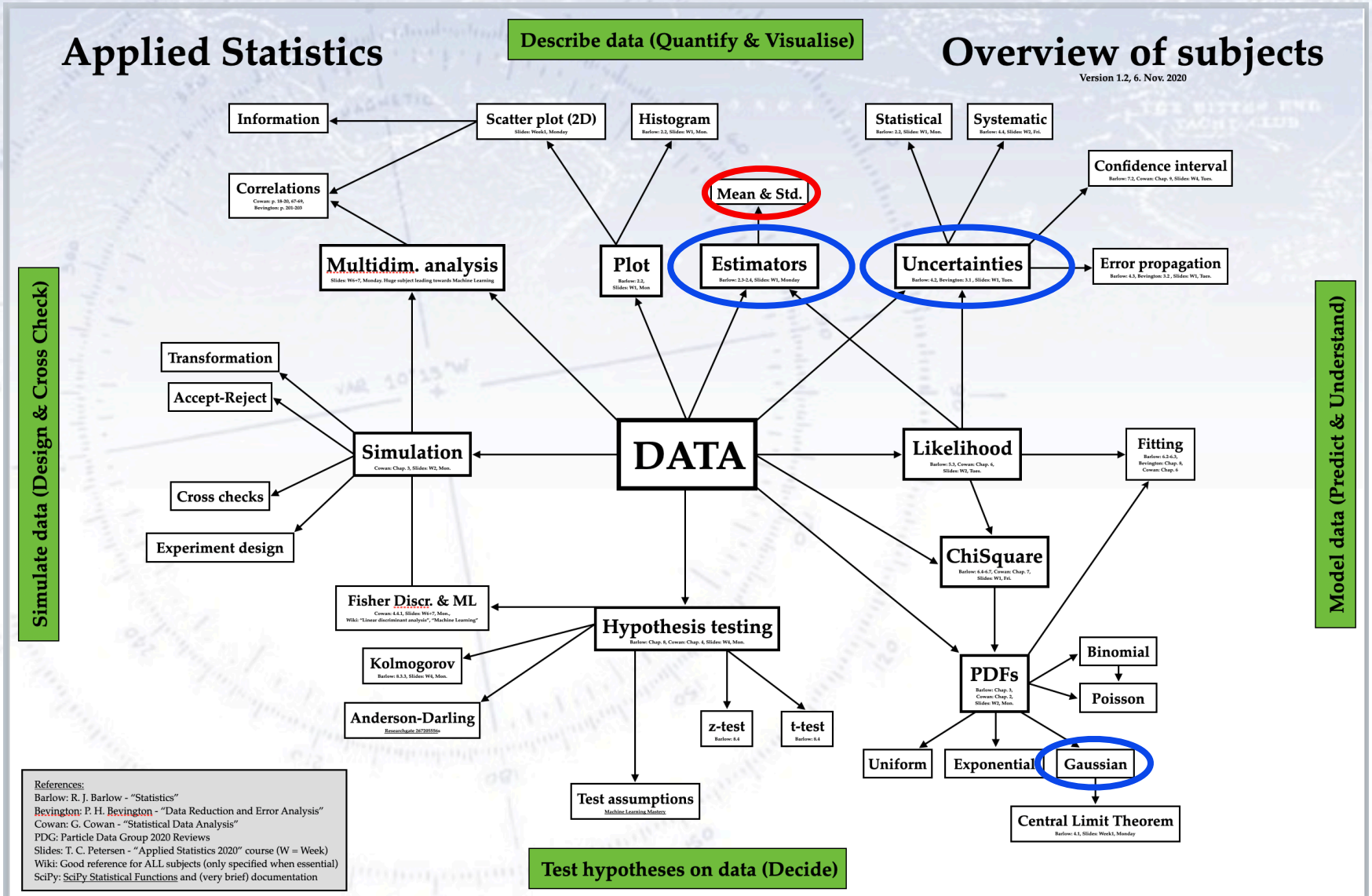





Troels C. Petersen (NBI)




