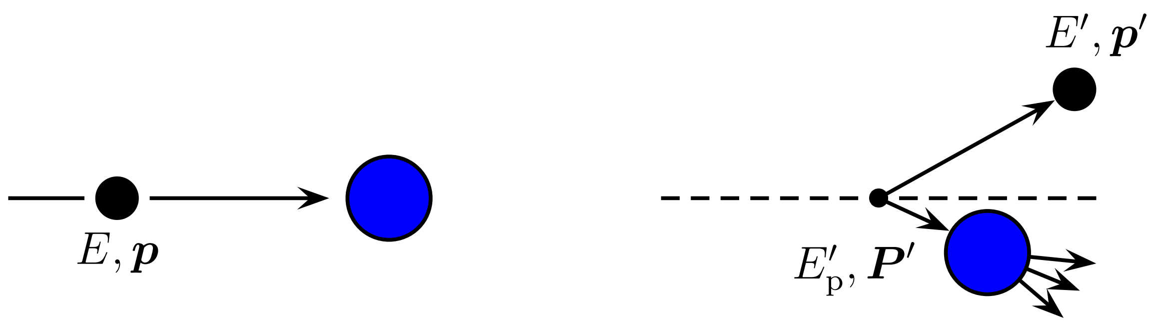
DEEP INELASTIC SCATTERING

Electron scattering off nucleons (Fig 7.1):

1) Elastic scattering: $E' = E'(\theta)$

2) Inelastic scattering: No 1-to-1 relationship between E' and θ

Inelastic scattering: nucleon gets excitet $\Rightarrow$ *nucleon resonances*



Invariant mass of nucleon resonance:

$$W^2c^2 = P'^2 = (P+q)^2 = M^2c^2 + 2Pq + q^2 = M^2c^2 + 2M\nu - Q^2$$

Here the Lorentz-invariant quantity ν is defined by:

$$\nu = \frac{Pq}{M}$$

Since in lab. frame

4-mom of incomming proton: $\quad P = (Mc, \mathbf{0})$

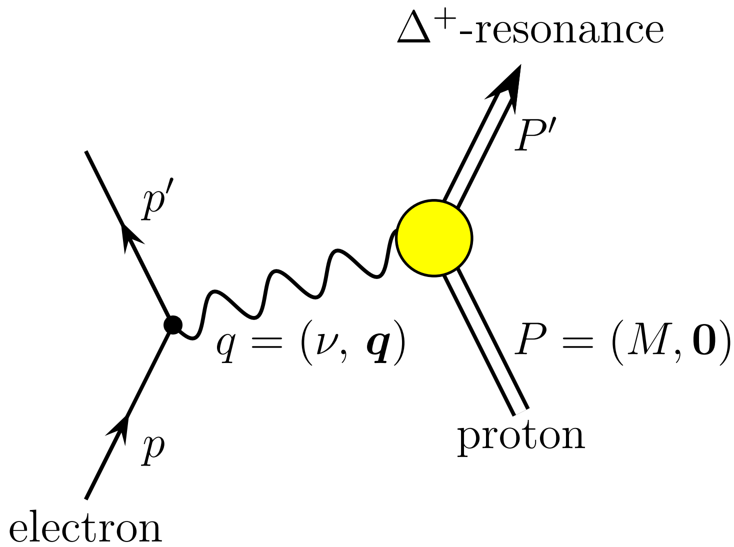
4-mom of exchanged photon : $\quad q = ((E' - E)/c, \boldsymbol{q})$

the energy transfer by the photon to the proton in this frame is

$$\nu = E - E'$$

THE $\mathbf{\Delta}$(1232) RESONANCE

Pronounced resonance observed with mass $W = 1232$ MeV/c^2.



Δ-resonance exists in 4 charge states: Δ^{++}, Δ^+, Δ^0, Δ^-. Here Δ^+

Resonance width $\Gamma \simeq 120$ MeV

$$\Rightarrow \text{ lifetime:} \qquad \tau = \frac{\hbar}{\Gamma} = 5.5 \times 10^{-24} \text{ s.}$$

☞ Typical lifetime for <u>strong</u> interaction processes ($c\tau \simeq 1.5$ fm!)

