Possible measurement of alpha in $B \to \rho\pi$ decays.

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Abstract

Using a simulation of $B \to \rho\pi$ decays, we have studied the possibility of measuring the angle alpha in the unitarity triangle through this decay channel. The analysis has been done using different strategies and for varies scenarios of experimental results.

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1 Introduction

The decay $B \to \rho\pi$, first proposed for analysis by A. Snyder and H. Quinn [1], gives an unambiguous possibility for measuring the angle alpha in the unitarity triangle, see Figure 1.

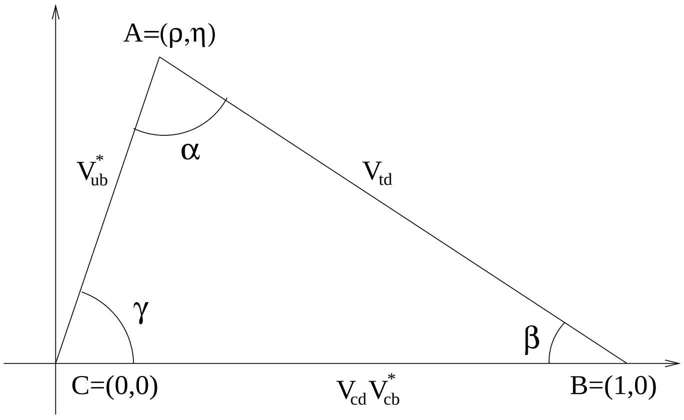

Figure 1: Unitarity Triangle.

Unitaity of the 3×3 CKM matrix applied to first and third row yields $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \doteq V_{ub}^* + V_{cd}V_{cb}^* + V_{td} = 0$, which is represented by the unitarity triangle, here in the Wolfenstein parameterization.

The amplitudes for $B \to \rho\pi$ are in the Standard Model of the form

$$\mathcal{A} = -e^{i\gamma}T + e^{-i\beta}P \tag{1}$$

where T are tree amplitudes, P are penguin amplitudes and the minus sign for convenience. For each of the processes $B^0 \to \rho^+\pi^-, B^0 \to \rho^-\pi^+$ and $B^0 \to \rho^0\pi^0$, there is a corresponding amplitude, $\mathcal{A}^{+-}, \mathcal{A}^{-+}$ and $\mathcal{A}^{00}$ respectively, and equivalently the amplitudes $\bar{\mathcal{A}}^{+-}, \bar{\mathcal{A}}^{-+}$ and $\bar{\mathcal{A}}^{00}$ for the $\bar{B}^0$ decay. Using $\alpha + \beta + \gamma = \pi$ and redefining the basic amplitudes to be

$$e^{i\beta}\mathcal{A} = -e^{i(\gamma+\beta)}T + P \tag{2}$$

$$e^{i\beta}e^{-2i\beta}\mathcal{A} = -e^{-i(\gamma+\beta)}T + P \tag{3}$$

one obtains [2]

$$\mathcal{A}^{+-} = e^{-i\alpha}T^{+-} + P^{+-} \quad , \quad \bar{\mathcal{A}}^{+-} = e^{i\alpha}T^{-+} + P^{-+} \tag{4}$$

$$\mathcal{A}^{-+} = e^{-i\alpha}T^{-+} + P^{-+} \quad , \quad \bar{\mathcal{A}}^{+-} = e^{i\alpha}T^{+-} + P^{+-} \tag{5}$$

$$\mathcal{A}^{00} = e^{-i\alpha}T^{00} + P^{00} \quad , \quad \bar{\mathcal{A}}^{+-} = e^{i\alpha}T^{00} + P^{00} \tag{6}$$

These amplitudes all interfere, and so they have a trivial common phase. From the purely $\Delta I = 1/2$ character of the penguin amplitudes, we also have that

$$P^{00} = \bar{P}^{00} = -\frac{1}{2}(P^{+-} + P^{+-}) \tag{7}$$

So the originally 12 amplitudes have been reduced to 10 from one common phase and an isospin constraint. Therefore the describtion can be made in terms of complex T's and P's together with α minus one common phase.