*"Statistics is merely a quantisation of common sense"*

# Mean & Width

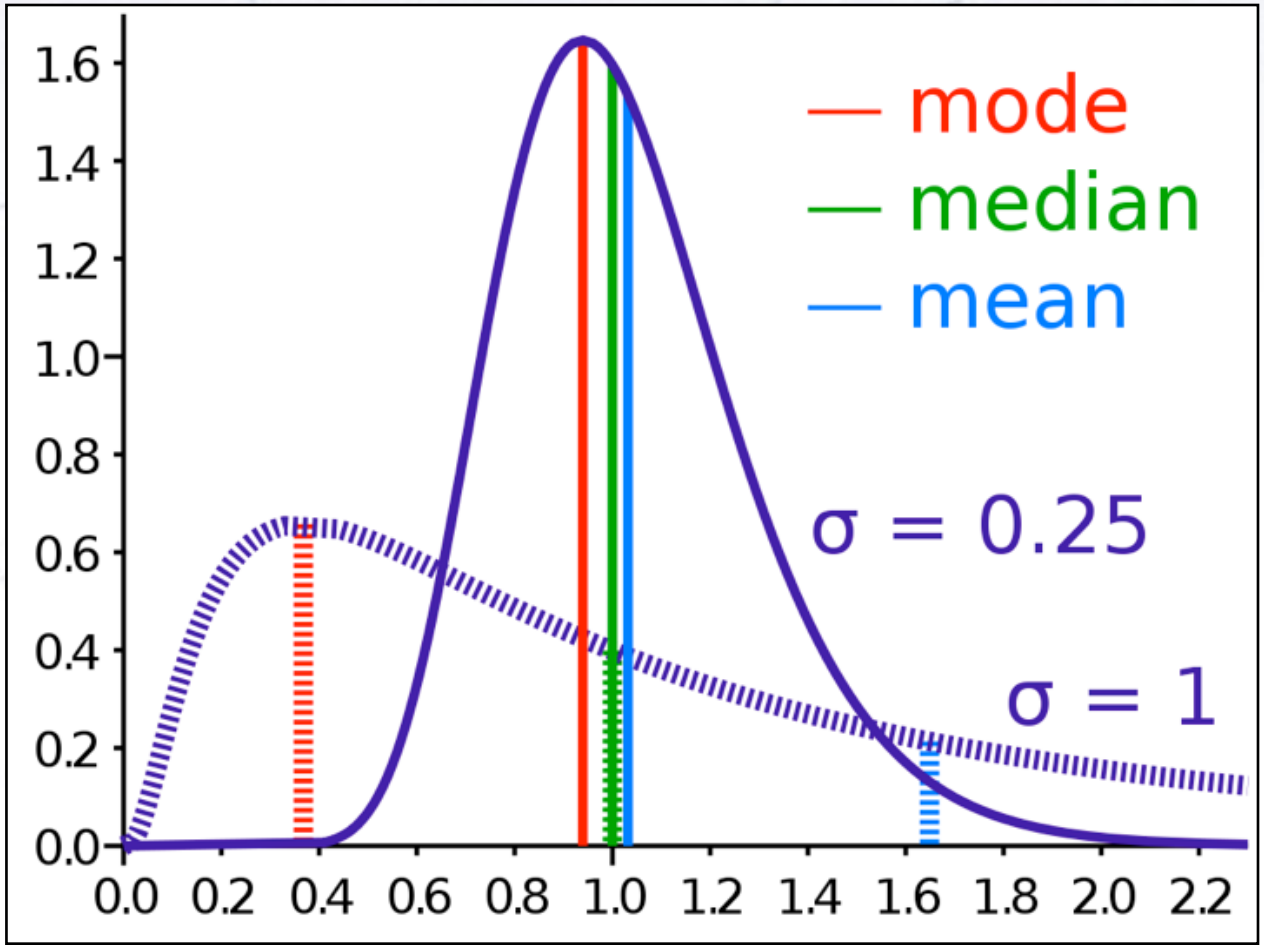

Applied Statistics
Describe data (Quantify & Visualise)
Overview of subjects
Version 1.2, 6. Nov. 2020
Simulate data (Design & Cross Check)
Model data (Predict & Understand)
Information
Scatter plot (2D)
Slides: Week1, Monday
Histogram
Barlow: 2.2, Slides: W1, Mon.
Statistical
Barlow: 2.2, Slides: W1, Mon.
Systematic
Barlow: 4.4, Slides: W2, Fri.
Confidence interval
Barlow: 7.2, Cowan: Chap. 9, Slides: W4, Tues.
Correlations
Cowan: p. 18-20, 67-69, Bevington: p. 203-203
Mean & Std.
Multidim. analysis
Slides: W6+7, Monday. Huge subject leading towards Machine Learning
Plot
Barlow: 2.2, Slides: W1, Mon
Estimators
Barlow: 2.3-2.4, Slides: W1, Monday
Uncertainties
Barlow: 4.2, Bevington: 3.1 , Slides: W1, Tues.
Error propagation
Barlow: 4.3, Bevington: 3.2 , Slides: W1, Tues.
Transformation
Accept-Reject
Simulation
Cowan: Chap. 3, Slides: W2, Mon.
DATA
Likelihood
Barlow: 5.3, Cowan: Chap. 6, Slides: W2, Tues.
Fitting
Barlow: 6.2-6.3, Bevington: Chap. 8, Cowan: Chap. 6
Cross checks
Experiment design
ChiSquare
Barlow: 4.4-4.7, Cowan: Chap. 7, Slides: W1, Fri.
Fisher Discr. & ML
Cowan: 4.4.1, Slides: W6+7, Mon., Wiki: "Linear discriminant analysis", "Machine Learning"
Hypothesis testing
Barlow: Chap. 8, Cowan: Chap. 4, Slides: W4, Mon.
Kolmogorov
Barlow: 8.3.3, Slides: W4, Mon.
Anderson-Darling
Researchgate 267205516
z-test
Barlow: 8.4
t-test
Barlow: 8.4
PDFs
Barlow: Chap. 3, Cowan: Chap. 2, Slides: W2, Mon.
Binomial
Poisson
Uniform
Exponential
Gaussian
Central Limit Theorem
Barlow: 4.1, Slides: Week1, Monday
Test assumptions
Machine Learning Mastery
References:
Barlow: R. J. Barlow - "Statistics"
Bevington: P. H. Bevington - "Data Reduction and Error Analysis"
Cowan: G. Cowan - "Statistical Data Analysis"
PDG: Particle Data Group 2020 Reviews
Slides: T. C. Petersen - "Applied Statistics 2020" course (W = Week)
Wiki: Good reference for ALL subjects (only specified when essential)
SciPy: SciPy Statistical Functions and (very brief) documentation
Test hypotheses on data (Decide)

