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Simple method for extracting $(1 - 2w_{tag})$ and $\sin(2\beta)$.

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Abstract:

We introduce a simple method for measuring $D \equiv (1 - 2w_{tag})$ and $\sin(2\beta)$, which can serve as a check and allows for a direct computation of systematic uncertainties. The idea is to use one optimally placed bin in the time distribution, hence the name OneBin. The method apply to both the TagMix and the CP fitting, which will therefore be treated simultaneously. The statistical uncertainties resulting from the method are compared to the likelihood result; it is within 10% of this.

This means that with a very simple tool, one can obtain almost optimal results along with a direct calculation of both statistical and systematic uncertainties. The goal is not to rival the likelihood method, but to provide a clear-cut view of what is going on.

1 Measuring $D \equiv (1 - 2w_{tag})$ and $\sin(2\beta)$

The measurement of $\sin(2\beta)$ requires knowledge of the mistag rate w_{tag}, or rather the dilution $D \equiv (1 - 2w_{tag})$, as one actually measures $D\sin(2\beta)$ in $B \rightarrow J/\Psi K_S^0$ decays, i.e. the uncertainty on D propagates directly to the uncertainty on $\sin(2\beta)$. The optimal way of extracting D and $\sin(2\beta)$ along with their statistical uncertainties is provided by an unbinned likelihood fit.

However, fitting the full set of data with a likelihood fit is not trivial and highly non-transparent, as one cannot see how the individual events contribute to the result. Addressing this problem, this note introduces a slightly less optimal but extremely simple and transparent method of extracting the two quantities and their statistical and systematic uncertainties.

Essentially, the methods consists of using one optimally placed bin, and then determine result and uncertainty from the count of events in this bin. In the introduction both the perfect and the z-smeared cases are treated, whereas backgrounds are omitted in the discussion for the sake of simplicity, but they are included later. The method is described and its performance is compared to the optimal likelihood fit. Thus, this note is somehow complementary to the BAD notes #5 [1] and #17 [2].

2 The distributions to fit

The measurement of the separation D comes from the distribution of events, where one B is fully reconstructed and the other B is tagged. The distribution of these so-called TagMix (TM)

events in time looks as follows:

$$f_{\pm}^{TM} \equiv \frac{1}{4\tau} e^{-\frac{|t|}{\tau}} [1 \pm D \cos(\Delta m t)], \tag{1}$$

where, here and in the following, $t \equiv t_{tag} - t_{reco}$ is the time difference between the decays of the tagged and the reconstructed B^0, τ and Δm are the lifetime and mass difference respectively of the B_d^0 meson, and a plus sign holds for unmixed events (opposite type B mesons), while a minus holds for mixed events (same type B mesons). Similarly, the measurement of $\sin(2\beta)$ comes from fitting the asymmetry in $B^0/\overline{B^0} \to J/\Psi K_S^0$ decays (CP events):

$$f_{\pm}^{CP} \equiv \frac{1}{4\tau} e^{-\frac{|t|}{\tau}} [1 \pm D \sin(2\beta) \sin(\Delta m t)], \tag{2}$$

where the sign depends on the tag of the other B; plus (minus) if it was tagged as a B^0 ($\overline{B}^0$).

Taking z-smearing into account, which will be done from this point and on, these distributions change and become:

$$F_{\pm}^{TM} \equiv \frac{1}{4\tau} e^{-\frac{|t|}{\tau}} [1 \pm D \cos(\Delta m t)] \otimes G(t), \tag{3}$$

$$F_{\pm}^{CP} \equiv \frac{1}{4\tau} e^{-\frac{|t|}{\tau}} [1 \pm D \sin(2\beta) \sin(\Delta m t)] \otimes G(t), \tag{4}$$

where G is a linear combination of two Gaussian functions. Both the perfect and the smeared distributions are shown in figure 1, where for illustration the values $D = 0.6$, $\sin(2\beta) = 0.7$ and $G \equiv 0.6\ G_1(\sigma{=}100\mu m) + 0.4\ G_2(\sigma{=}200\mu m)$ were used.

