Reservoir Modeling Combining Geostatistics with Markov Chain Monte Carlo Inversion

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1 Introduction

Reservoir modeling conditioned by recorded seismic reflection data is the most prominent geophysical technique to investigate the unknown properties of the subsurface. However, even if seismology produces good quality tomographic images, it still remains challenging to obtain a good picture of some particular properties such as porosity or permeability that are of most interest for oil and gas exploration. The link between elastic parameters and such properties lies in the complex relationships between, among others, intrinsic properties of rocks, mineralogy, and interaction with fluids which are usually described by a rock physics model [1]. Since these relationships are usually nonlinear and affected by uncertainty, it is difficult to invert seismic data directly for, e.g., porosity employing the standard optimization approaches because they generally rely on linearised models and simple scaling laws. Here we propose an approach based on a Markov chain Monte Carlo (McMC) technique which is able to combine rock physics modeling and reflection seismology to invert for porosity and facies of the subsurface. It takes into account the nonlinearities deriving from the rock physics model and moreover it provides an estimation of uncertainties on the unknown properties. Similar approaches have been studied before, see e.g., [2–4].

2 Overview of the Markov Chain Monte Carlo Inverse Method

We follow a probabilistic approach, in which all information is represented by probabilities, as described in [5], where the inverse problem consists in performing an

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E. Pardo-Igúzquiza et al. (eds.), *Mathematics of Planet Earth*,
Lecture Notes in Earth System Sciences, DOI: 10.1007/978-3-642-32408-6_148,

indirect measurement of unobservable parameters of the subsurface given some measured quantities on the surface of the Earth. The solution to the inverse problem is the posterior distribution, a combination of the prior and likelihood functions describing all possible models and relative probabilities.

Our aim is then to explore the model space in order to obtain a collection of models which all fit the measured data and are consistent with the a priori information. Moreover we are interested in estimating the uncertainty on unknown model parameters. Markov chain Monte Carlo algorithms represent a natural choice to fulfill these requirements, so we construct a multi-step algorithm capable of sampling the posterior distribution. The ingredients necessary to sample solutions to this inverse problem are essentially two [6]: (I) an algorithm generating samples from a proposal distribution according to the available prior information and (II) a sampler of the likelihood function. The prior geological information is represented by one or multiple training images which supply the necessary information about geological patterns to the algorithm. The posterior distribution is finally sampled employing the extended Metropolis algorithm [6, 7] based on the degree of fit between measured and calculated seismograms. We consider Gaussian uncertainties and hence we utilize an L_2-norm for the misfit function.

Importance of Informed Priors: Geostatistics

One difficulty arising in high-dimensional space sampling is that a tremendous computational effort is needed to properly sample the posterior distribution. The huge size of model space, in fact, hampers the adoption of this kind of methodology in several cases. However, the use of proper informed priors can significantly improve the situation, reducing drastically the size of the model space to be sampled. This is obtained by employing an algorithm which generates models adhering to the prior knowledge so that only plausible models are taken into account in the sampling process. One recently introduced technique consists in generating realizations of a model exploiting the multiple-point statistics contained in prototype models. Specifically, the sequential Gibbs sampling method (see [8] and references therein) uses a sequential simulation approach where the algorithm learns the statistics from a training image which is scanned searching for recurring patterns. In principle, to increase the number of patterns, multiple training images may be used. A randomly selected hyper-rectangular volume of the model is then chosen to be re-simulated at each iteration of the Markov chain to propose a new model, where voxels are re-computed using sequential simulation conditioned on the rest of voxels [9].

3 Numerical Experiments

The target of our study is a synthetic reservoir model derived (but modified) from the Stanford VI-E model [10]. It consists of a 3D arrangement of $38 \times 50 \times 20$ voxels with size of 100, 100 and 4 m each respectively. Each voxel is parameterised with facies and porosity as the unknown parameters. Using the reservoir model derived from the Stanford VI-E model we constructed some "synthetic observations" by computing the seismograms to be inverted. In our case the forward model calculation consists of several steps. The first is the computation of the elastic properties from the facies and porosity of the subsoil. Then we compute the synthetic seismograms using a convolution approach.

The target zone of the reservoir is constituted by two facies, one representing sand (channel in a fluvial deposition system and oil-saturated) and the other representing shale (floodplain and brine-saturated). We assume the mineralogy to be known and describe it as consisting of four minerals (clay, quartz, feldspar, rock fragments) with known volume fraction in each facies but unknown porosity. The link between porosity and other petrophysical properties with the elastic moduli of the bulk rock for sand facies is modeled using the constant cement model [11] and the usual formula for isotropic V_P. An empirical law from [12] is used instead to compute V_P for shale facies.