Δ^+ decays:

$$\Delta^+ \rightarrow \text{p} + \pi^0$$
$$\Delta^+ \rightarrow \text{n} + \pi^+$$

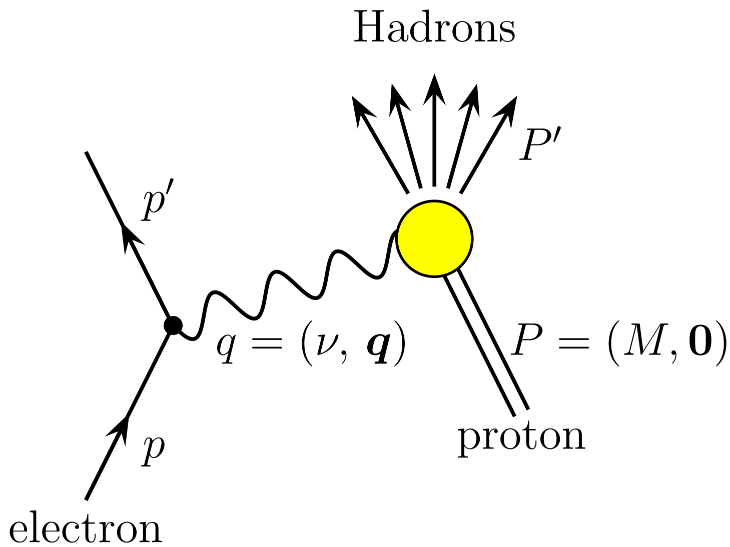
Pions (π^+, π^0, π^-):

- ☞ lightest hadrons (strongly interacting particles): ~ 140 MeV/c^2
- ☞ bosons: spin-0

STRUCTURE FUNCTIONS

For $W \gtrsim 2.5\ \mathrm{GeV}/c^2$ individual resonances not distinguishable:

☞ many hadrons produced:



Describe dynamics by form factors: W_1 and W_2 structure functions

☞ In *elastic* scattering, only one free variable: $E' = E'(\theta)$

Since $W = M$, we have relationship

$$2M\nu - Q^2 = 0.$$

☞ In *inelastic* scattering, one extra degree of freedom:

Exitation energy of proton

Since now $W > M$, we have relationship

$$2M\nu - Q^2 > 0.$$

Cross section is function of two variables: $(E',\ \theta)$ or $(Q^2,\ \nu)$

Cross section formula (a la Rosenbluth formula for elastic case):

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^*_{\mathrm{Mott}} \left[W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \frac{\theta}{2}\right]$$

W_2 describes electric interaction; W_1 describes magnetic interaction

DEEP INELASTIC SCATTERING EXPERIMENTS

First experiments in late 60's at SLAC: 25 GeV/c electron beam.
At fixed scattering angle (4°) measure scattered electron energy E'

Experimental observation by varying beam energy (Fig. 7.2):

- ☞ Rapid fall-off of cross-section with Q^2 in resonance region
- ☞ Slower fall-off of cross-section with Q^2 above resonance region

Behaviour above resonance region was surprising!

Remember for *elastic* scattering:

- ☞ Off pointlike particle (Mott formula): $\mathrm{d}\sigma/\mathrm{d}Q^2 = \text{constant}$
- ☞ Off real nucleon (dipole fit): $\mathrm{d}\sigma/\mathrm{d}Q^2 \propto 1/Q^8$

Here, in Deep Inelastic Scattering (Fig. 7.3):
Above resonances $\mathrm{d}\sigma/\mathrm{d}Q^2 \simeq \text{constant} \Rightarrow$ point-like constituents ...

Change of variables:

1) Introduce Lorentz-invariant dimensionless Bjorken scaling variable

$$x \equiv \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}$$

x gives a measure of the inelasticity of the process:

For elastic scattering: $2M\nu - Q^2 = 0 \quad \Rightarrow \quad x = 1$

For inelastic scattering: $2M\nu - Q^2 > 0 \quad \Rightarrow \quad 0 \leq x \leq 1$

2) Change to dimensionless structure functions:

$$F_1(x,\, Q^2) = Mc^2\, W_1(Q^2,\, \nu)$$
$$F_2(x,\, Q^2) = \nu\, W_2(Q^2,\, \nu)$$

From experimental data:

Observation: Structure functions F_1 and F_2 depend only weakly on Q^2 for fixed x (Fig. 7.4)

Conclusion: Electrons scatter off a point charge.