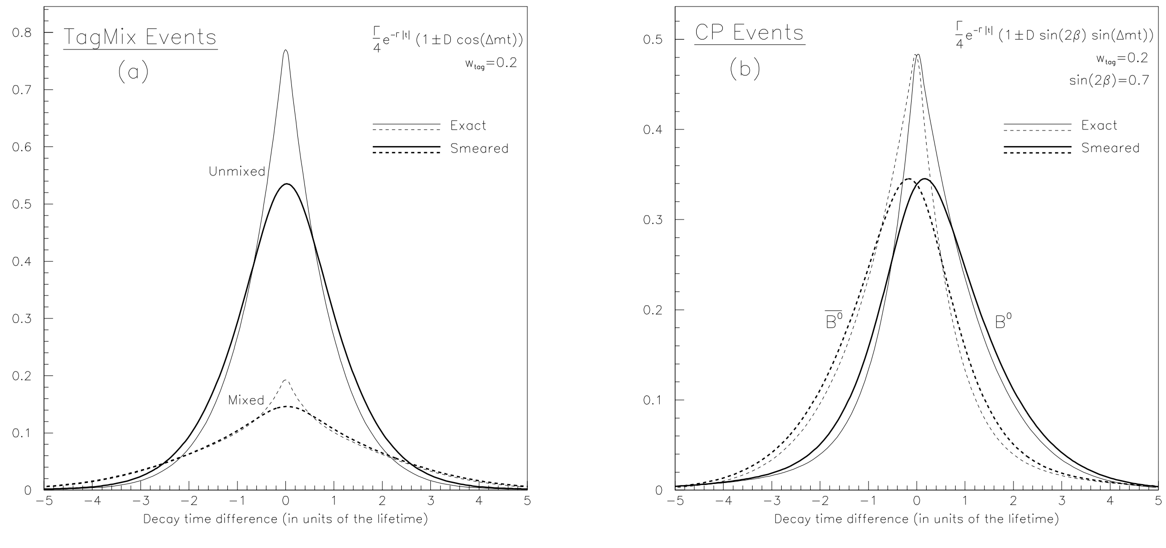

Figure 1: *Distribution of (a) TagMix events and (b) CP events. The thin lines show the perfect case, whereas the thick lines are when vertex resolution has been taken into account.*

However, in the following, the parameters of G (relative fractions, widths and biases) will be obtained from the data (see section 5).

3 The OneBin method extraction of D and $\sin(2\beta)$

The optimal way of fitting these distributions to data is, as already mentioned, with an unbinned likelihood fit. However, this is not a very transparent method and a simpler method is therefore advocated as a cross check of the likelihood result, as a basis for comparison between different fitting codes and, last but not least, to be able to directly compute the systematics.

3.1 The CountAll method — an introduction to counting

Before introducing the OneBin method, we consider a fully time integrated situation, i.e. when no timing information is available (or used). This method applies only to the TagMix case, since the CP asymmetry disappears when integrating over time:

$$\int_{-\infty}^{\infty} \frac{f_+^{TM} - f_-^{TM}}{f_+^{TM} + f_-^{TM}} \equiv 1 - 2\chi_d, \qquad \int_{-\infty}^{\infty} \frac{f_+^{CP} - f_-^{CP}}{f_+^{CP} + f_-^{CP}} \equiv 0, \tag{5}$$

where χ_d is defined by the number of mixed events (events with two B mesons of same sign, which is governed by the function f_-^{TM}) over the total number of events ($f_+^{TM} + f_-^{TM}$). The idea is that by comparing the time integrated rates of mixed and unmixed events (see Fig. 1b), one can extract the wrong tag fraction using the following formula:

$$\chi_{\text{measured}} = (1 - w_{\text{tag}})\chi_d + \ w_{\text{tag}}(1 - \chi_d) \quad \Rightarrow \quad w_{\text{tag}} = \frac{\chi_{\text{measured}} - \chi_d}{1 - 2\chi_d}. \tag{6}$$

However, as can also be seen in Fig. 1b, the "outer" regions in time tend to dilute the ratio of mixed to unmixed events. It would therefore be desirable not to include the regions which contains no information about the mixing, i.e. the regions for which $\int_{t_0}^{\infty} F_+^{TM} - F_-^{TM} dt = \frac{1}{2\tau}\int_{t_0}^{\infty} e^{-|t|/\tau}\cos(\Delta m t) dt = 0$. The idea of the OneBin method is exactly to exclude this region, and thus find one optimized central bin in which the number of mixed and unmixed events are counted.

3.2 The OneBin method

In the TagMix case, the central region of time contains most of the information about D, and so one could imagine a measurement of D by simply counting the numbers of unmixed (u) and mixed (m) events within some time interval, $I_{\text{TagMix}} \equiv [-t_0, t_0]$. This can be expressed as:

$$\boxed{D \equiv (1 - 2w_{tag}) = k_{TM}\ \tfrac{u-m}{u+m},} \tag{7}$$

$$u, m \equiv \int_{I_{\text{TagMix}}} F_{\pm}^{TM}\ dt, \quad k_{TM} \equiv \frac{\int_{I_{\text{TagMix}}} e^{-\frac{|t|}{\tau}} \otimes G\ dt}{\int_{I_{\text{TagMix}}} e^{-\frac{|t|}{\tau}} \cos(\Delta m t) \otimes G\ dt}. \tag{8}$$

In words, one counts the number of unmixed and mixed events within some time bin, and since k_{TM} is just a constant, D can be directly calculated from these counts.

In the case of CP events we are looking for an asymmetry, and so one takes a bin on each side of $t = 0$, $I_{\text{CP}}^- \equiv [-t_2, -t_1]$ and $I_{\text{CP}}^+ \equiv [t_1, t_2]$. With these intervals one can define a positive (p) bin and a negative bin (n) as follows:

$$\boxed{\nu \equiv D\sin(2\beta) = k_{CP}\ \tfrac{p-n}{p+n},} \tag{9}$$

$$p, n \equiv \int_{I_{CP}^-} F_{\mp}^{CP}\ dt + \int_{I_{CP}^+} F_{\pm}^{CP}\ dt, \quad k_{CP} \equiv \frac{\int_{I_{\text{CP}}^- + I_{\text{CP}}^+} e^{-\frac{|t|}{\tau}} \otimes G\ dt}{\int_{I_{\text{CP}}^- + I_{\text{CP}}^+} e^{-\frac{|t|}{\tau}} \sin(\Delta m t) \otimes G\ dt}. \tag{10}$$

The expression for p and n are slightly more complicated, since the distributions are asymmetric, thus one has to take into account that the B^0 and the $\overline{B^0}$ have opposite asymmetries.
The place of the cuts, t_0 for TagMix and t_1 and t_2 for CP events, is determined by minimizing the statistical uncertainty, and the results are shown in figure 2 below.