1.1 Time dependence of decay rate

The decay rate Γ is time dependent, which allows for a time dependent fit. Let p and q define the two B mass eigenstates

$$|B^0\rangle_{Light} = p|B^0\rangle + q|\bar{B}^0\rangle \tag{8}$$
$$|B^0\rangle_{Heavy} = p|B^0\rangle - q|\bar{B}^0\rangle \tag{9}$$

and assume that $|p/q| = 1$. Defining

$$\lambda \equiv \frac{q\langle f|\mathcal{H}|B^0\rangle}{p\langle \bar{f}|\mathcal{H}|\bar{B}^0\rangle} \equiv \frac{q\bar{\mathcal{A}}}{p\mathcal{A}} \tag{10}$$

where f is some final state, we have [3]

$$\Gamma(B^0_{t=0}, f_{t>0}) = e^{-\Gamma t}\left|\cos\tfrac{1}{2}\Delta m t\bar{\mathcal{A}} + i\frac{p}{q}\sin\tfrac{1}{2}\Delta m t\mathcal{A}\right|^2 = \tag{11}$$
$$e^{-\Gamma t}|\mathcal{A}|^2\left[\tfrac{1}{2}(1+|\lambda|^2) - \tfrac{1}{2}\cos\Delta m t(1-|\lambda|^2) + \mathrm{Im}\lambda\sin\Delta m t\right] \tag{12}$$

with variations of signs depending on which state was tagged first (written here as $t = 0$ and $t > 0$) and whether we consider B^0 or $\bar{B}^0$. It is the term $\mathrm{Im}\lambda$ that contains $\sin 2\alpha$, which in the absence of penguins/weak phases is the only measurable quantity containing α.

2 Simulation and reconstruction

From the simulation of $B \to \rho\pi$ decays, which assumed knowledge of all parameters, events are generated and then fitted with a maximum likelihood fit by Minuit [4] assuming knowledge of the time dependence in the events[1].

To specifically test the mirror solution $\frac{\pi}{2} - \alpha$, the fit is done with both α and $\frac{\pi}{2} - \alpha$ as starting point. The most likely fit is chosen and compared to the original parameters. The values chosen for the parameters in reference case are shown in Table 1

[1] Appropriate detector smearing of the time information was applied

below. In the figure next to is shown a plot of the likelihood of the fit as a function of α for an experiment with 1200 events. As can be seen from the plot, there are other local minima, than the correct one, most prominently at the mirror solution $\pi/2-\alpha$.

Parameter:	Value:
α	1.347
T^{+-}	1.00
T^{-+}	0.47
T^{00}	0.14
P^{+-}	$-0.20\ e^{-0.5i}$
P^{-+}	$0.15\ e^{2.0i}$
$c_{\rho 1450}$	-0.228

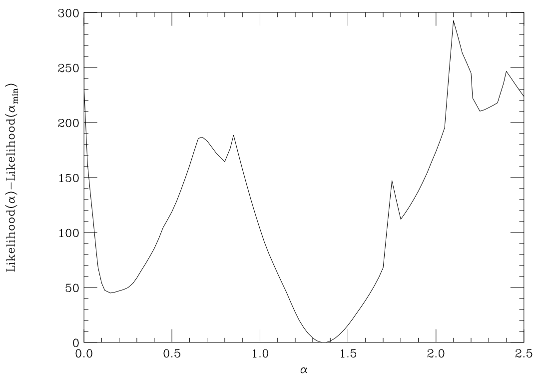

Table 1: Parameter setting and example of simulation.
Parameters used as true values in the simulation, unless otherwise stated (see subsections 3.4, 3.5 & 3.6), except for the $\rho(1450)$ coefficient, $c_{\rho_{1450}}$, which was only used (i.e. nonzero) in subsection 3.3. The graph shows an example of the likelihood as a function of α in a simulation with these parameters.

A mirror solution can be defined either as a solution originating from the starting point $\frac{\pi}{2} - \alpha$, or one that lies closer to that value, than to the true α. Whereas the former is in some sense most correct, since it allows for disentangling the two peaks, the later was used, since the information about starting point was not saved. If the two peaks do not overlap, the two definitions coincide, which is a good approximation for most cases, except when high precision is required or when α is close to $\frac{\pi}{4}$, see subsection 3.6. In the most recent version of the program, the initial starting point of the fit was recorded in the data file, and so one has the possibility to choose either definition.

