Presentation of Article:
Too good to be true: When overwhelming evidence fails to convince

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Introduction

- Normally:
  More measurements agree with your model -> better confidence in your model
- The article introduces a ”*hidden failure state*”
- This causes your confidence in the model to decrease with increasing agreement with data.
- This is known as: *Verschlimmbesserung* or *disimprovement*.
- The article analyzes this through Bayesian analysis:

Theory – Bayesian Analysis

Bayes' Law

Without hidden failure state:

$$P[H_i|\mathbf{X}] = \frac{P[\mathbf{X}|H_i]P[H_i]}{P[\mathbf{X}]},$$

Including a hidden failure state defined by the variable F :

$$= \frac{\sum_{f} P[\mathbf{X}|H_i, f]P[H_i, F = f]}{\sum_{f,H_k} P[\mathbf{X}|H_k, f]P[H_k, F = f]}.$$

$$= \left(1 + \frac{\sum_{f=0}^{1} P[\mathbf{X}|H_{1-i}, F = f]\ P[H_{1-i}, F = f]}{\sum_{f=0}^{1} P[\mathbf{X}|H_i, \quad F = f]\ P[H_i, \quad F = f]} \right)^{-1}$$

Determining Origin of Roman Pot

Roman pot found in Britain – we wish to determine whether a specific pot was made in Roman occupied Britain or transported from Italy to Britain.

- **Two hypotheses: H_0: Italy, H_1: Britain**
  - Flat prior – both are equally likely
  - Test for certain trace element found in British clay: error rate $p_e = 0.3$
- **Hidden Failure State – introduction of trace element during manufacturing process**
  - Rate of contamination: $p_c = 0.01$
  - 50 / 50 distribution of contaminated pots between Britain and Rome
  - If contaminated, the trace element will be measured with 90 % probability

Determining Origin of Roman Pot

Information table:

$P[F, H_i]$

Contaminated		Origin: Italy H_0	Origin: Britain H_1
Y	$F=0$	0.005 ($\frac{1}{2} p_c$)	0.005 ($\frac{1}{2} p_c$)
N	$F=1$	0.495 ($\frac{1}{2}(1-p_c)$)	0.495 ($\frac{1}{2}(1-p_c)$)

Each square represents an outcome

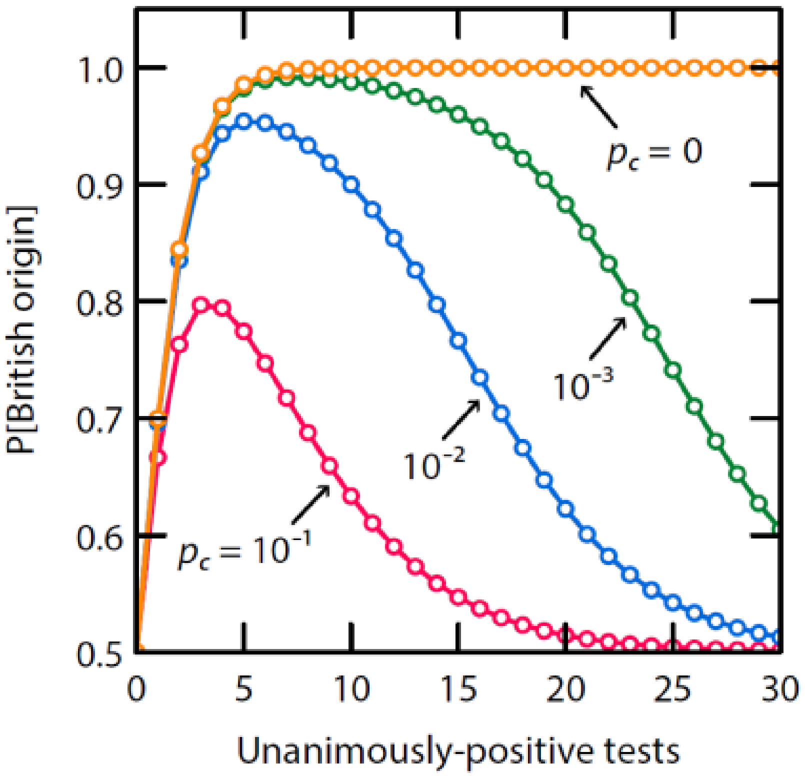
$P[\text{Positive result} \mid F, H_i]$

Contaminated		Origin: Italy H_0	Origin: Britain H_1
Y	$F=0$	0.9	0.9
N	$F=1$	0.3 (p_e)	0.7 ($1-p_e$)

Each square represents a distribution

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x}$$

Plotting the resulting PMF:



Identifying Suspect in Identity Parade

We want to estimate the probability of correctly identifying a suspect as the perpetrator through the use of identity parades.