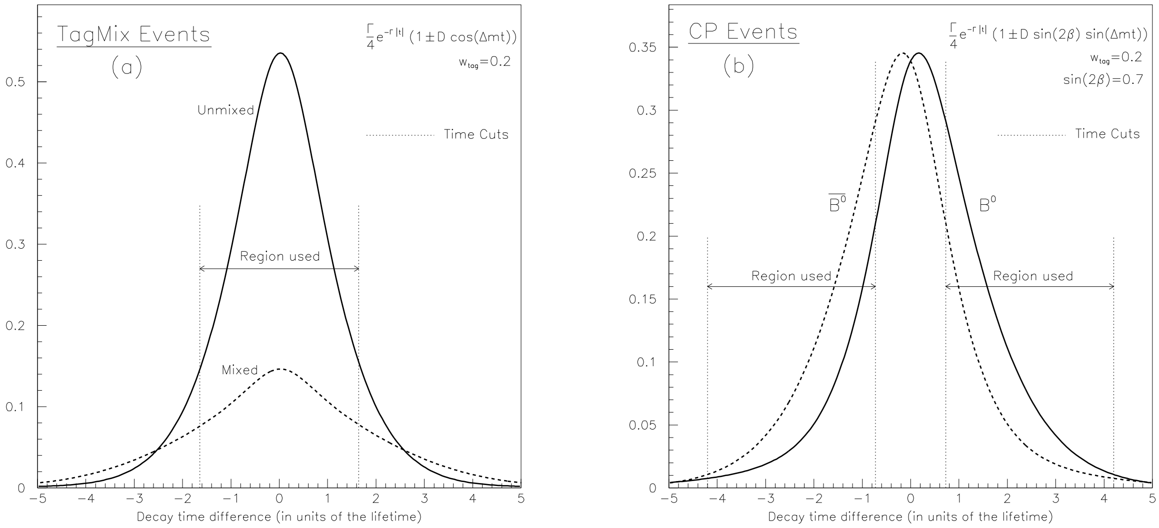

Figure 2: *Optimal placing of bins for determining D and $\nu \equiv D\sin(2\beta)$ using two counts of events.*

As can be seen from the figure, the bins chosen in the TagMix case are *not* at the points $|t| = 2.5\tau$, where the density of mixed events begin to exceed the density of unmixed, but a little before, at $|t| = 1.6\tau$. This turns out to be when the number of mixed events beyond that point matches the number of unmixed, such that this domain as a whole contains no information about mixing, i.e.

$$\int_{t_0}^{\infty} e^{-\frac{|t|}{\tau}} \cos(\Delta m t) \otimes G \; dt = 0 \quad \Rightarrow \quad t_0 \doteq 1.6 \; \tau \doteq 2.5 \; ps. \tag{11}$$

In the CP fitting case, it is not as easy to argue for the placing of the cuts, and one must rely on numerical optimization. However, events with $t \sim 0$ does not contain very much information about the asymmetry, and so the optimum of the inner cut lies a little away from the center, as seen on the figure. The asymmetry does not change sign until t is high ($|t| \sim 5\ \tau$), which makes the outer cut lie far out ($|t| \sim 4.2\ \tau$).

Other more refined methods have been considered, such as the so-called TrigBin and TwoBion methods, and at a slight cost of simplicity one can obtain statistical power even closer to that of the likelihood method. However, since the goal is to provide a simple and transparent method, the OneBin method was preferred. A short description of TrigBin and Two bin is givenin Appendix A.

4 Performance of the OneBin method

As mentioned before, the likelihood method gives the optimal fit, and so the statistical uncertainties of other methods are necessarily larger. To see how much (in fact how little!) information is lost by using a simple method, the statistical uncertainty achieved by the various methods (including the TrigBin and TwoBin methods) relative to the statistical uncertainty of the likelihood is considered.

4.1 Statistical uncertainties

The statistical uncertainty of the bin methods are calculated using the following formula:

$$\sigma_\theta^{Bin\ methods} = \left|\frac{\partial\theta}{\partial r}\right|\sqrt{\frac{r(1-r)}{N}}\ \text{, where}\quad r_{TM} = \frac{u}{u+m}\ \ \&\ \ r_{CP} = \frac{p}{p+n}. \tag{12}$$

Assuming that the likelihood method is efficient[1], the statistical uncertainty of it is:

$$\sigma_\theta^{Likelihood} = \frac{1}{\sqrt{\frac{\partial^2 \log L}{\partial\theta^2}}}, \tag{13}$$

where L is the likelihood function, and θ is the variable fitted for (i.e. D and ν).