But since nucleon is an extended object:

Nucleons have a sub-structure made up of point-like constituents

F_1 results from magnetic interaction $\Rightarrow$ vanishes for spin-0 particles
For spin-$\frac{1}{2}$ particles one can show the relation

$$2xF_1(x) = F_2(x).$$

Since data confirm this (Fig. 7.5):

The point-line constituents of the nucleon have spin-$\frac{1}{2}$.

THE PARTON MODEL

Model by Feynman and Bjorken: Partons ≡ constituents of proton

Today: ☞ charged partons = quarks

☞ neutral partons = gluons

The model: look at the proton in a fast moving system

⇒ transverse momentum and rest masses of partons can be neglected

⇒ proton structure given by longitudinal momenta of partons

Impulse approximation: proton decomposed into free moving partons

☞ valid for short duration process: neglect inter-parton interactions

☞ valid in DIS since short distance inter-parton interactions are weak (*asymptotic freedom!*)

☞ electron interaction with proton viewed as incoherent sum of interaction with individual partons

☞ Parton/electron interaction regarded as elastic scattering process

Interpretation of Bjorken x when neglecting parton masses:

☞ x is fraction of proton 4-momentum carried by struck parton.

Thus, DIS in lab-frame:

☞ Parton with 4-mom. xP interacts with photon with $q = (\nu/c,\, \boldsymbol{q})$

Interpretation of Q^2 variable (in Breit frame (Fig. 7.6)):

$$\frac{\lambda}{2\pi} = \frac{\hbar}{|\boldsymbol{q}|} = \frac{\hbar}{\sqrt{Q^2}}$$

λ: wavelength of virtual photon.

Thus, Q^2 fixes spatial resolution with which structures can be studied

STRUCTURE FUNCTIONS IN THE PARTON MODEL

Assume nucleon is build from different types of quarks f with electric charges $z_f e$

Probability for photon interacting with quark $\propto z_f^2$.

Partons inside nucleon:

- ☞ *Valence quarks:* responsible for quantum numbers
- ☞ *Sea quarks:* virtual quark-antiquark pairs
- ☞ *Gluons:* bind quarks together

Distribution function of momentum:

- quarks $q_f(x)$
- antiquarks $\bar{q}_f(x)$
- gluons $g(x)$

Structure function F_2 (electric interaction) is then

$$F_2(x) = x \sum_f z_f^2 \left[q_f(x) + \bar{q}_f(x) \right]$$

Also DIS experiments based on alternate beam particles:

muons: as electrons, sensitive to electric charges of quarks

neutrinos: sensitive to the *weak charges* of quarks

Combining DIS results (electrons, muons, neutrinos, antineutrinos):
$\Rightarrow$ separate F_2 into contributions from valence and sea quarks.

Observations (Fig. 7.7):

- ☞ Sea quarks contribute only at small x; negligible above $x = 0.35$;
- ☞ Valence quarks contrinute maximally at $x \simeq 0.20$; distribution goes to zero for $x \to 0$ and $x \to 1$
- ☞ For large x, F_2 become small: unlikely that one quark alone carries major fraction of momentum

THE QUARK STRUCTURE OF NUCLEONS

From DIS: Nucleons have constituents (quarks) which are

- ✔ electrically charged
- ✔ point-like
- ✔ spin-1/2

Should be able to explain quantum numbers of nucleons from those of quarks

		u	d	p (uud)	n (udd)
Charge	z	$+2/3$	$-1/3$	1	0
Isospin	I	1/2		1/2	
	I_3	$+1/2$	$-1/2$	$+1/2$	$-1/2$
Spin	s	1/2	1/2	1/2	1/2

In total six different quarks – six "flavours".