# Defining the mean

There are several ways of defining "a typical" value from a dataset:

a) Arithmetic mean  b) Mode (most probably)  c) Median (half below, half above)

d) Geometric mean  e) Harmonic mean  f) Truncated mean (robustness)



# Mean and Width

It turns out, that the best estimator for the **mean** is (as you all know):

$$\hat{\mu} = \frac{1}{N} \sum_i x_i = \bar{x}$$

The second (central) moment of the data is called the **variance**, defined as:

$$\hat{V} = \frac{1}{N} \sum_i (x_i - \mu)^2$$

Note the “hat”, which means “estimator”. It is sometimes dropped...

# Mean and Width

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$$\hat{\sigma} = \sqrt{\frac{1}{N}\sum_i (x_i - \mu)^2}$$

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$$\hat{s} = \sqrt{\frac{1}{N-1} \sum_i (x_i - \bar{x})^2}$$

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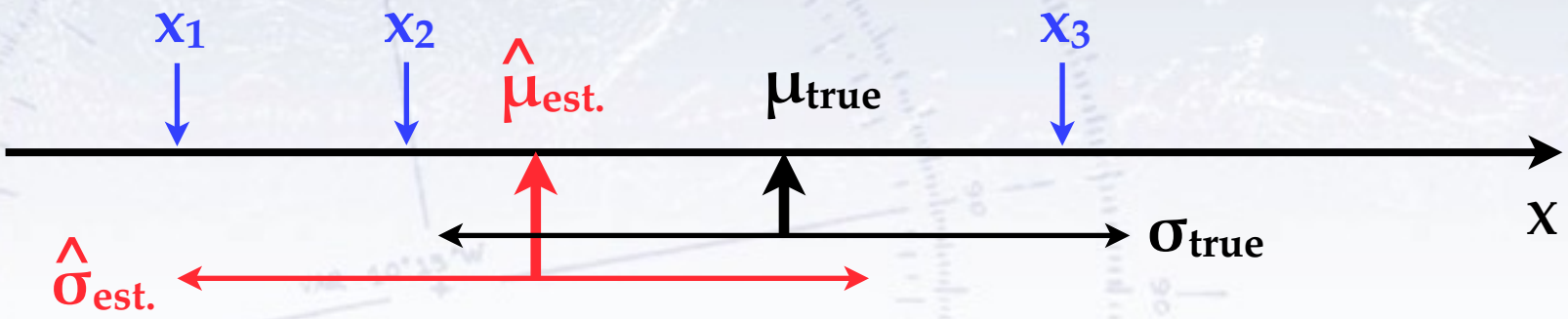
# Why not "just" the naive SD?

Imagine taking 3 independent measurements, and then the mean and SD:

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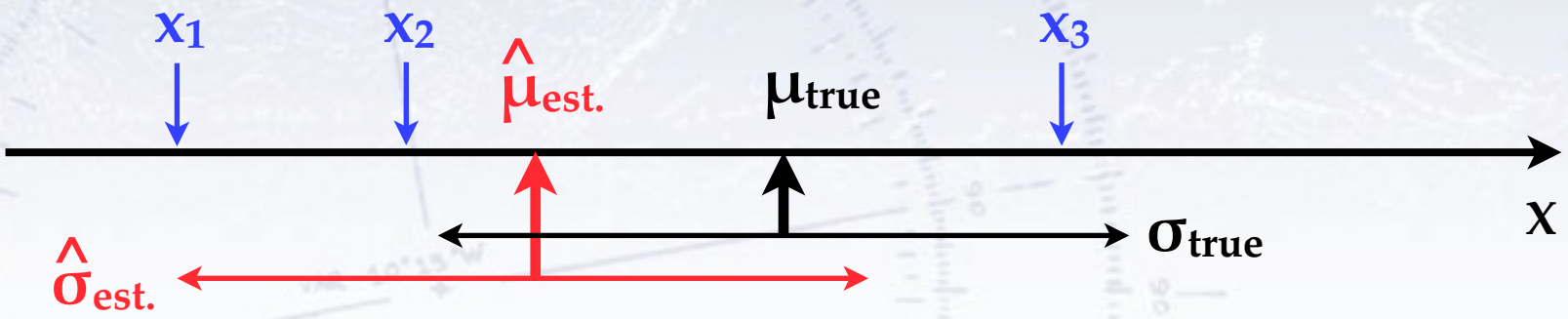