4.2 Comparison with likelihood

The ratio $\sigma_{Bin\ method}/\sigma_{likelihood}$ as a function of w_{tag} is shown in figure 3 below for the various methods and the results summarized in table 1.

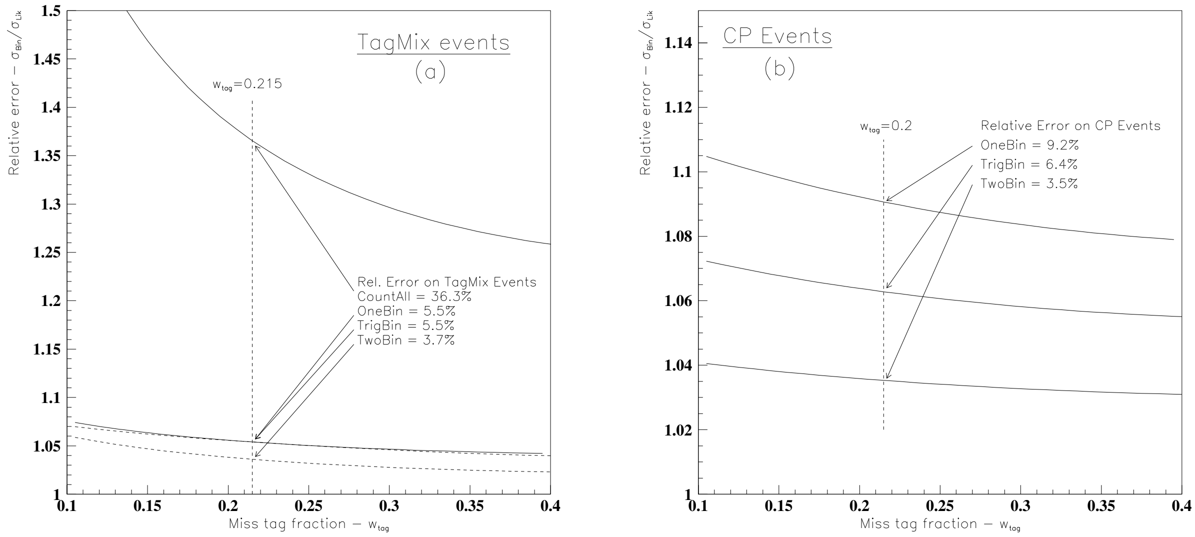

Figure 3: *Statistical uncertainty of the OneBin, TrigBin and TwoBin methods relative to the likelihood method* $\frac{\sigma_{Bin}}{\sigma_{likelihood}}$ *for (a) TagMix and (b) CP fitting. Note the difference in scales.*

For a mistag fraction of $w_{tag} = 0.2$, the TagMix result of OneBin yields a statistical uncertainty 5.5% larger than the likelihood one, whereas the CP result yields 9.2% increase. This is a small price to pay for gaining both simplicity and ability to directly compute systematic uncertainties (see section 7). In table 1 below, the results are summarized for the various methods:

5 Validation

In order to investigate the expected results and uncertainties of the OneBin method relative to the likelihood method and thereby validate the OneBin method, 1000 toy MC experiments

[1] i.e. that the variance $V[\theta] = -(1+\frac{\partial b}{\partial\theta})^2/E\frac{\partial^2 \log L}{\partial\theta^2}$, where b is the bias and E is the estimator (the RCF inequality), which is almost always the case for a sufficiently large data sample.

were generated both in the TagMix and the CP case. For the resolution function we used the following parameters:

$$f_{core} = 0.69, \quad \sigma_{core} = 124\mu m, \quad \delta_{core} = -20\mu m \tag{14}$$

$$f_{tail} = 0.31, \quad \sigma_{tail} = 283\mu m, \quad \delta_{tail} = -70\mu m, \tag{15}$$

which were taken from [3].

5.1 Validation in the TagMix case

In the TagMix case, 1000 toy MC experiments are carried out for each tagging category according to their expected tagging fraction and efficiency given the sample of fully reconstructed B-mesons. For each MC the result and the statistical uncertainty of the likelihood is compared with that of the OneBin method. In figure 4 below the results can be seen for the first tagging category:

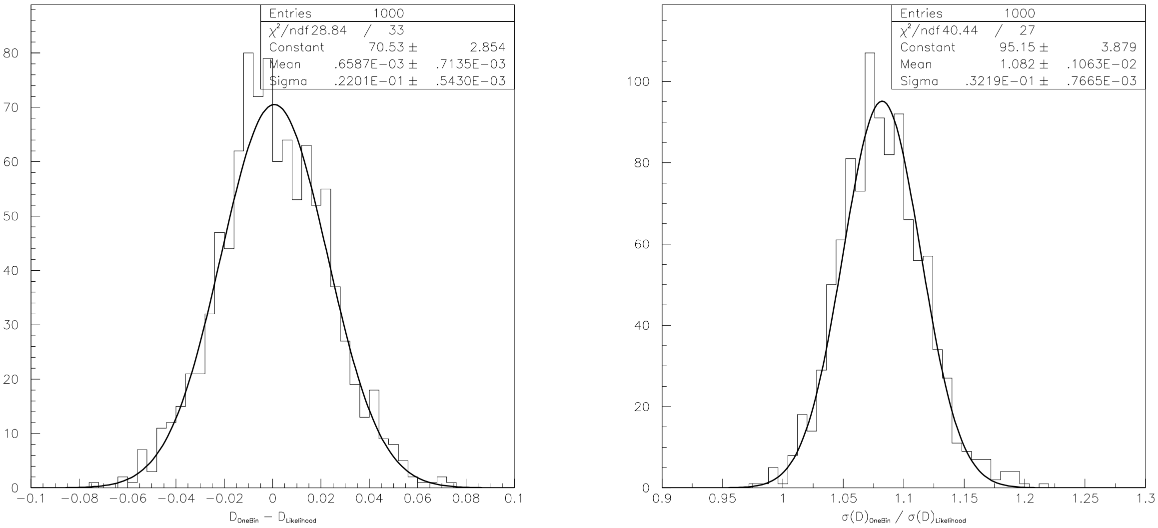

Figure 4: *Comparison of the OneBin method to the likelihood method in the TagMix case. The first figure shows the difference between the OneBin and the likelihood result, and since the central value is consistent with zero, the OneBin method has no bias relative to the likelihood method. The second figure shows the ratio between the OneBin and the likelihood error, and thus one expects* 8% *more error from the OneBin method in the TagMix case.*

We see that the OneBin method is unbiased compared to the likelihood method, and that for the first tagging catagory, which have a low wrong tag fraction, one expects 8% additional error, in accordance with 3.

$w_{tag} = 0.215$	TagMix	CP
OneBin	5.4%	9.1%
TrigBin	5.4%	6.3%
TwoBin	3.6%	3.4%

Table 1: *The quoted numbers represent the relative increase of the statistical uncertainties compared to the likelihood method from using the simpler methods described in the text, e.g.* 5.5% *means that* $\sigma_{bin} = 1.055\ \sigma_{likelihood}$.

5.2 Validation in the CP case

In the CP case, the error from the likelihood method was larger than expected, so in order for the toy MC to reproduce the error obtained from the likelihood fit, only sample experiments with a likelihood error compatible with the one obtained from the Osaka data sample were used. The CP events were distributed between the four tagging categories accordingly (see previous section 5.1), and in figure 5 below are shown the result of the toy MC:

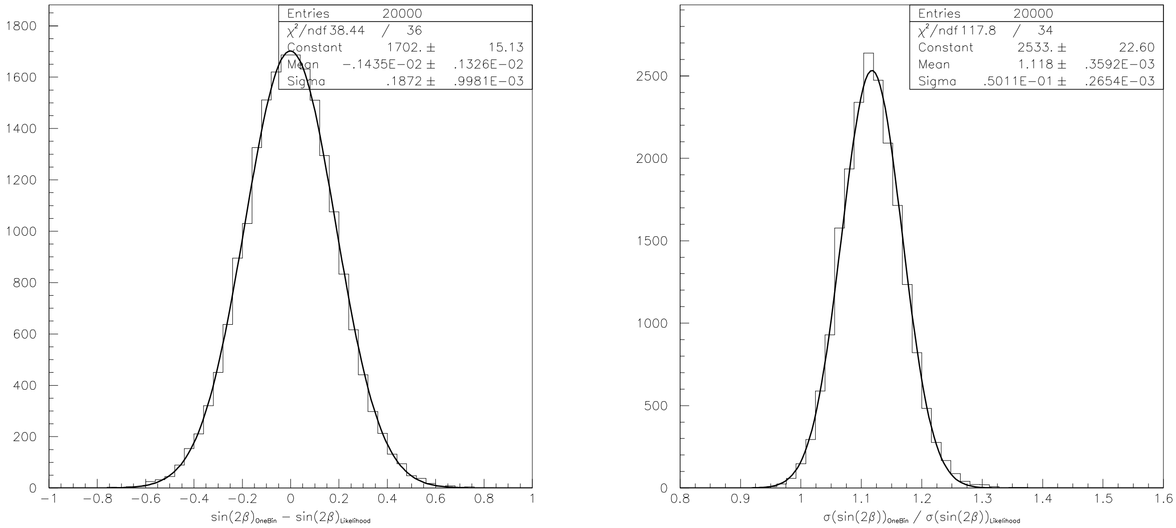

Figure 5: *Comparison of the OneBin method to the likelihood method in the CP case. The first figure shows the difference between the OneBin and the likelihood result, and since the central value is consistent with zero, the OneBin method has no bias relative to the likelihood method. The second figure shows the ratio between the OneBin and the likelihood error, and thus one expects* 12% *more error from the OneBin method in the CP case.*

Once again we see that the OneBin method is not biased, and that in the CP case the expected additional error is 12% — a little higher than calculated due to an increase from averaging over dilutions.

6 Results on the Osaka data sample and comparison with the likelihood fit

6.1 Tagmix result

The OneBin method was applied first to the sample of fully reconstructed B mesons, which have only combinatorial background, and then to the more complicated $D^*\ell\nu$ sample, and the results were compared with those of the likelihood fit. The results on the fully reconstructed sample are summarized in table 6.1 below. They are consistent with the results in the Osaka paper [4] taking into account that OneBin uses the same resolution for all events (the global resolution function G) while the likelihood fit uses the event by event error.