- **Again two hypotheses: H_0: Innocent, H_1: Guilty**
  - Flat prior: 50 / 50
  - False-Negative rate: the probability of falsely accusing an innocent suspect when perpetrator is in the parade $p_{fn} = 0.48$
  - False-Positive rate: the probability of falsely accusing an innocent suspect when perpetrator is not in the parade $p_{fp} = 0.133$
- **Hidden Failure State: bias in the conduction of the parade**
  - Small probability p_c that the parade is biased
  - If the identity parade is biased, the suspect is identified as guilty 90 % of the time regardless of guilt.

Identifying Suspect in Identity Parade

Information table:

$P[F, H_i]$

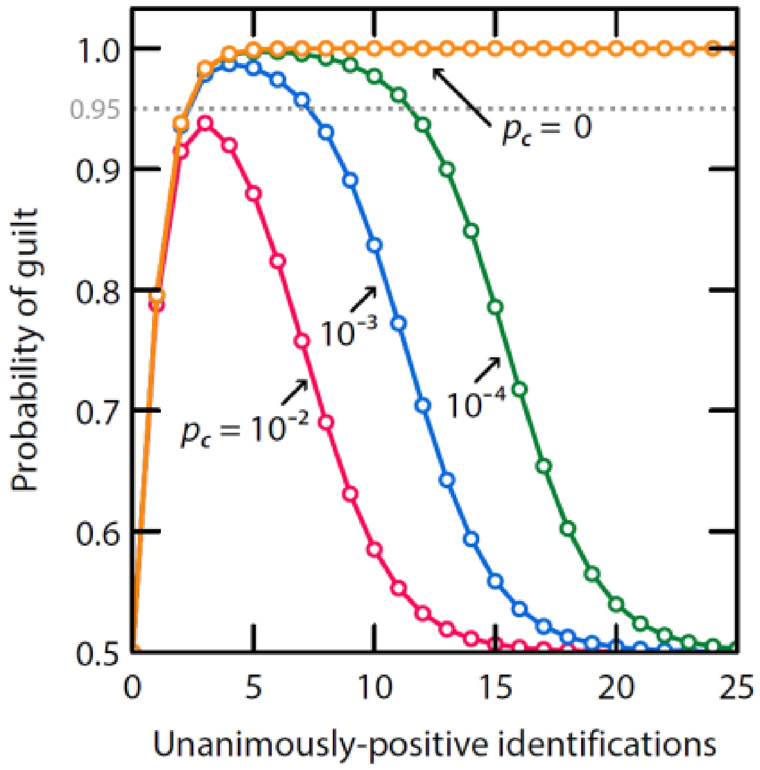
Biased parade		Suspect is… Innocent H_0	Suspect is… Guilty H_1
	Y $F=0$	0.005 $\frac{1}{2} p_c$	0.005 $\frac{1}{2} p_c$
	N $F=1$	0.495 $\frac{1}{2}(1-p_c)$	0.495 $\frac{1}{2}(1-p_c)$

Each square represents an outcome

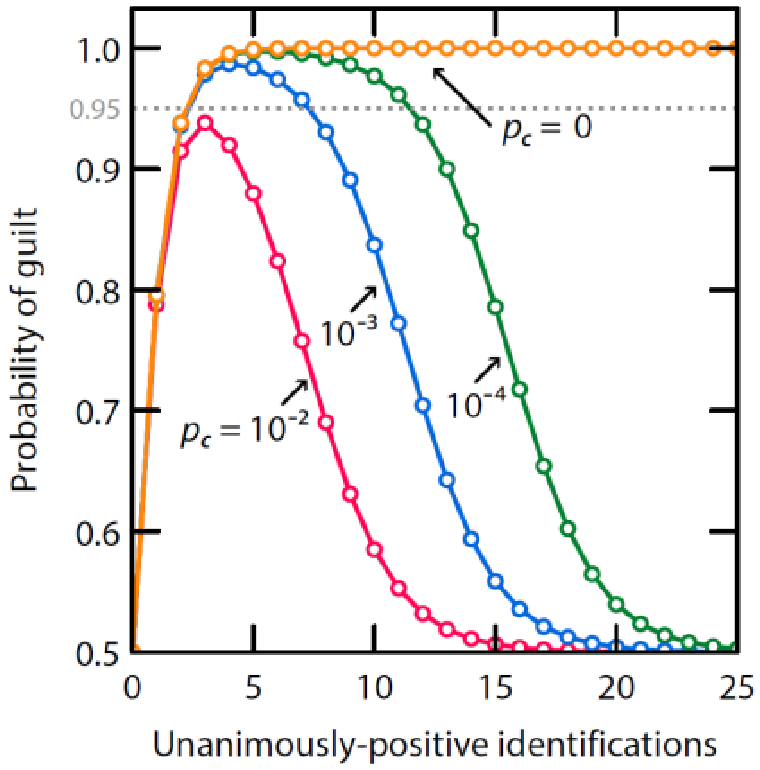
$P[\text{Identification} \mid F, H_i]$

Biased parade		Suspect is… Innocent H_0	Suspect is… Guilty H_1
	Y $F=0$	0.9	0.9
	N $F=1$	0.13 p_{fp}	0.52 $1-p_{fn}$