2.1 Backgrounds

Apart from the real decays, two types of background are included. As they are thought to be of equal size and interfere roughly in the same manner, they are set to the same value, in order to limit the dimensions of the analysis.

- *Background from $B \to \pi\pi\pi$ decays,* which resembles the signal $B \to \rho\pi$, since it has three pions in the final state with reconstructed mass of the b-meson. However, the 3π decays lie uniformly distributed in a Dalitz plot, whereas the $\rho\pi$ decays tends to concentrate in bands around the edges (because of the ρ) and in the corners (because of spin correlation).
- *Continuum background,* where other events kinematically resemble the $B \to \pi\pi\pi$ decays. These events also tend to concentrate in bands around the edges in a Dalitz plot, but this is because they mainly are two jet events, where two pions in the same jet have a small invariant mass. However, the time dependence of continuum background is very much different from the $B \to \rho\pi$ decays time dependence, which allows for a very good discrimination.

2.2 Approximations and constrains

In this analysis, it was assumed that the backgrounds could be fitted well, and therefore the backgrounds were set to the correct size, *and not fitted for*! This assumption was very timesaving, as the dimensionality of the fit went down, and it was tested separately, that the backgrounds in fact could be fitted well (within 10%) as shown in Table 4. However, for a full understanding of the backgrounds influence, one should include them as parameters in the fit, despite their time consumption.

If a solution fall outside the interval $[-\frac{\pi}{4}, \frac{3\pi}{4}]$, it can be questioned whether such solutions should be included, since they mostly come from data with no well defined minimum. The errors on such solutions were often, though not always, very large, and the effects on the result therefore correspondingly small, and so they were included in the weighted average. However, it is important to keep this in mind at later stages, since a full understanding of the results would include more rigorously treatment of such cases.

3 Results

The simulation is run with various settings of number of events and backgrounds, and the two things of most interest are the fraction of the time a mirror solution was obtained and the value of the average of alpha along with its uncertainty and the width of the distribution.

The fraction of the time that the mirror solution $\frac{\pi}{2} - \alpha$ is chosen over the correct solution α shows what the chances are that an experiment will get the mirror solution and not the correct answer for α. In the results, the number of correct solutions are counted and compared to the total number of runs, and also the number of solutions outside the $[-\frac{\pi}{4}, \frac{3\pi}{4}]$ interval is indicated (in parenthesis).

The value of the average of α, $<\alpha>$, along with the uncertainty of the result, δ_α, and the width of the distribution, σ_α, was obtained from a weighted average of repeated experiments taken *only over correct solutions* (i.e. not mirror solutions), using the errors, σ_i, provided by Minuit. Finally, the $\chi^2/dof.$ was calculated to check that the errors from Minuit were indeed realistic. This was done as follows:

$$
\begin{aligned}
<\alpha> &= \frac{1}{\sum_i 1/\sigma_i^2}\sum_i \alpha_i/\sigma_i, && (13)\\
\delta_\alpha &= \frac{1}{\sqrt{\sum_i 1/\sigma_i^2}}, && (14)\\
\sigma_\alpha &= \sqrt{\sum_i \alpha_i^2/\sum_i 1 - \left(\sum_i \alpha_i/\sum_i 1\right)^2}, && (15)\\
\chi^2/dof. &= \sum_i \left(\frac{\alpha_i - <\alpha>}{\sigma_i}\right)^2 /(N_{exp}-1) && (16)
\end{aligned}
$$

Deviations from the true value of α used (1.347) indicate shortcomings in the procedure for reconstruction and fitting, which has to be understood. For that reason, the uncertainty in the average has been included in the results, so that one knows how precise the average is. Also the width of the distribution is given, to give an idea of what the errors will be in an actual experiment[2].

To also include the mirror solutions in the analysis, one can choose to look at $\sin(2\alpha)$, which is the dominant form of influence on the amplitudes, and which doesn't distinguish between the two solutions. The error analysis has to be done with a little caution, since the usual method of simply translating a result in alpha, $\alpha \pm \delta\alpha$, into $\sin(2\alpha) \pm 2\cos(2\alpha)\delta\alpha$ does not lead to a succesful results. For $\alpha = \pi/4$ the error would then be zero, and for $\delta\alpha > 1$ the error could become larger the allowed region of the parameter, both of which do not make sense.