☞ Three generations (families) of doublets with increasing masses:

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix} \qquad \begin{matrix} \longleftarrow Q/e = 2/3 \\ \longleftarrow Q/e = -1/3 \end{matrix}$$

Mass pattern (*constituent masses*):

$$\begin{pmatrix} 0.3 \\ 0.3 \end{pmatrix} \begin{pmatrix} 1.3 \\ 0.45 \end{pmatrix} \begin{pmatrix} 175 \\ 4.2 \end{pmatrix} \qquad [\mathrm{GeV}/c^2]$$

Quarks inside nucleons:

- ☞ Valence quarks: u, d
- ☞ Sea quarks: $\mathrm{u} + \bar{\mathrm{u}}$, $\mathrm{d} + \bar{\mathrm{d}}$, $\mathrm{s} + \bar{\mathrm{s}}$; (c, b, t quarks too heavy)

Quark Charges

Structure functions of proton and neutron in electron scattering:

$$F_2^{\mathrm{e,p}}(x) = x \cdot \left[\frac{1}{9}(d_{\mathrm{v}}^{\mathrm{p}} + d_{\mathrm{s}}^{\mathrm{p}} + \bar{d}_{\mathrm{s}}^{\mathrm{p}}) + \frac{4}{9}(u_{\mathrm{v}}^{\mathrm{p}} + u_{\mathrm{s}}^{\mathrm{p}} + \bar{u}_{\mathrm{s}}^{\mathrm{p}}) + \frac{1}{9}(s_{\mathrm{s}}^{\mathrm{p}} + \bar{s}_{\mathrm{s}}^{\mathrm{p}})\right]$$
$$F_2^{\mathrm{e,n}}(x) = x \cdot \left[\frac{1}{9}(d_{\mathrm{v}}^{\mathrm{n}} + d_{\mathrm{s}}^{\mathrm{n}} + \bar{d}_{\mathrm{s}}^{\mathrm{n}}) + \frac{4}{9}(u_{\mathrm{v}}^{\mathrm{n}} + u_{\mathrm{s}}^{\mathrm{n}} + \bar{u}_{\mathrm{s}}^{\mathrm{n}}) + \frac{1}{9}(s_{\mathrm{s}}^{\mathrm{n}} + \bar{s}_{\mathrm{s}}^{\mathrm{n}})\right]$$

Assume **isospin** symmetry: Neutron and proton related through quark symmetry operation: u $\leftrightarrow$ d:

$$u_{\mathrm{i}}^{\mathrm{p}}(x) = d_{\mathrm{i}}^{\mathrm{n}}(x) \equiv u_{\mathrm{i}}(x) \qquad \text{and} \qquad d_{\mathrm{i}}^{\mathrm{p}}(x) = u_{\mathrm{i}}^{\mathrm{n}}(x) \equiv d_{\mathrm{i}}(x)$$

Also drop small s-quark terms, thus

$$F_2^{\mathrm{e,p}}(x) = x \cdot \left[\frac{1}{9}(d_{\mathrm{v}} + d_{\mathrm{s}} + \bar{d}_{\mathrm{s}}) + \frac{4}{9}(u_{\mathrm{v}} + u_{\mathrm{s}} + \bar{u}_{\mathrm{s}})\right]$$
$$F_2^{\mathrm{e,n}}(x) = x \cdot \left[\frac{1}{9}(u_{\mathrm{v}} + u_{\mathrm{s}} + \bar{u}_{\mathrm{s}}) + \frac{4}{9}(d_{\mathrm{v}} + d_{\mathrm{s}} + \bar{d}_{\mathrm{s}})\right]$$

Define for each quark and anti-quark $q \equiv q_{\mathrm{v}} + q_{\mathrm{s}}$. Then

$$F_2^{\mathrm{e,p}}(x) = x \cdot \left[\frac{1}{9}(d + \bar{d}) + \frac{4}{9}(u + \bar{u})\right]$$
$$F_2^{\mathrm{e,n}}(x) = x \cdot \left[\frac{1}{9}(u + \bar{u}) + \frac{4}{9}(d + \bar{d})\right]$$

Now, "average" nucleon form factor:

$$F_2^{\mathrm{e,N}}(x) \equiv \frac{F_2^{\mathrm{e,p}}(x) + F_2^{\mathrm{e,n}}(x)}{2} = \frac{5}{18}\, x \sum_{f=u,d} [q_f(x) + \bar{q}_f(x)]$$

In neutrino scattering, no dependence on quark charges:

$$F_2^{\nu,\mathrm{N}}(x) = x \sum_f [q_f(x) + \bar{q}_f(x)]$$

Thus, measure (sum of squares of) quark charges through comparison
Assignment of charges $+2/3$ and $-1/3$ for u- and d-quarks confirmed

Quark momentum distributions (Fig. 8.1)