Breco sample	Lepton tag	Kaon tag	NT1 tag	NT2 tag
OneBin	11.0 ± 3.2	20.4 ± 2.1	15.3 ± 3.5	34.6 ± 3.7
Likelihood	11.2 ± 3.1	19.8 ± 2.0	15.2 ± 3.4	34.2 ± 3.6

As can be seen, the $5-6\%$ larger error for OneBin compared to those of the likelihood are in perfect agreement with the calculation in section 4.2.
The analysis of the $D^*\ell\nu$ sample is more complicated, since many backgrounds enter. The numbers describing the backgrounds, which are obtained from data, were taken from [3]. Once again the OneBin method was applied, and the results were then compared to that of the likelihood method in the Osaka paper:

$D^*\ell\nu$	Lepton tag	Kaon tag	NT1 tag	NT2 tag
OneBin	10.2 ± 2.0	21.1 ± 1.6	21.8 ± 2.8	37.1 ± 2.6
Likelihood	8.4 ± 2.0	19.9 ± 1.6	21.0 ± 2.8	36.1 ± 2.5

Here the errors have to be slightly rescaled, since the likelihood fit has a floating value for Δm, while the OneBin fit uses the PDG value.

6.2 CP result

For the CP-fitting, the result of the OneBin and the likelihood fit were:

$$\sin(2\beta)_{\text{Likelihood}} = 0.12 \pm 0.37, \tag{16}$$
$$\sin(2\beta)_{\text{OneBin}} = -0.11 \pm 0.41. \tag{17}$$

Given the somewhat high likelihood error of 0.37, the sigma of the distribution of difference in result of the OneBin and the likelihood fit is 0.19. Since the OneBin result lies within $(0.12-(-0.11)/0.19 \doteq 1.2$ sigma of the likelihood result, they are perfectly compatible.
Thus, the OneBin method serves the purpose of checking the result of the likelihood fit. Furthermore, in the present situation of a low value of $\sin(2\beta)$, the OneBin method reassures the low result, since one by simply counting the number of events on each side of $t=0$ obtain a negative result for $\sin(2\beta)$.

6.3 Word about backgrounds

The inclusion of backgrounds can be done in the following way:

$$\begin{array}{lcl} u^{true} \equiv u^{measured} - u^{background} & & p^{true} \equiv p^{measured} - p^{background} \\ m^{true} \equiv m^{measured} - m^{background} & \& & n^{true} \equiv n^{measured} - n^{background} \end{array} \tag{18}$$

The size of the various backgrounds and their respective uncertainties will thus be an input parameters, which will then be treated when calculating the uncertainty on the result.

7 Systematic uncertainties

The systematic uncertainties can be difficult to grasp when doing a likelihood fit. A great advantage of the simple fitting methods described above is that many of the systematic uncertainties become directly calculable. Returning to the expressions (8) and (10), we see that while the statistical uncertainty is entirely embedded in u, m and p, n, the systematic uncertainties in the results are contained in the constants $k_{TM}(\tau, \Delta m, \sigma_z)$ and $k_{CP}(\tau, \Delta m, \sigma_z)$.

Therefore, by looking at how these two constants change (contradiction in terms, but the point should be clear) with lifetime τ, mass difference Δm and z-smearing the systematic uncertainties from these effects can be determined. To simplify the calculation of the derivatives of the "constants" k_{TM} and k_{CP}, we introduce the following notation[2]:

$$C \equiv \frac{1}{4\tau}\int_{-t_0}^{+t_0} e^{-\frac{|t|}{\tau}}\cos(\Delta m t)\otimes G\ dt, \quad E_C \equiv \frac{1}{4\tau}\int_{-t_0}^{+t_0} e^{-\frac{|t|}{\tau}}\otimes G\ dt, \tag{19}$$

$$S \equiv \frac{1}{2\tau}\int_{t_1}^{t_2} e^{-\frac{|t|}{\tau}}\sin(\Delta m t)\otimes G\ dt, \quad E_S \equiv \frac{1}{2\tau}\int_{t_1}^{t_2} e^{-\frac{|t|}{\tau}}\otimes G\ dt, \tag{20}$$

which leads to simplified expressions for u, m and p, n as well as k_{TM} and k_{CP}:

$$u, m \equiv E_C \pm DC, \quad k_{TM} \equiv \frac{E_C}{C}, \tag{21}$$

$$p, n \equiv E_S \pm \nu S, \quad k_{CP} \equiv \frac{E_S}{S}. \tag{22}$$

The derivatives of k_{TM} and k_{CP} can now be expressed in terms of these four constants and their derivatives:

$$\frac{\partial k_{TM}}{\partial \Delta m} = \frac{1}{C^2}\left(\frac{\partial E_C}{\partial \Delta m}C - \frac{\partial C}{\partial \Delta m}E_C\right), \quad \frac{\partial k_{CP}}{\partial \Delta m} = \frac{1}{S^2}\left(\frac{\partial E_S}{\partial \Delta m}S - \frac{\partial S}{\partial \Delta m}E_S\right), \tag{23}$$