Instead, the average of the values $|\sin(2\alpha) - \sin(2(\alpha \pm \delta\alpha))|$ is used as the error on $\sin(2\alpha)$, which does not suffer from any of the two failures of the textbook method, though it still converges to it, when $\delta\alpha$ becomes small. Having chosen the value of α close to $\pi/2$, makes a weighted average of $\sin(2\alpha)$ biased, since the smallest errors giving the largest contributions will come from the edges of the interval $[-\frac{\pi}{4}, \frac{3\pi}{4}]$, where $\cos(2\alpha)$ is small. For this reason, the average of $\sin(2\alpha)$ was done without weights, and the uncertainty in the result simply taken to be $\sigma_\alpha/\sqrt{N-1}$.

[2]In the limit of many experiments with equal error, $\delta_\alpha \to \sigma_\alpha/N_{exp}$.

3.1 Reference simulation

Using 200, 600 and 1200 events (N_{ev}) and background and continuum ($B\&C$) of 0, 6 and 12 times the signal, 300 experiments were simulated. This both gives an idea of what results can be achieved given number of events and backgrounds and serves as a reference for the following cases. The results are shown in Table 2 below:

N_{ev}	$B\&C$	Correctness	$\bar{\alpha} \pm \delta_\alpha \pm \sigma_\alpha$	$\chi^2/dof.$
200	0.0	209 : 300 (0)	$1.380 \pm 0.009 \pm 0.131$	0.82
	6.0	182 : 300 (3)	$1.392 \pm 0.018 \pm 0.260$	0.85
	12.0	198 : 300 (7)	$1.413 \pm 0.020 \pm 0.310$	0.90
600	0.0	252 : 300 (0)	$1.369 \pm 0.005 \pm 0.088$	1.16
	6.0	210 : 300 (0)	$1.422 \pm 0.011 \pm 0.214$	1.35
	12.0	220 : 300 (1)	$1.410 \pm 0.012 \pm 0.230$	1.17
1200	0.0	287 : 300 (0)	$1.360 \pm 0.003 \pm 0.066$	1.24
	6.0	253 : 300 (0)	$1.371 \pm 0.008 \pm 0.141$	1.37
	12.0	235 : 300 (0)	$1.376 \pm 0.010 \pm 0.207$	1.59

Table 2: Reference results.
Shown as a function of number of events, N_{ev}, and background and continuum ($B\&C$) is the value of α, its uncertainty δ_α, and its width, σ_α along with the number of mirror solutions out of the total number of experiments run.

In general, the error decreases with number of events and increases with background, as would be expected. The chance of obtaining af mirror solution follows the same pattern, and roughly 900 events are needed to get the true value for alpha with 90% confidence, assuming no background (1900 with background 6.0 and 3400 with background 12.0).

A striking feature is that all the obtained averages are above the true value, and that they increase with background and to some extend limited number of events. This could mean, that in order to get the right answer, the cause for this shift has to be understood and corrected for. It was checked, that with sufficiently many events, the result converged to the correct value (100 experiments with 10000 events in each and no background gave the result 1.3450 ± 0.0019 with no mirror solutions.)

The average of $\sin(2\alpha)$, its uncertainty, $\delta_{\sin(2\alpha)}$, and its sigma, $\sigma_{\sin(2\alpha)}$, is shown in Table 3, along with sine of $\bar{\alpha}$ for comparison.

The fit to both types of solution through $\sin(2\alpha)$ is successful. The average of the sine fits the sine of the average within errors, and the $\chi^2/dof.$ seem reasonable.