Each square represents a distribution



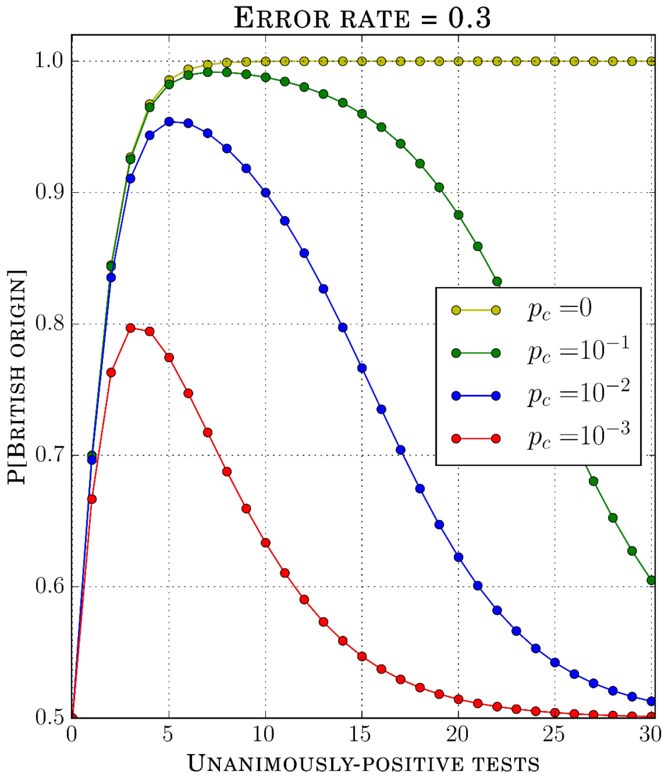
Identifying Suspect in Identity Parade



Even with just p_c = 1%,
the probability of guilt is never > 95%.