Integration of experimental momentum distribution:

$$\int_0^1 F_2^{\nu,\mathrm{N}}(x)\,\mathrm{d}x \simeq \frac{18}{5}\int_0^1 F_2^{\mathrm{e,N}}(x)\,\mathrm{d}x \simeq 0.5.$$

$\Rightarrow$ Only half of nucleon momentum carried by quarks! Rest by gluons

Consider ratio of neutron and proton structure functions, $F_2^{\mathrm{n}}/F_2^{\mathrm{p}}$:

☞ Goes to 1 for low x: sea quarks dominate (Fig. 8.2)

☞ Goes to 1/4 for $x \to 1$

- would have expected $2/3 = (2z_\mathrm{d}^2 + z_\mathrm{u}^2)/(2z_\mathrm{u}^2 + z_\mathrm{d}^2)$
- observation looks like $z_\mathrm{d}^2/z_\mathrm{u}^2$

Constituent quarks

DIS experiments: ☞ half of nucleon momentum carried by quarks
☞ other half carried by gluons

DIS: probing quarks deep inside hadrons
☞ large $Q^2 \Rightarrow$ small length scale ($\lambda \simeq h/\sqrt{Q^2}$)

Quark masses estimated from DIS (*current quark masses*) are small:

$$m_{\mathrm{u}}^{\mathrm{cur}} = 1.5 - 5 \ \mathrm{MeV}/c^2$$
$$m_{\mathrm{d}}^{\mathrm{cur}} = 3 - 9 \ \mathrm{MeV}/c^2$$

For spectroscopy (magnetic moments, hadron exitation energy, ...):
☞ Combine sea quarks and gluons with valence quarks
☞ Constituent quarks (or effective quarks)
☞ Each valence quark drags along a lot of "junk" ⇒ larger masses

$$m_{\mathrm{u}}^{\mathrm{eff}} \simeq 300 \ \mathrm{MeV}/c^2$$
$$m_{\mathrm{d}}^{\mathrm{eff}} \simeq 300 \ \mathrm{MeV}/c^2$$

d-quark heavier than u-quark:
☞ proton (uud) mass: 938.3 MeV/c^2
☞ neutron (udd) mass: 939.6 MeV/c^2
In addition Coulomb repulsion larger for proton than neutron

Quarks in hadrons

Hundreds of unstable hadrons known.

From their study: information on the strong interaction.

Baryons

Lowest mass states (ground states): neutron and proton

3 quarks ➪ half-integral spins

Many exited states:

- ☞ Parallel to atomic spectra
- ☞ BUT: energy gap between states of same order as nucleon mass

⇒ Classify exited states as individualo particles

Baryons produced in pairs: baryon + anti-baryon

⇒ Conserved quantum number: *baryon number* B

Baryon:	$B = +1$	Quark:	$B = +1/3$
Anti-baryon:	$B = -1$	Anti-quark:	$B = -1/3$

Experiment: B conserved in <u>all</u> reactions.

➪ No proton decay observed:

$$\mathrm{p} \nrightarrow \pi^0 + \mathrm{e}^+ \qquad \tau > 5.5 \times 10^{32} \text{ years}$$

Mesons

Lightest hadrons are pions: π^-, π^0, π^+

Masses: ~ 140 MeV/c^2

➪ Considerably smaller than constituent quark mass

Quark-antiquark pairs:

$$|\pi^+\rangle = |\mathrm{u\bar{d}}\rangle, \qquad |\pi^-\rangle = |\mathrm{\bar{u}d}\rangle, \qquad |\pi^0\rangle = \frac{1}{\sqrt{2}}\left\{|\mathrm{u\bar{u}}\rangle + |\mathrm{d\bar{d}}\rangle\right\}$$

Spin: all mesons have integer spin; pions are spin-0

All mesons decay: no meson number

$$\pi^- \to \mu^- \bar{\nu}_\mu \qquad \pi^0 \to \gamma\gamma$$

QUARK-GLUON INTERACTION

Quark colour

Total baryon wavefunction

$$\psi(\text{total}) = \phi(\text{space})\,\alpha(\text{spin})\,\chi(\text{flavour})$$

Apply to Δ^{++}:

- ☞ Charge ++: uuu quark state
- ☞ Lowest lying spin-3/2 state: $L = 0$ and parallel quark spin

➪ ϕ, α, and χ all symmetric ➪ ψ symmetric

Seems impossible since quarks are *fermions*: Violates Pauli principle!