N_{ev}	$B\&C$	$\overline{\sin 2\alpha} \pm \delta_{\sin 2\alpha} \pm \sigma_{\sin 2\alpha}$	$\sin 2\bar{\alpha} \pm \delta_{\sin 2\bar{\alpha}}$	$\chi^2/dof.$
200	0.0	$0.357 \pm 0.014 \pm 0.250$	0.372 ± 0.017	0.72
	6.0	$0.339 \pm 0.025 \pm 0.439$	0.350 ± 0.034	1.06
	12.0	$0.237 \pm 0.030 \pm 0.526$	0.310 ± 0.038	2.17
600	0.0	$0.377 \pm 0.010 \pm 0.170$	0.393 ± 0.009	1.12
	6.0	$0.308 \pm 0.021 \pm 0.357$	0.293 ± 0.021	2.81
	12.0	$0.315 \pm 0.022 \pm 0.385$	0.317 ± 0.023	2.05
1200	0.0	$0.394 \pm 0.007 \pm 0.124$	0.410 ± 0.006	1.26
	6.0	$0.381 \pm 0.014 \pm 0.250$	0.389 ± 0.014	1.63
	12.0	$0.364 \pm 0.021 \pm 0.356$	0.381 ± 0.018	2.05

Table 3: Results in terms of $\sin(2\alpha)$.
The average, uncertainty and width of the distribution of $\sin(2\alpha)$ was calculated including mirror solutions, and the result was compared to $\sin(2\alpha)$.

Notice the good agreement between $\delta_{\sin 2\alpha}$ and $\delta_{\sin 2\bar{\alpha}}$. This suggests, that the mirror solutions indeed have the same distribution as the correct ones mirrored in $\pi/4$. The offset from the true value of $\sin(2\alpha)$ reflects the slight shift in the average for α.

3.2 Fitting backgrounds

In the case of fitted backgrounds, only 100 experiments are simulated and only for 200 and 600 events. The resuls are the following:

N_{ev}	$B\&C$	Cor:Tot	$\bar{\alpha} \pm \delta_{\alpha} \pm \sigma_{\alpha}$	$c_{BoS} \pm \sigma_{BoS}$	$c_{CoS} \pm \sigma_{CoS}$
200	0.0	$76:100$	$1.377 \pm 0.014 \pm 0.158$	-0.06 ± 0.15	0.03 ± 0.13
	6.0	$59:100$	$1.393 \pm 0.034 \pm 0.265$	5.87 ± 0.50	5.78 ± 0.56
	12.0	$68:100$	$1.374 \pm 0.035 \pm 0.302$	11.52 ± 1.20	11.41 ± 1.37
600	0.0	$90:100$	$1.363 \pm 0.008 \pm 0.094$	-0.01 ± 0.03	0.00 ± 0.03
	6.0	$70:100$	$1.362 \pm 0.019 \pm 0.213$	5.92 ± 0.27	5.86 ± 0.33
	12.0	$75:100$	$1.394 \pm 0.021 \pm 0.266$	11.76 ± 0.74	11.61 ± 0.75
1200	0.0	$98:100$	$1.356 \pm 0.006 \pm 0.051$	-0.00 ± 0.01	0.00 ± 0.01
	6.0	$81:100$	$1.372 \pm 0.013 \pm 0.142$	5.92 ± 0.25	5.93 ± 0.25
	12.0	$79:100$	$1.337 \pm 0.015 \pm 0.197$	11.76 ± 0.51	11.71 ± 0.52

Table 4: Fitted backgrounds.
Results when including the size of the background (c_{BoS}) and the continuum (c_{CoS}) as parameters in the fit.

As can be seen, the results did not change considerably, and there is good agreement especially between the two widths of the distribution, whereas the uncertainty increased a little. If anything, the results got a little closer to the correct value of α, and the backgrounds were fitted fairly well.

3.3 Including both $\rho(770)$ and $\rho(1450)$.

Higher mass ρ's exist, and if they are not taken into account, the analysis could be compromised. Therefore a simulation including the $\rho(1450)$ with the strength (see Table 1) measured by ALEPH in τ decays[3] [5] was done, and the coefficient $c_{\rho_{1450}}$ subsequently fitted for. Since the $\rho(1450)$ resonance is visible in a Dalitz plot, not surpricingly, it could be fitted very accurately.