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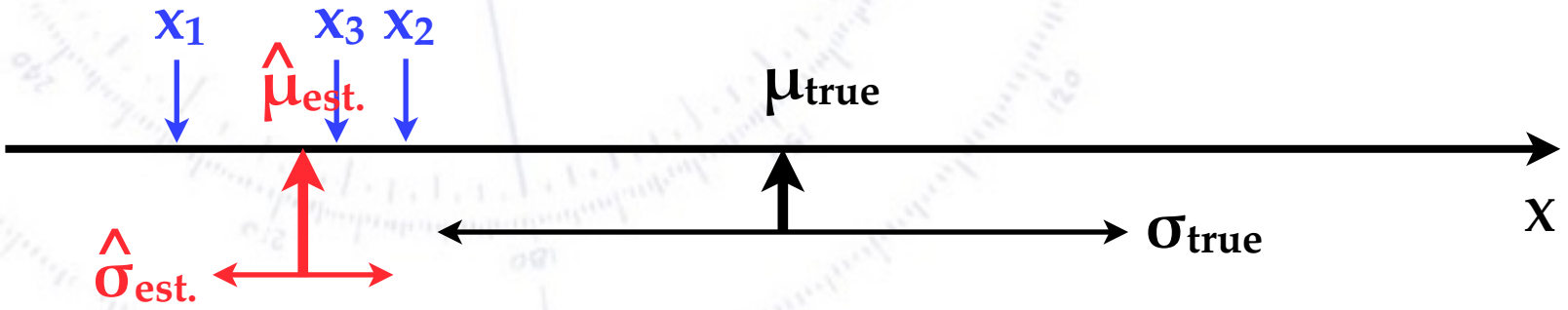
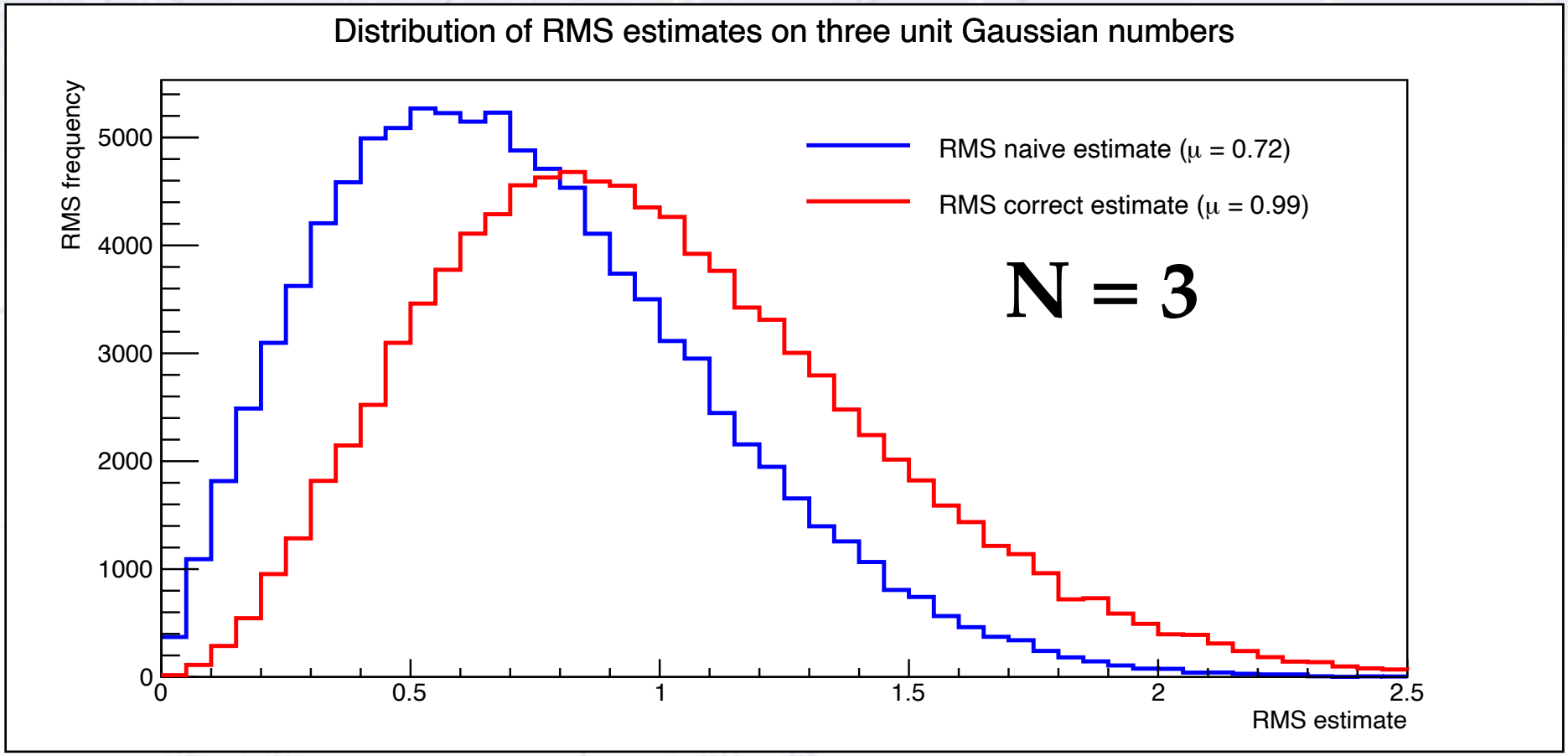



However, now the mean is off (not terribly so) and the SD way off (terribly so!). If we had used the true mean in the formula, it would not have been a problem.