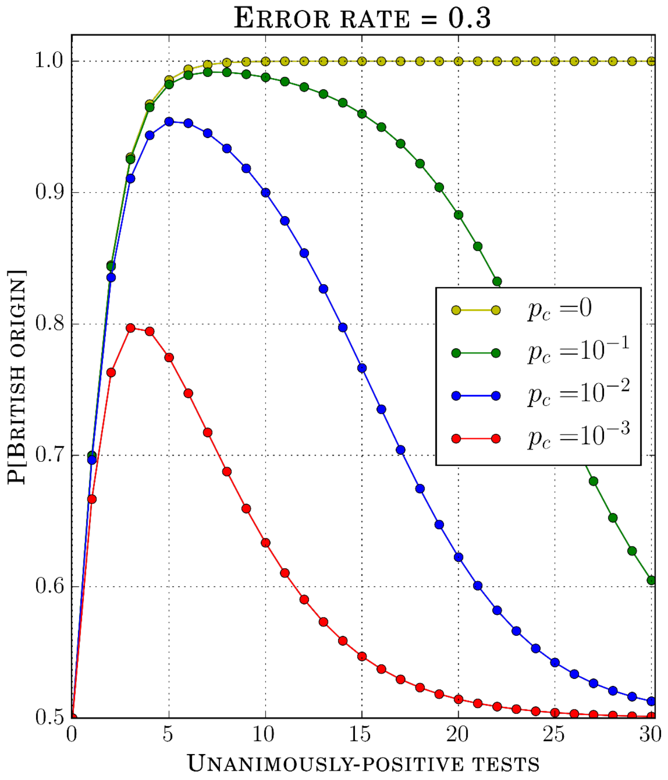
Determining Origin of Roman Pot

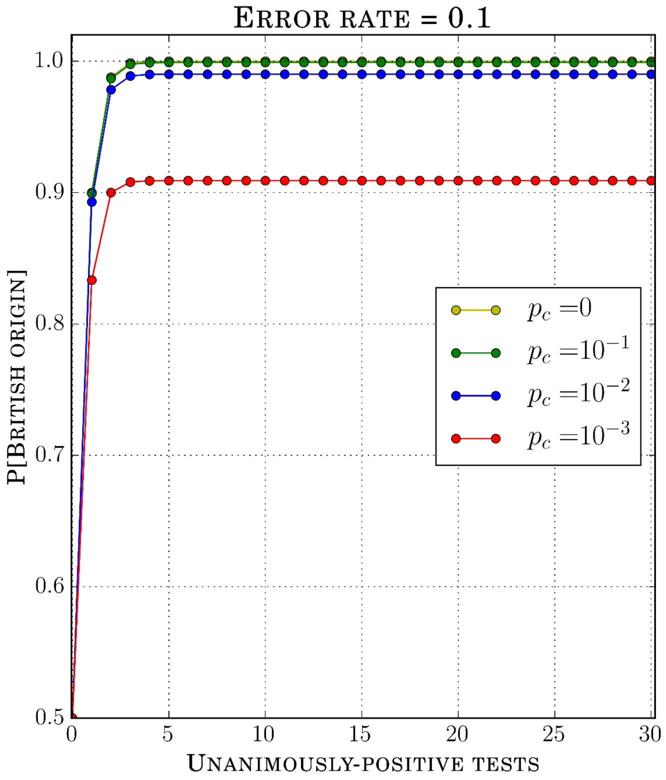
What if the success rate (1-p_e) is larger than the success rate while contaminated (90 %)



Determining Origin of Roman Pot

What if the success rate ($1-p_e$) is larger than the success rate while contaminated (90 %)





Determining Origin of Roman Pot

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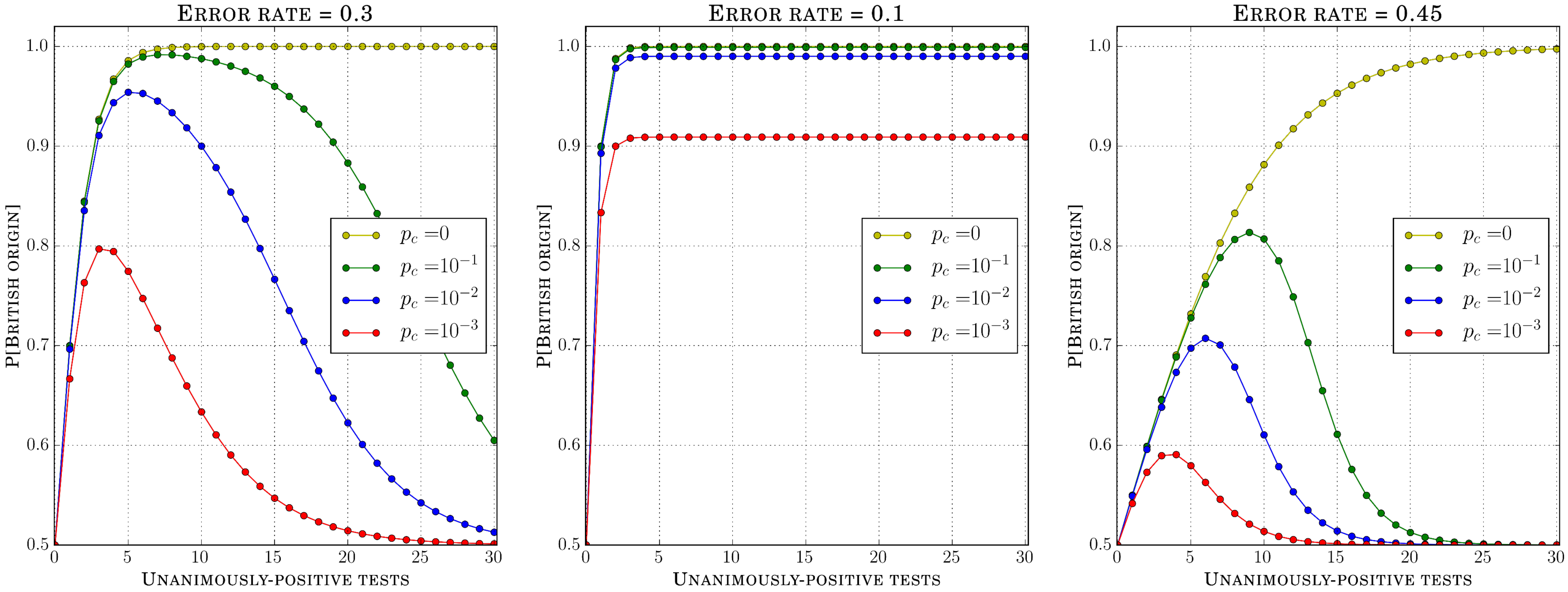

Conclusion

- Including hidden failure states highly changes the probabilistic nature of the problem
- Even a small probability of bias (hidden failure state) can drastically reduce the confidence of our test
- In real life, the ratio between (1- p_e) and p_{fp} high determines the significance of the hidden failure state

Questions?