Unfortunately, this introduces an extra dimension in the fit, and the simulation becomes very time consuming. Therefore only a subset of the settings in the reference case was considered, specifically only the case of 200 and 600 events, and only simulation of 100 experiments were conducted. The results are listed in Table 5:

N_{ev}	$B\&C$	Corr.	$\bar{\alpha} \pm \delta_\alpha \pm \sigma_\alpha$	$c_{\rho_{1450}} \pm \sigma_{\rho_{1450}}$	$\chi^2/dof.$
200	0.0	95:100	$1.364 \pm 0.012 \pm 0.102$	-0.230 ± 0.023	0.82
	6.0	83:100	$1.351 \pm 0.021 \pm 0.249$	-0.228 ± 0.069	0.69
	12.0	78:100	$1.377 \pm 0.026 \pm 0.294$	-0.229 ± 0.079	1.63
600	0.0	100:100	$1.351 \pm 0.007 \pm 0.076$	-0.226 ± 0.015	1.14
	6.0	95:100	$1.364 \pm 0.012 \pm 0.118$	-0.227 ± 0.027	0.86
	12.0	91:100	$1.386 \pm 0.015 \pm 0.173$	-0.226 ± 0.033	0.80

Table 5: Including $\rho(1450)$.
Results when including the resonance ρ_{1450} in the simulation and using the coefficient of it, $c_{\rho_{1450}}$, as parameters in the fit.

Clearly, the $\rho(1450)$ resonance can be fittet quite well, compared to the true value of -0.228, and so one should not worry much about the resonances, as long as they are included in the fit. In fact, it seems that the extra parameter gives a degree of freedom to discriminate solutions, resulting in a much better fit. Both the distance to the true solution, σ_α and the percentage of mirror solutions has decreased significantly, and it may turn out, that the $\rho(1450)$ resonance ease the analysis.

[3] The dominantly higher resonance is $\rho(1450)$, whereas $\rho(1770)$ only contributes little

3.4 Ignoring penguin contributions in final analysis.

Given a very limited number of events, one could imagine doing a fit with fewer parameters. One obvious reduction is to assume that the penguin amplitudes are negligible, which reduces the number of parameters to six instead of ten. A simulation was done, where penguin contributions in the fit were initially set to zero and fixed in Minuit, while the true values remained the same as quoted in Table 1. The results are shown below in Table 6.

N_{ev}	$B\&C$	Correctness	$\bar{\alpha} \pm \delta_\alpha \pm \sigma_\alpha$	$\chi^2/dof.$
200	0.0	130:200	$1.474 \pm 0.006 \pm 0.068$	0.84
	6.0	128:200	$1.489 \pm 0.009 \pm 0.144$	1.31
	12.0	129:200	$1.503 \pm 0.011 \pm 0.180$	1.23
600	0.0	163:200	$1.476 \pm 0.003 \pm 0.039$	0.88
	6.0	140:200	$1.486 \pm 0.005 \pm 0.060$	0.85
	12.0	143:200	$1.488 \pm 0.006 \pm 0.068$	0.85
1200	0.0	182:200	$1.481 \pm 0.002 \pm 0.029$	0.93
	6.0	171:200	$1.494 \pm 0.003 \pm 0.039$	0.67
	12.0	158:200	$1.490 \pm 0.004 \pm 0.046$	0.67

Table 6: Assuming no penguins.
Results when the fit assumes no penguin contributions, though still including them in the simulation.

As can be clearly seen, the results are consistantly shifted much more from the true value that the uncertainties allow for, and so the procedure is invalid. Even if it were somehow established that the penguin contributions are not as large as the values used in the simulation (see Table 1), the ten-dimensional fit still does an equally good or even better job, and so penguin contributions cannot and should not be neglected. Also, it is of interest to measure the penguin contributions, which is what the ten-dimensional fit does.

3.5 Assuming only real decay amplitudes.

Another reduction of the fits dimensionality is to assume that the final-state-interactions are negligible, which makes all of the amplitudes real. This is a reduction of four parameters, since the common phase is no longer there. The simulation was carried out with all of the imaginary parts of the parameters set to zero, and the analysis was then done with the corresponding reduced number of dimensions. The results are listed in Table 7.