# How incorrect is the naive SD?

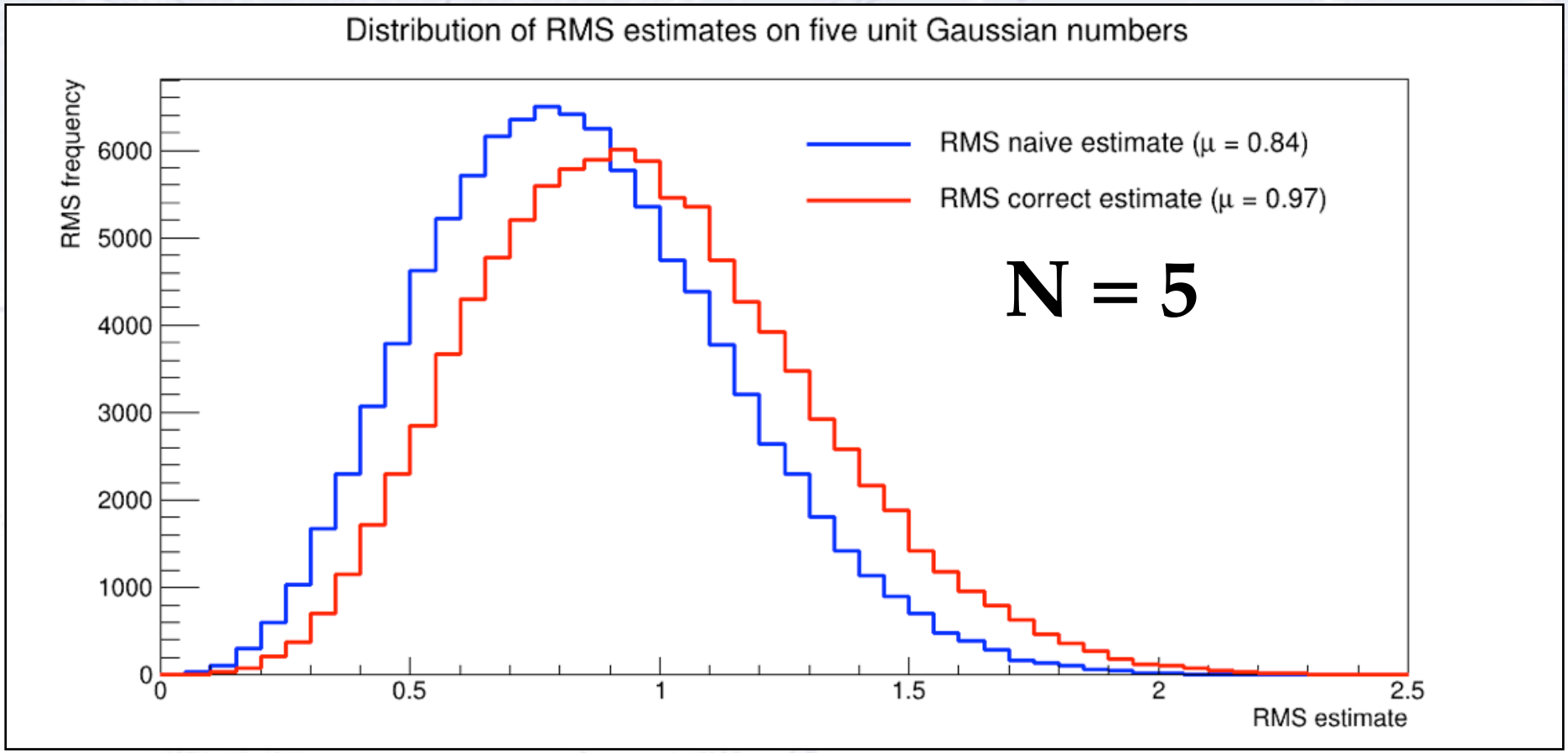
Such questions can most easily be answered by a small simulation…
Produce N=3 numbers from a unit Gaussian, and calculate the SD estimate:




So, the "naive" SD underestimates the uncertainty significantly…

# How incorrect is the naive SD?

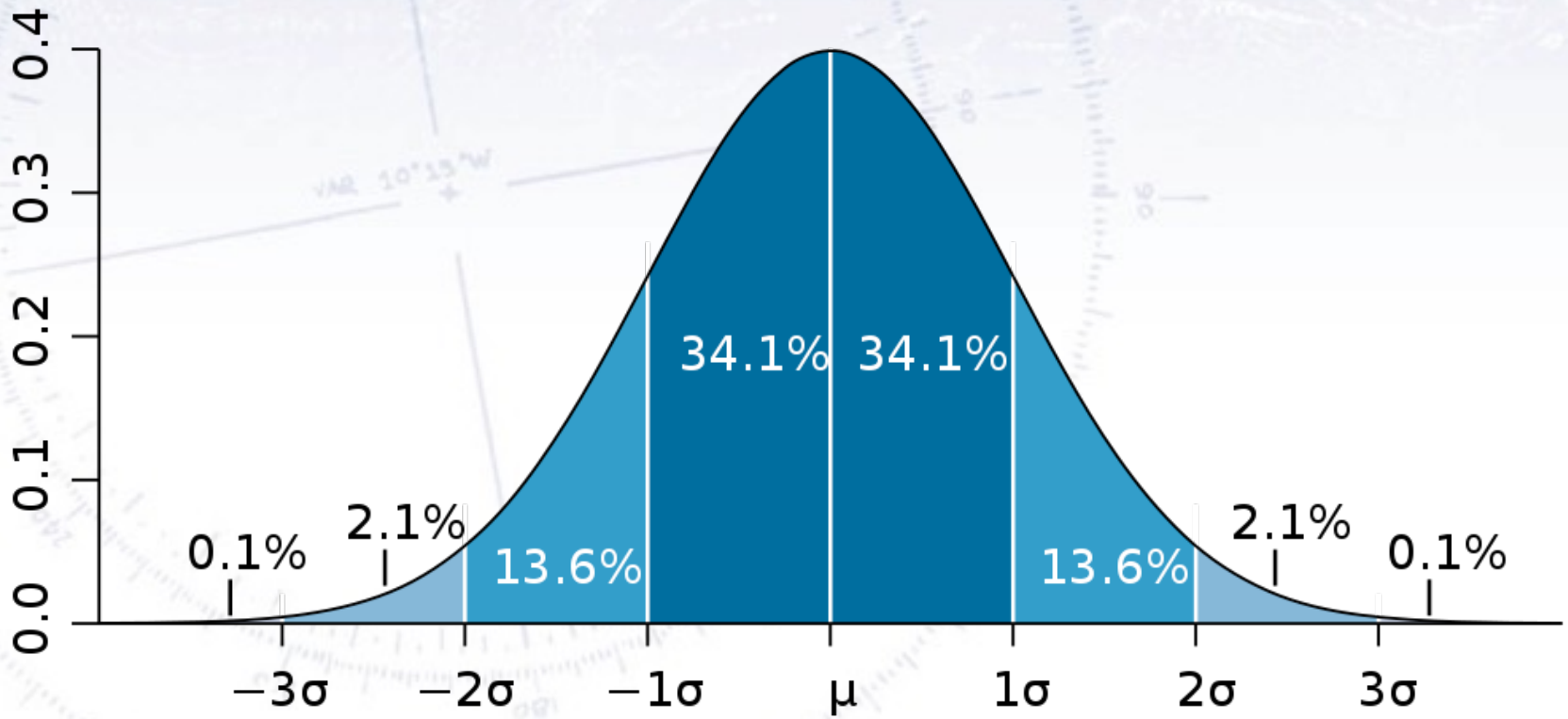
Such questions can most easily be answered by a small simulation…
Produce N=5 numbers from a unit Gaussian, and calculate the SD estimate:




Here, the "naive" SD underestimates the uncertainty a bit...

# SD and Gaussian σ relation

When a distribution is Gaussian, **the Std. corresponds to the Gaussian width σ**:



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Example:

**Cavendish Experiment**

(measurement of Earth's density)

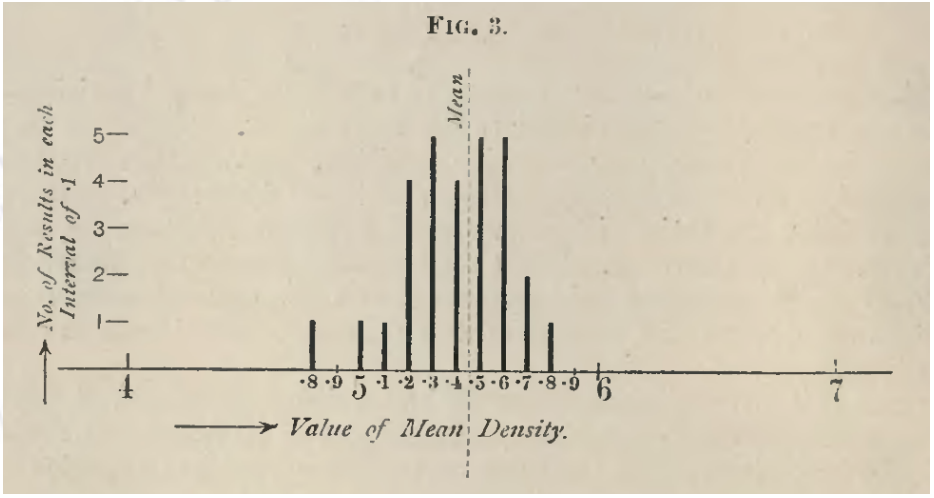
N = 29

mu = 5.42

sigma = 0.333

sigma(mu) = 0.06

**Earth density = 5.42 ± 0.06**



# Mean and Width

What is the **uncertainty on the mean?** And how quickly does it improve with more data?

$$\hat{\sigma}_{\mu} = \hat{\sigma} / \sqrt{N}$$

Please commit to memory now!

