Approximate Bayesian Computational Methods

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- not on closed form (= intractable)
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MUST be able to simulate from likelihood function.

Also requires a known prior.

Algorithm 1

Algorithm 1 Likelihood-free rejection sampler 1

for $i = 1$ to N **do**
  repeat
    Generate $\boldsymbol{\theta}'$ from the prior distribution $\pi(\cdot)$
    Generate $\mathbf{z}$ from the likelihood $f(\cdot|\boldsymbol{\theta}')$
  until $\mathbf{z} = \mathbf{y}$
  set $\boldsymbol{\theta}_i = \boldsymbol{\theta}'$,
end for

Algorithm 1

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end for

This NEVER happens!

Algorithm 2

Algorithm 2 Likelihood-free rejection sampler 2

for $i = 1$ to N **do**
  repeat
    Generate $\boldsymbol{\theta}'$ from the prior distribution $\pi(\cdot)$
    Generate $\mathbf{z}$ from the likelihood $f(\cdot|\boldsymbol{\theta}')$
  until $\rho\{\eta(\mathbf{z}), \eta(\mathbf{y})\} \leq \varepsilon$
  set $\boldsymbol{\theta}_i = \boldsymbol{\theta}'$,
end for

Example

Moving average model (order q):

$$y_k = u_k + \sum_{i=1}^{q} \theta_i u_{k-1} \tag{1}$$

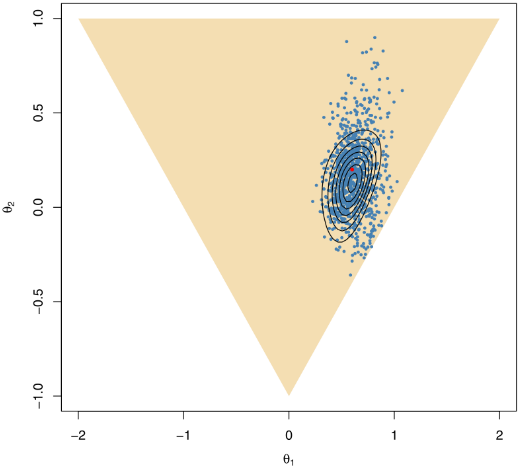
With

$$u_k \sim \mathcal{N}(0,1)$$

For $q = 2$, prior is uniform in the triangle:

$$-2 < \theta_1 < 2, \quad \theta_1 + \theta_2 > -1, \quad \theta_1 - \theta_2 < 1$$

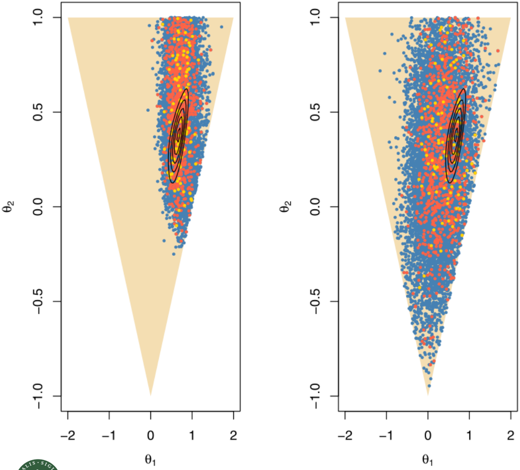
Example



- Prior
- True values (0.6, 0.2)
- Contour of true distr.
- Sampled points

Example

Different metrics and tolerances - 1



Prior

Contour of true distr.

$\varepsilon = 1\%$

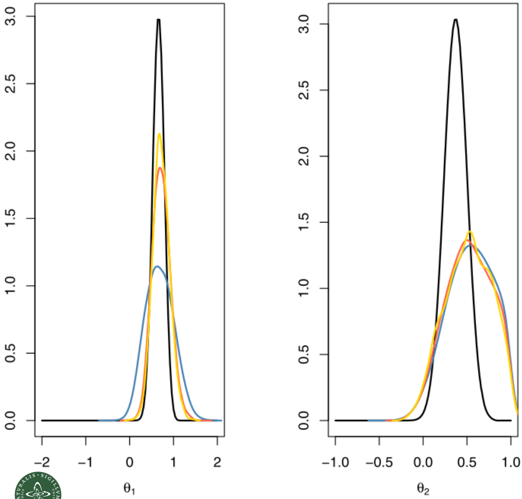
$\varepsilon = 0.1\%$

$\varepsilon = 0.01\%$

(left) autocovariance
(right) raw distance

Example

Different metrics and tolerances - 2



- $\varepsilon = 1\%$
- $\varepsilon = 0.1\%$
- $\varepsilon = 0.01\%$
- Contour of true distr.

Algorithm 3

MCMC-ABC

Algorithm 3 Likelihood-free MCMC sampler

Use Algorithm 2 to get a realisation $(\boldsymbol{\theta}^{(0)}, \mathbf{z}^{(0)})$ from the ABC target distribution $\pi_\varepsilon(\boldsymbol{\theta}, \mathbf{z}|\mathbf{y})$

for $t = 1$ to N **do**

Generate $\boldsymbol{\theta}'$ from the Markov kernel $q(\cdot|\boldsymbol{\theta}^{(t-1)})$,

Generate $\mathbf{z}'$ from the likelihood $f(\cdot|\boldsymbol{\theta}')$,

Generate u from $\mathcal{U}_{[0,1]}$,

if $u \leq \frac{\pi(\boldsymbol{\theta}')q(\boldsymbol{\theta}^{(t-1)}|\boldsymbol{\theta}')}{\pi(\boldsymbol{\theta}^{(t-1)})q(\boldsymbol{\theta}'|\boldsymbol{\theta}^{(t-1)})}$ and $\rho\{\eta(\mathbf{z}'), \eta(\mathbf{y})\} \leq \varepsilon$ **then**

set $(\boldsymbol{\theta}^{(t)}, \mathbf{z}^{(t)}) = (\boldsymbol{\theta}', \mathbf{z}')$

else

$(\boldsymbol{\theta}^{(t)}, \mathbf{z}^{(t)}) = (\boldsymbol{\theta}^{(t-1)}, \mathbf{z}^{(t-1)})$,

end if

end for