Solution (Greenberg, 1964): quarks possess another attribute, *colour*
Quarks exist in three colours: red, green, blue

Pauli principle applies to whole wavefunction including $\chi(\text{colour})$

$$\psi(\text{total}) = \phi(\text{space})\,\alpha(\text{spin})\,\chi(\text{flavour})\,\chi(\text{colour})$$

Colour never observed in nature:

- ➪ baryons and mesons are colour singlets
- ➪ $\chi(\text{colour})$ is always anti-symmetric for hadrons!
- ➪ total wavefunction anti-symmetric if and only if

$$\phi(\text{space})\,\alpha(\text{spin})\,\chi(\text{flavour})$$

is symmetric!

Colour: the charge of the strong interaction
Interquark force independent of quark color: colour SU(3) symmetry

Gluons

QED: ♦ Photon is exchange particle that couples to *electric* charge

♦ Massless vector boson: $J^P = 1^-$

♦ Electrically neutral

QCD: ♦ Gluon is exchange particle that couples to *colour* charge

♦ Massless vector boson: $J^P = 1^-$

♦ Colour charged!

Gluon colour charge: colour-anticolour ➩ 3×3 combinations:

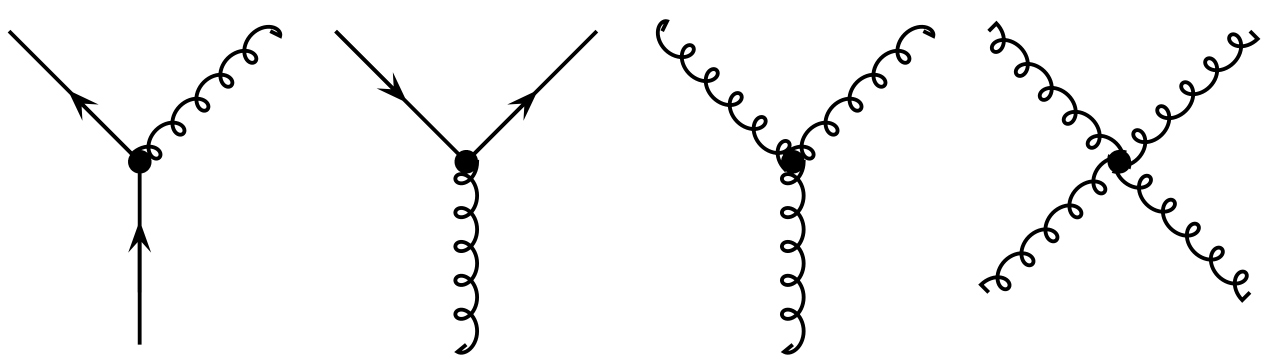
➩ Octet of physical states:

$$\mathrm{r\bar{g}},\ \mathrm{r\bar{b}},\ \mathrm{g\bar{b}},\ \mathrm{g\bar{r}},\ \mathrm{b\bar{r}},\ \mathrm{b\bar{g}},\ \sqrt{1/2}(\mathrm{r\bar{r}} - \mathrm{g\bar{g}}),\ \sqrt{1/6}(\mathrm{r\bar{r}} + \mathrm{g\bar{g}} - 2\mathrm{b\bar{b}})$$

➩ Fully symmetric singlet ⇒ no effect in colour space (unphysical):