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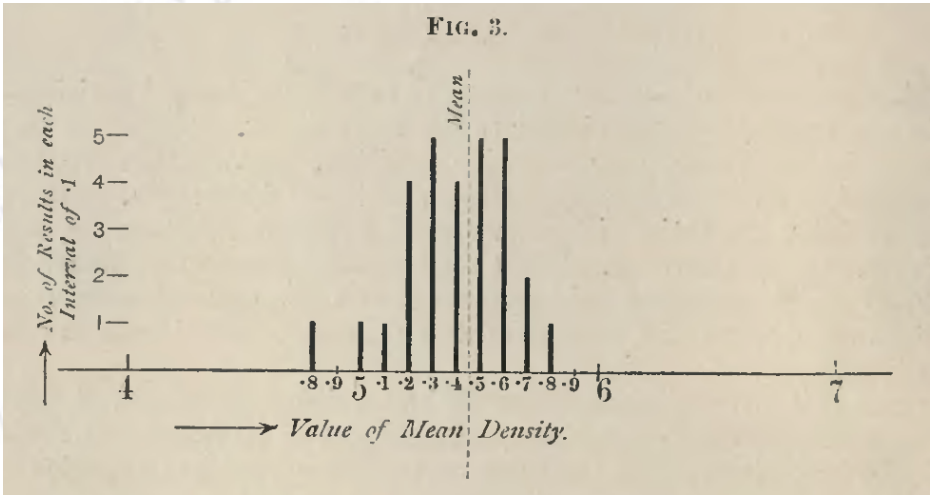


# Weighted Mean

What if we are given data, which has different uncertainties?
How to average these, and what is the uncertainty on the average?

$$\hat{\mu} = \frac{\sum x_i/\sigma_i^2}{\sum 1/\sigma_i^2}$$

For measurements with varying uncertainty, there is no meaningful SD!
The uncertainty on the mean is:

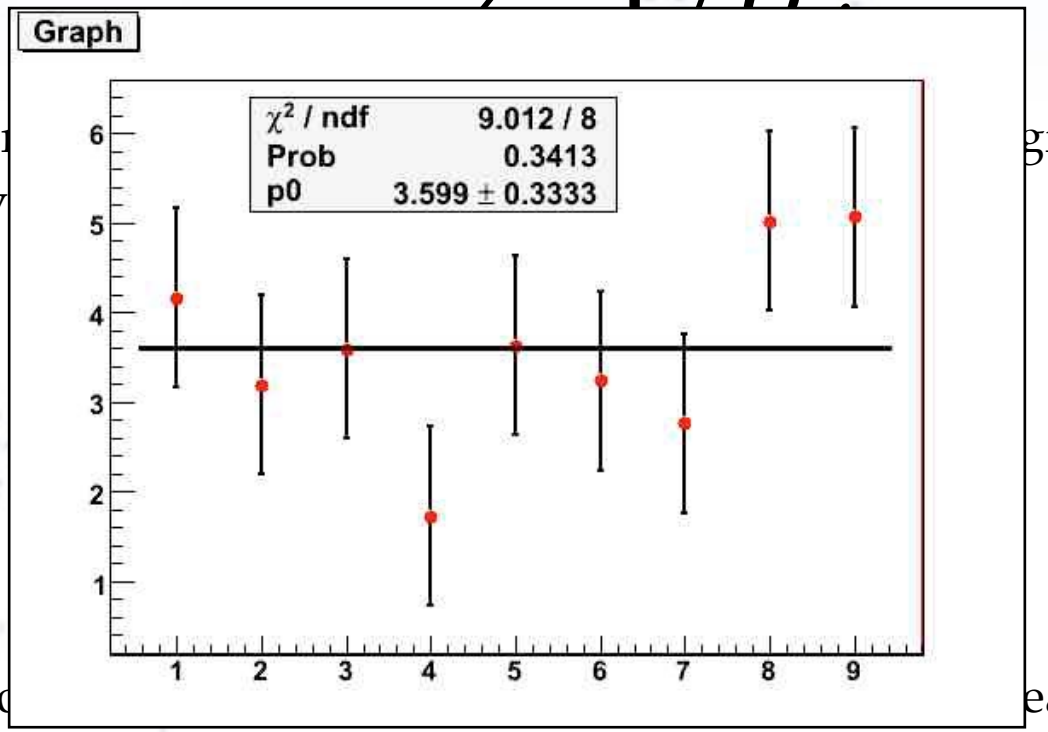
$$\hat{\sigma}_\mu = \sqrt{\frac{1}{\sum 1/\sigma_i^2}}$$

Can be understood intuitively, if two persons combine 1 vs. 4 measurements

# Weighted Mean

What if we are given data, which has different uncertainties?
How to ave

Note that when doing a weighted mean, one should check if the measurements agree with each other!
This can be done with a ChiSquare test.




For measureme gful SD!
The uncertainty

Can be understo easurements

# Resolution using InterQuantile Range

A useful measure of resolution is the InterQuantile Range (IQR), as this is not affected by long tails.