Seismic modeling is carried out in the framework of the acoustic approximation, where the basic ingredients are the P-wave velocity and the density model. The seismic data are "recorded" at the surface on top of each pixel column as a zero-offset section. This in reality can correspond to data recorded at different source-receiver offset that have been processed such that they represent an equivalent zero-offset section which is easier to interpret. The wavelet is constructed from a Ricker function with 50 Hz peak frequency and is assumed to be known in the inversion process.

4 Results and Discussion

We ran $2 \cdot 10^6$ iterations, obtaining about $7 \cdot 10^5$ models, of which only one every 10^2 was retained to ensure independence of samples. Figure 1a shows one particular model from the solutions. We ended up with a collection of models representing samples of the posterior distribution which can be used to estimate subsurface properties and their relative probabilities/uncertainties. The solutions are used as a database that can be queried to obtain information on several different aspects since it represents the complete solution of the inverse problem. Here we show two examples of the kind of information which can be retrieved from the collection of models. The first is to compute the value of porosity at two different locations, obtaining histograms of possible values (Fig. 1b). The histogram tells us which range of values is most probable and, moreover, gives us an estimation of the uncertainty. The two histograms show a different behavior, one having a more pronounced peak, reflecting the different degree of resolving power. The second example is a map of the probability of having the sand facies on a slice of the 3D model at $z = 40$ m (Fig. 1c). The continuity of structures depicted in Fig. 1c is due to the prior information deriving from the geostatistical algorithm which takes into account the spatial continuity present in the training image. This example shows

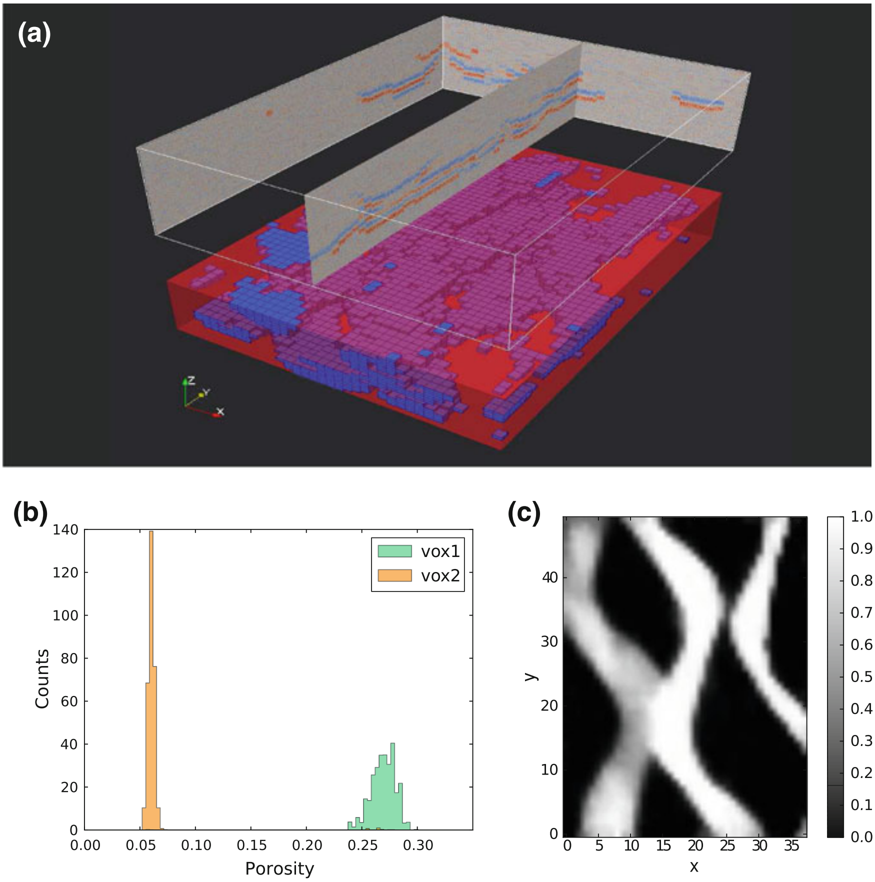

Fig. 1 **a** An example of a two-facies reservoir model from the collection of solutions with some slices through the volume of observed seismograms plotted on *top*. **b** Histogram of porosity for two voxels, one located at $(x, y, z) = (1500, 3500, 20)$ m and the other at $(1000, 1000, 48)$ m. **c** Probability of having sand (and hence a channel) on a 2D slice of the model at $z = 40$ m

how it is possible to retrieve more sophisticated information from the database of solutions that can result very useful for real problems applications. Again, the uncertainty, clearly imaged in this probability plot, is an integral part of the answer we were searching for.

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