$$\sqrt{1/3}(\mathrm{r\bar{r}} + \mathrm{g\bar{g}} + \mathrm{r\bar{r}})$$

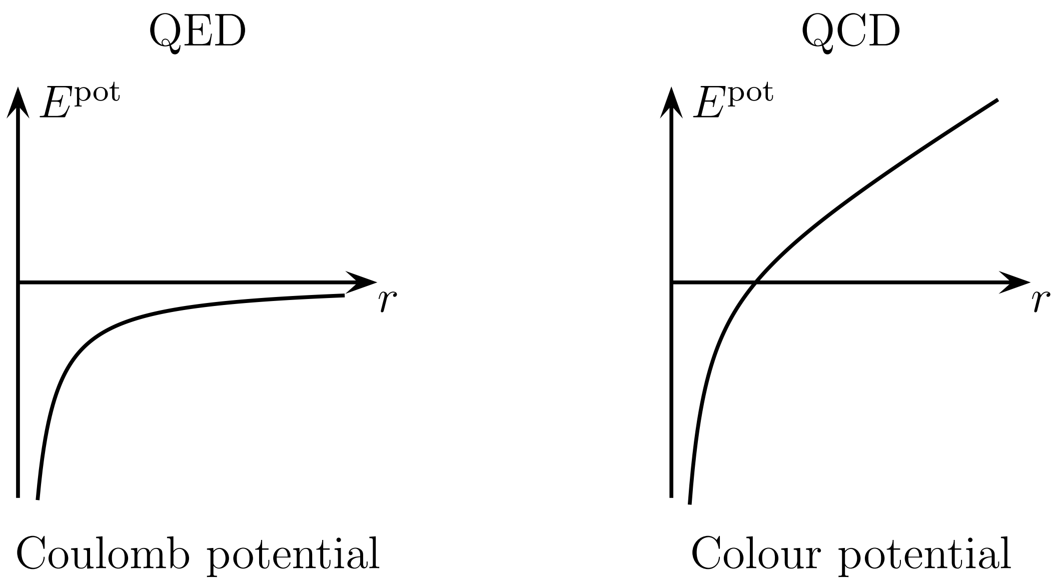
Quark-gluon and gluon-gluon interactions:



Hadrons as colour-neutral objects

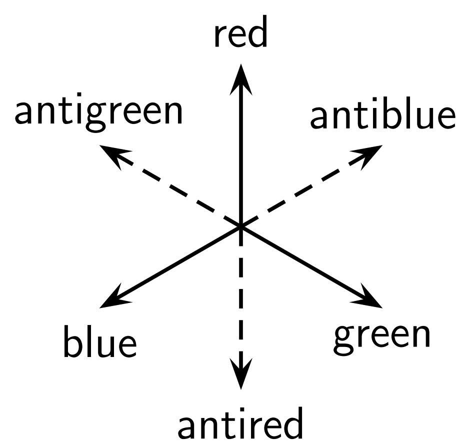
Only colourless particles occur in Nature: No free quarks observed!

Potential between quarks increases limitless for increasing separation:
➩ *Confinement*



QCD situation different from QED due to gluon-gluon interactions

Colour SU(3) symmetry:



Colourless (white) combinations possible in two ways:
☞ Baryon way: r + g + b
☞ Meson way: $(r + \bar{r})$ or $(g + \bar{g})$ or $(b + \bar{b})$

Physical (colourless) hadrons:

☞ Mesons: colour + anticolour

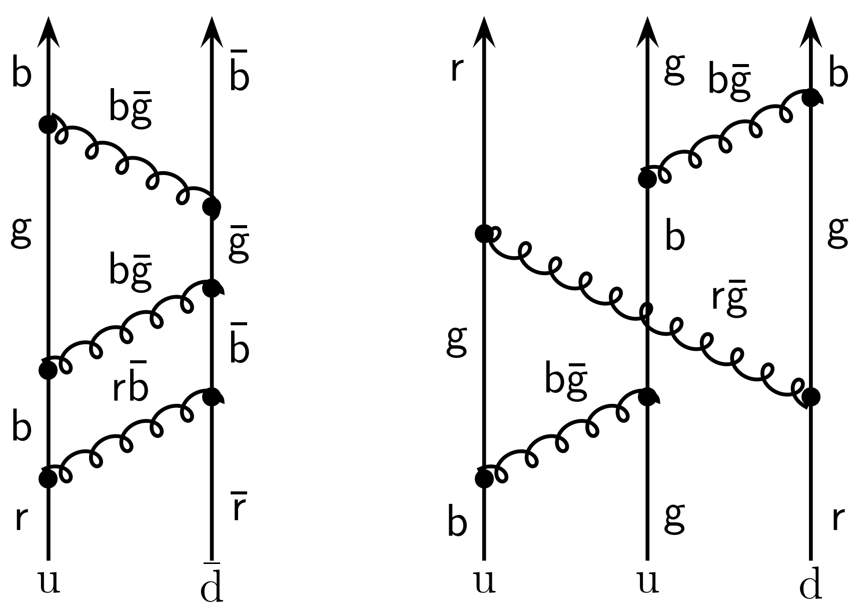
$$|\pi^+\rangle = \begin{cases} |\mathrm{u_r \bar{d}_{\bar{r}}}\rangle \\ |\mathrm{u_g \bar{d}_{\bar{g}}}\rangle \\ |\mathrm{u_b \bar{d}_{\bar{b}}}\rangle \end{cases}$$