$$\frac{\partial k_{TM}}{\partial \tau} = \frac{1}{C^2}\left(\frac{\partial E_C}{\partial \tau}C - \frac{\partial C}{\partial \tau}E_C\right), \quad \frac{\partial k_{CP}}{\partial \tau} = \frac{1}{S^2}\left(\frac{\partial E_S}{\partial \tau}S - \frac{\partial S}{\partial \tau}E_S\right), \tag{24}$$

$$\frac{\partial k_{TM}}{\partial \sigma_z} = \frac{1}{C^2}\left(\frac{\partial E_C}{\partial \sigma_z}C - \frac{\partial C}{\partial \sigma_z}E_C\right), \quad \frac{\partial k_{CP}}{\partial \sigma_z} = \frac{1}{S^2}\left(\frac{\partial E_S}{\partial \sigma_z}S - \frac{\partial S}{\partial \sigma_z}E_S\right). \tag{25}$$

The derivatives above can easily be calculated, and the systematic uncertainties due to lifetime τ, mass difference Δm and z-smearing then determined. Since the numbers are results of simple calculations, they can hardly be discussed and so this gives a firm basis for the discussion of systematic uncertainties.

8 Summary

A simple method for extracting D and $\sin(2\beta)$ has been introduced and compared to the likelihood method. The method has a statistical uncertainty within 10% of the likelihood result, and its simplicity allows for direct and indisputable calculation of the systematic uncertainties. The method is intuitive and therefore suitable for presenting first results as well as cross checking the result of the likelihood fit.

8.1 Inclusion in fitting package

The OneBin method has been implemented into the fitting package *tfit* of Soeren Prell [5]. In later versions of this note a brief user guide will be included in the appendix.

References

[1] C. Yeche M. Zito F. Le Diberder, S. Versille. TagMix: Measuring and improving the performances of the tagging algorithm with data. 1999.

[2] R. Cahn. TagMixZ and its Application to the Analysis of CP Violation. 2000.

[3] H.M. Schune F. Le Diberder, S. Plaszczynski, T. Petersen. Measurement of the mistag fraction. BAD #22 2000.

[2]The normalization of S and E_S is due to not taking the interval $[-t_2, -t_1]$ into account. However, with no biases the distribution F_+^{CP} and F_-^{CP} are exactly symmetric, and the expressions simplify to those of (20).

[4] The BaBar collaboration A study of time-dependent CP-violation asymmetries in $B^0 \to J/\psi K_S$ and $B^0 \to \psi(2S)K_S$ decays ICHEP, Osaka, Japan October 31, 2000.

[5] S. Prell. http://www.slac.stanford.edu/~prell/tFit/tFit.html. 2000.

A TrigBin and TwoBin methods

A.1 TrigBin method

If one wants to refine the OneBin method, the events outside the cuts can be included in the computation. To include them in an efficient manner still using just *one cut*, one can divide events not according to the time, but according to $\cos(\Delta mt)$ for TagMix events and $\sin(\Delta mt)$ for CP events.

This method called *TrigBin* uses the fact that if there were no smearing, $\cos(\Delta mt)$ determines the ratio of unmixed and mixed events and $\sin(\Delta mt)$ determines the asymmetry. By dividing the events into bins according to their value of $\cos(\Delta mt)$ (TagMix case) and $\sin(\Delta mt)$ (CP case), one exploits that at larger times the number of mixed events exceeds the number of unmixed and the asymmetry has changed side, and in this way one can use the entire range of times, since they are evaluated according to the value of $\cos(\Delta mt)$ and $\sin(\Delta mt)$.

Effectively this means that the time range is divided into regions composed of several disconnected parts, where in each part the number of events is computed, and the final result is then constructed from the parts. This is what is illustrated in figure 6, for (a) TagMix fitting and (b) CP fitting.

In the TagMix case, the number of unmixed and mixed events is compared in each region, whereas in the CP case the number of events with $t > 0$ is compared to the number of events with $t < 0$ in each region. In this way, the TrigBin method is essentially the OneBin method applied to each region, which in turn are determined by the value of $\cos(\Delta mt)$ or $\sin(\Delta mt)$. Once again, one finds the optimal value at which to divide the events by minimizing the statistical uncertainty, and the resulting positions of the bins are also shown in figure 6.