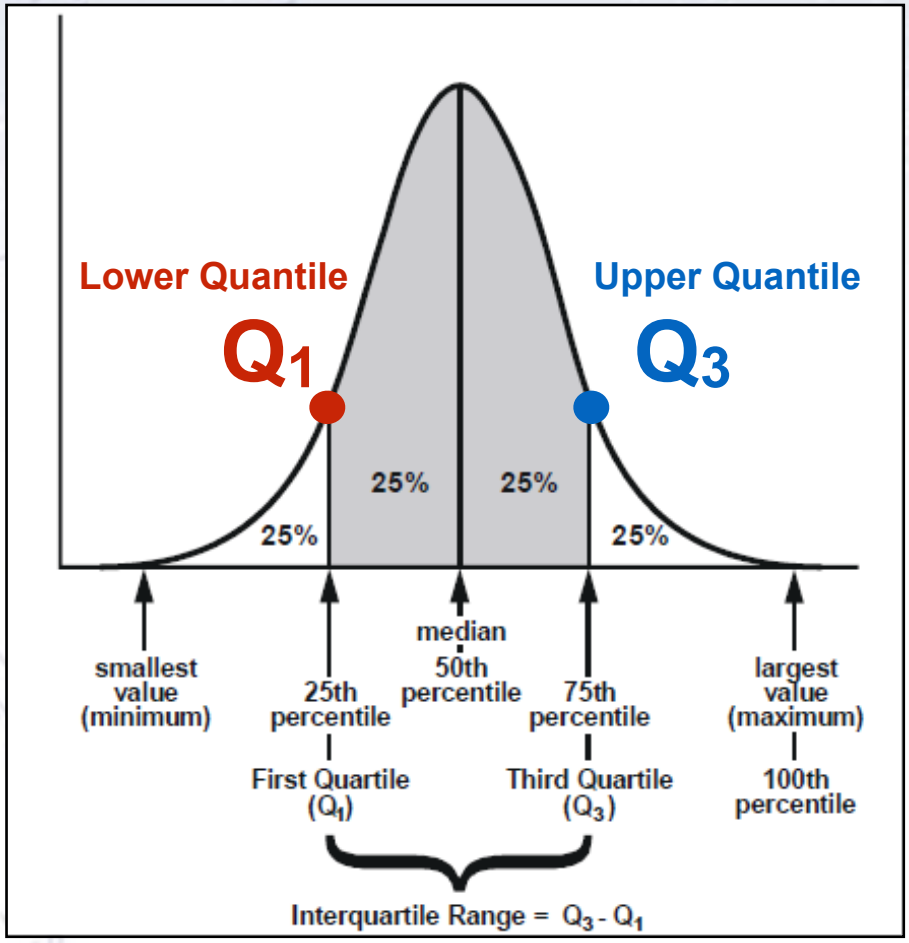
IQR measures **statistical dispersion**, calculated as the difference

$$IQR = Q_3 - Q_1$$

The InterQuantile Efficiency (IQE) is defined as:

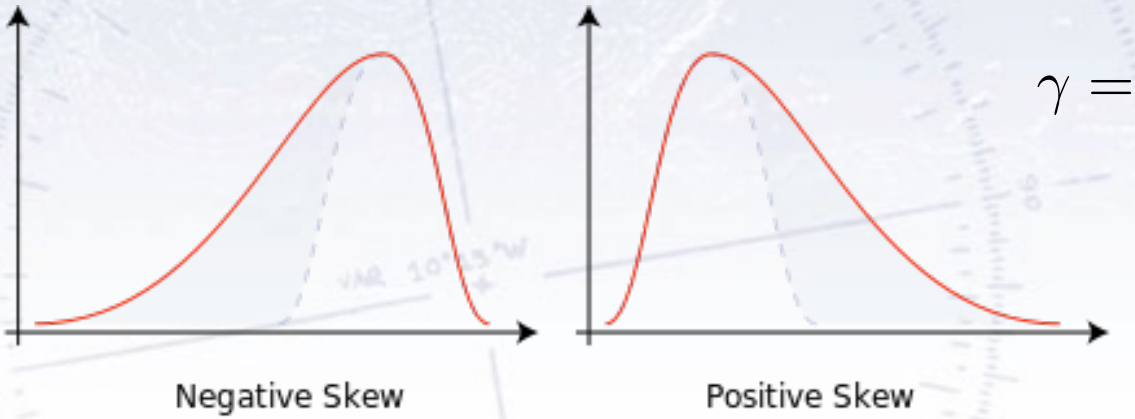
$$IQE = IQR / 1.349$$

The factor $1.349 = 2\,\Phi^{-1}(0.75)$ ensures that IQR = 1 for a unit Gaussian.



# Skewness and Kurtosis

Higher moments reveal something about a distributions asymmetry and tails:




$$\gamma = \frac{\frac{1}{N}\sum_i (x_i - \bar{x})^3}{(\frac{1}{N}\sum_i (x_i - \bar{x})^2)^3/2}$$

$$\kappa = \frac{\frac{1}{N}\sum_i (x_i - \bar{x})^4}{(\frac{1}{N}\sum_i (x_i - \bar{x})^2)^2} - 3$$