☞ Baryon: red + green + blue

$$|p\rangle = \begin{cases} |\mathrm{u_r u_g d_b}\rangle \\ |\mathrm{u_b u_r d_g}\rangle \\ \vdots \end{cases}$$

External colour of hadrons constant: **colourless!!!**

Colour of constituents (inter-)change continously:



π^+ proton

The strong coupling constant $\boldsymbol{\alpha}_\mathrm{s}$

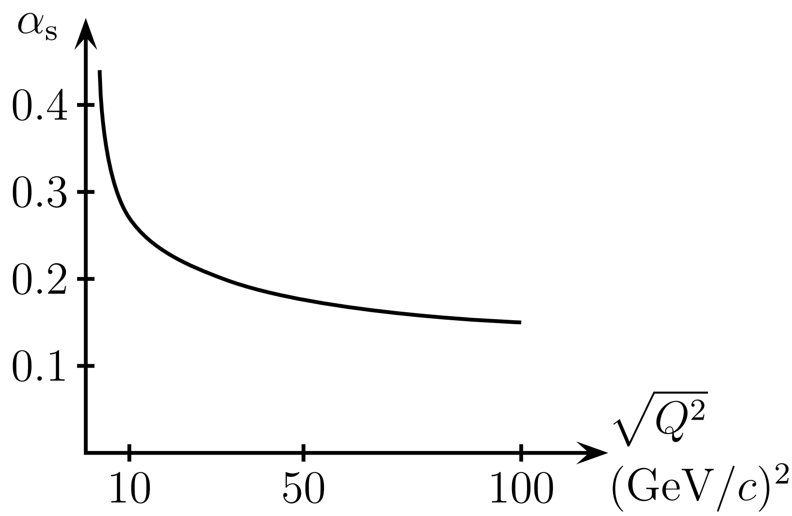
In Quantum Field Theories coupling constants are not constant:

$$\alpha = \alpha(Q^2)$$

In QED, dependence is weak
In QCD, dependence is strong (due to gluon-gluon interaction):

$$\alpha_\mathrm{s}(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda^2)}$$

n_f: number of active quark flavours at this Q^2 $[n_f = 3 - 6]$
Λ: Fixes overall scale of strong coupling
Determined by experiment: $\Lambda \simeq 250$ MeV/c^2



Dependence is equivalent to dependence on separation:
☞ large $Q^2 \Rightarrow$ small separation: small coupling (*asymptotic freedom*)
☞ small $Q^2 \Rightarrow$ large separation: strong coupling (*confinement*)

Perturbative QCD only valid for $\alpha_\mathrm{s} \ll 1$ or equivalently $Q^2 \gg \Lambda^2$

Scaling Violations

We have shown that F_2 was independent of Q^2: $F_2(x, Q^2) = F_2(x)$

However, high-precision measurements show that F_2 indeed has some Q^2 dependence (Fig. 8.4):

☞ Increases with Q^2 at small x

☞ Decreases with Q^2 at large x

Or, in other words, when we increase Q^2 we see (Fig. 8.5):

☞ Fewer quarks with high momentum fraction x

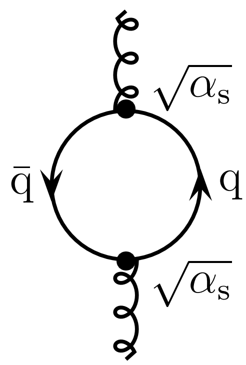
☞ More quarks with small momentum fraction x

At large Q^2 we are probing deeper into nucleon ($\lambda \simeq h/\sqrt{Q^2}$):

➩ Resolving more virtual quark-antiquark pairs

➩ More partons to share same momentum

Creation of virtual quark pairs $\propto \alpha_s$:



➩ Extract α_s from scaling violation measurement:

$$\alpha_s(Q^2 = 100\,(\mathrm{GeV}/c)^2) \simeq 0.16$$

Conclusion on nucleon structure:

The nucleon interior is an extremely busy place with plenty of quantum fluctuations. The deeper we probe the more fluctuations are visible to us.