Heavy charged particle passage through matter

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Content

Bohrs argument

- The momentum transferred to an atomic electron by the passage of a heavy charged particle is in the impulse approximation:

$$\Delta P = \int F dt = e \int E_{\perp} dt = e \int E_{\perp} \frac{dx}{v},$$

since the longitudinal components cancel.

- Use Gauss' law on a long cylinder with the impact parameter, b as radius and the projectile line-of-flight as axis:

$$\int E_{\perp} 2\pi b dx = 4\pi z e,$$

Bohrs argument cont'd

- and hence

$$\begin{aligned}\Delta P &= \frac{2ze^2}{bv}\\ \Delta E &= \frac{2z^2e^4}{m_e v^2 b^2}.\end{aligned}$$

- The energy lost to all electrons located between b and $b + db$ in a thickness dx is

$$-\Delta E(b) = \frac{4\pi z^2 e^4}{m_e v^2}\rho_e \frac{db}{b} dx,$$

valid between some b_{min} and b_{max}.

Bohrs argument cont'd

- Integrating over impact parameters, we get

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2}\rho_e \ln \frac{b_{\max}}{b_{\min}}.$$

The maximal transferable energy in a head-on collision is $2\gamma^2 m_e v^2$, whereby

$$\begin{aligned} \frac{2z^2e^4}{m_e v^2 b_{\min}^2} &= 2\gamma^2 m_e v^2 \\ b_{\min} &= \frac{ze^2}{\gamma m_e v^2}. \end{aligned}$$

Bohrs argument cont'd

- The condition for b_{max} is more tricky. The argument is that the interaction time must be short compared with the orbital period of the bound electron:

$$\frac{b}{\gamma v} \leq \tau,$$

leading finally to

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} \rho_e \ln \frac{\gamma^2 m v^3 \tau}{z e^2}.$$

Penetration range

The distance travelled by a charged particle before it stops is not an exact function of kinetic energy. This is because head-on collisions with electrons and nuclei are of statistical nature, causing a spread in the range called straggling.

For not too high kinetic energies, the average range is approximately

$$R = 0.004[\mathrm{g/cm^2}]\, E_{\mathrm{kin}}^{1.75} \quad (E \text{ in MeV})$$

for protons. This is more or less what we expect from the β^{-2} behaviour of $\frac{dE}{dx}$ at low energies.

The Bragg peak

- Because of the β^{-2} behaviour, a proton or ion reaches its maximum ionization power just before it stops. This is called the Bragg peak.
- This make protons or light ions much better for cancer-therapy than the X-rays and electron guns we have in Denmark, since protons can deliver most of their energy at the Bragg peak to the sick tissue without destroying the healthy tissue in front. (anti-protons would be even better).

Equivalent dose

- The biological damage effects in average human tissue, however, vary with the kind of radiation. For example, 1 MeV neutrons causes 20 times more damage than electrons at the same dose. The equivalent dose is weighted by such factors. The unit is Sievert with the same dimension as Gray (J/kg).
- The biological damages are furthermore different (by up to a factor 20) in different organs of the body (the least in the skin and bone surfaces).
- The chemical or enzymatic restoration of damaged cells can not keep up with a sudden high dose. A sudden dose greater than 2-3 Sv is life-threatening.

Typical doses

- Ordinary people get most radiation from inhalation of Radon from the underground (typically 1.5 mSv/yr). Stewardesses get more from cosmic radiation. Physicists working at nuclear labs in general get negligible doses. However, a dosimeter monitoring how much you get is advisable, if you work with radiation on a daily basis.
- Most radioactive sources do not produce radiation with energy high enough to penetrate the skin. Therefore it is most important to avoid inhaling or eating radioactive substances, which brings the radiation directly to the cells. This may also be why you should not smoke.

Shielding

- A high Z material (e.g. Pb) is best for shielding against γ-rays and positrons.
- For electrons low-Z polystyrene or lucite work best in order to avoid bremsstrahlung (or a sandwich of low-Z/high-Z shield).
- For neutrons use paraffin, water or other hydrogeneous material.
- For high energy charged hadrons use the cheapest material (because you need a lot) with the highest density (i.e. Fe).