N_{ev}	$B\&C$	Correctness	$\bar{\alpha} \pm \delta_\alpha \pm \sigma_\alpha$	$\chi^2/dof.$
200	0.0	70:100	$1.311 \pm 0.014 \pm 0.124$	1.06
	6.0	66:100	$1.310 \pm 0.028 \pm 0.257$	1.05
	12.0	60:100	$1.374 \pm 0.036 \pm 0.319$	0.77
600	0.0	91:100	$1.344 \pm 0.007 \pm 0.084$	1.62
	6.0	77:100	$1.322 \pm 0.016 \pm 0.163$	1.19
	12.0	73:100	$1.318 \pm 0.021 \pm 0.180$	0.89
1200	0.0	91:100	$1.336 \pm 0.005 \pm 0.055$	1.56
	6.0	69:100	$1.360 \pm 0.013 \pm 0.123$	1.25
	12.0	76:100	$1.326 \pm 0.015 \pm 0.153$	1.12

Table 7: Real decay amplitudes.
Results in the absence of final state interactions, i.e. real decay amplitudes both in simulation and fit.

This time, the results are consistent with the true value, and the errors are smaller than in the reference simulation. Also the likelihood of a correct result is greater, though not significantly. However, keep in mind, that the assumption of real decay amplitudes simplify the situation greatly, and though final state interactions may be small, they are nonzero. Still, if final state interactions turns out to be small, the this simplification seems to be promising.

3.6 α closer to mirror solution.

If it turns out that the value of α is closer to $\frac{\pi}{4}$, so that the mirror solution lies closer to the right value, then one might fear that the two minima become indistinguishable. A simulation was carried out with the parameter $\alpha = 0.9627$, which is one third of the distance to $\frac{\pi}{4}$, compared to the earlier value used (1.347). At this new value of α and using 600 events, the actual value was still fairly well fitted, but the chance of obtaining a mirror solution greatly increased, and with background the two solutions became nearly indistinguishable!

However, the concern for a mirror solution close to the true value, is of less importance, as the latest constrains favor a value of α in the region of $\frac{\pi}{2}$ (see [6] p. 105).

4 Conclusions

The simulation of $B \to \rho\pi$ decays has been simulated under various assumptions. The reference case suggests that α can be obtained with 90% confidence with 900 events assuming no background. The following simulations showed that the existence of higher ρ resonances can be accommodated in the fit - maybe even improving it, and that one might be able to neglect final states interactions if they indeed prove to be small whereas penguins should not be ignored under any circumstances.

Having said that, one should begin to worry about the general offset of the reference solutions, which suggests that there are part of the analysis, that we are not taking into account, giving us a systematic error. One possiblity is that the skewness of the minima causes a shift in the average of the solution, but in order to analyze this idea, one has to establish the size of the effect and its uncertainty.

The consistent behavior of the results and errors along with the convergence of the result at higher number of events seem to verify, that we are indeed doing a realistic simulation, which we understand to some extent.

A Datafiles

The datafiles from the simulations can be found on `thsrv.lbl.gov`. Results from different scenarios are put in different directories, which also contains a simple fortran program for analysis. The directories are the following:

- `/home/petersen/rhopi/res_clean` for the reference results.
- `/home/petersen/rhopi/res_aleph` in the case of two ρ resonances.
- `/home/petersen/rhopi/res_pignore` where penguins were ignored.
- `/home/petersen/rhopi/res_real` assuming real decay amplitudes.
- `/home/petersen/rhopi/res_closemir` when $\alpha = 0.9627$.

The datafiles were given logical names of the form `dataij.txt`, where `i` indicates the number of events $(1 = 200, 2 = 600, 3 = 1200)$ and `j` refers to the ratio of background over signal $(1 = 0.0, 2 = 6.0, 3 = 12.0)$.

Also included is a small program to convert between different dataformats, `datamake.f`, and a program to analyze the datafiles, `avemake.f`. The job of running the simulation, accumulating results and writing them to files was carried out by `aleph_scan.f` which used `aleph_param.in` as input file and `aleph_basic.f` for subroutines, all in directory `/home/petersen/rhopi/`.

References

[1] Measuring CP asymmetry in $B \rightarrow \rho\,\pi$ decays without ambiguities, Arthur E. Snyder and Helen R. Quinn, Physical Review D, 1st September 1993.

[2] Uniqueness of Fits to $B \rightarrow \rho\pi$, Robert N. Cahn, Note, September 1998.

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