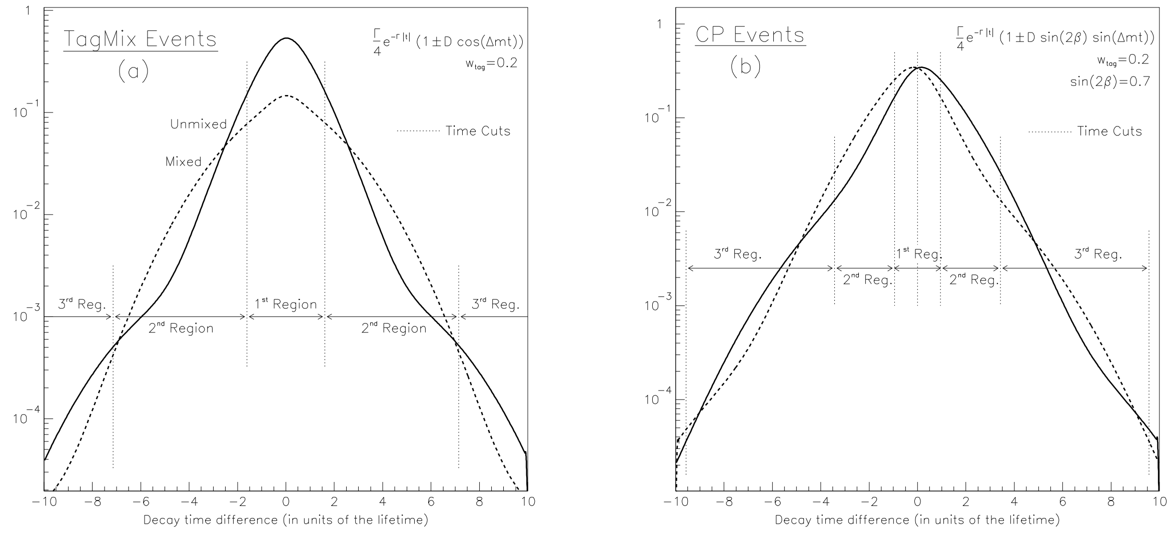

Figure 6: *Effective position of bins for TrigBin method, when dividing optimally according to the value of* $\cos(\Delta mt)$ *(TagMix) and* $\sin(\Delta mt)$ *(*CP*). A larger time interval and logarithmic scales are used, to illustrate the idea of cutting on* cos *and* sin*.*

In the TagMix case, the innermost bin doesn't change position very much compared to OneBin, but now the data outside is used. This contributes in principle, but since it has almost the same number of mixed and unmixed events, it does not have a great impact on the power of the method, as will be seen when discussing results.

In the case of CP fitting, the events with $t \sim 0$ carries only little information, but the statistics are good, and overall they contribute to some extend to the result. Using events with small t in the CP case also turns out to be the best way to include a second set of bins, which we shall discuss next.

A.2 TwoBin method

Another way to improve the power of the OneBin method is to include a second set of optimally placed bins. In the TagMix case, these two bins[3] should be placed in the regions where there are more mixed events than unmixed. In the CP case, the outer region beyond the bins from the OneBin method has too little statistics, and so the most information is extracted with two bins on each side of $t = 0$, one next to the other, in effect dividing the bin of the OneBin method.

Once again the statistical uncertainty on the result, which is obtained by combining the result in each set of bins, is minimized, and the result of this is shown in figure 7 below.

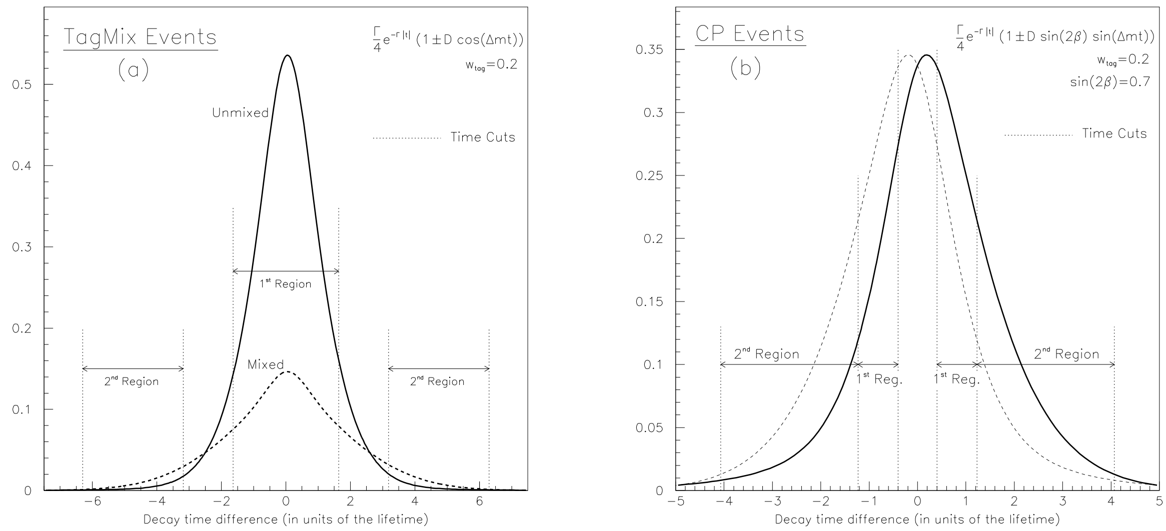

Figure 7: *Optimal placing of bins for TwoBin method — again obtained by minimizing the statistical uncertainty.*

As can be seen from the figure, the TwoBin method in the TagMix case discards events, which are at a point in time, where they contribute with little or no information, and so the central bin is essentially untouched compared to the OneBin method, whereas one additional bin with two parts has been placed on the sides.

The TwoBin method on CP events is able to include more of the statistically rich central region, resulting in a significant improvement of the methods power, as discussed in the following section.

It should be stressed once again, that the placing of the bins is simply a result of minimizing the statistical uncertainty. As the amount of data increases and systematic uncertainties become important, these should be taken into account. However, as section 7 will discuss, this is not difficult for these simple methods, as the systematic uncertainties can be computed directly.

[3]As the distribution is symmetric, the outer bin is composed of two disconnected parts.