Sum of Weighted Poisson Events

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Compound Poisson Distribution

- Describes the distribution of x: $x = w_1 + w_2 + ... + w_n$
- w_i are independent of each other and n.
- All weights are drawn from the same distribution.
- The amount of numbers drawn, n, is Poisson distributed (with mean λ)
- Problem of using the CPD when the underlying distribution of w is unknown.
- These cases usually approximated using a normal distribution
- Authors suggest a new approximation using a scaled Poisson distribution.

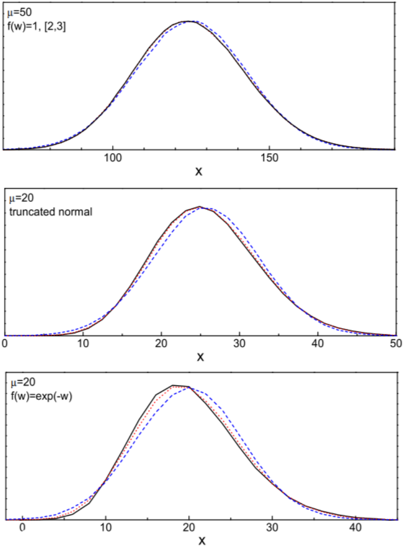
Scaled Poisson Distribution

- The SPD is defined with $\tilde{\lambda} = \lambda \frac{E(w)^2}{E(w^2)} = \mu \frac{E(w)}{E(w^2)} = \frac{\mu}{s}$ with scale $s = E(w^2)/E(w)$
- $\tilde{n}$ is taken from a Poisson distribution with mean $\tilde{\lambda}$.
- $\tilde{x} = s\tilde{n}$ ensuring $E(\tilde{x}) = E(x) = \mu$ and $var(\tilde{x}) = var(x) = \sigma^2$
- Skew and kurtosis of the SPD: $\tilde{\gamma}_1 = 1/\tilde{\lambda}^{1/2}$, $\tilde{\gamma}_2 = 1/\tilde{\lambda}$
- The ratio between the SPD and CPD $(\gamma_1/\tilde{\gamma_1}, \gamma_2/\tilde{\gamma_2})$ is found them to be greater than or equal to one.
- Both should be better than the normal distribution.

Monte Carlo Simulation Comparisons

- Comparing the CPD, SPD and normal distribution, using different weights.
- For the table $\lambda = 50$.

Type of weight	$\tilde{\lambda}$	γ_1	γ_2	$\tilde{\gamma}_1$	$\tilde{\gamma}_2$
$u[0,1]$	37.50	0.184	0.036	0.163	0.027
$u[1,2]$	48.21	0.149	0.023	0.144	0.021
$u[2,3]$	49.34	0.144	0.021	0.142	0.020
$\exp(-w)$	25.00	0.300	0.120	0.200	0.040
$\mathcal{N}_t(1,1)$	36.48	0.199	0.045	0.166	0.027
$1\ (p=0.5)$, 10	29.94	0.197	0.039	0.182	0.033
$1\ (p=0.8)$, 10	19.01	0.299	0.092	0.229	0.052

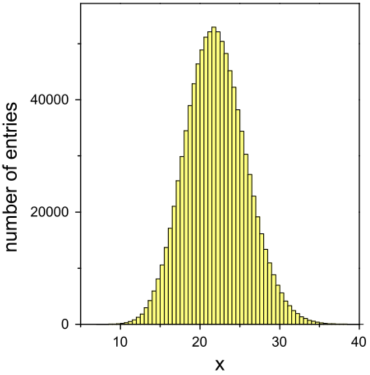


Poisson Bootstrap

- n observation $x_i, \ i = 1, 2, \ldots, n$
- $x = \sum_i x_i$
- Produce n Poisson distributed numbers n_i with mean 1
- $x_k = \sum_i x_i \cdot n_i$
- Parameters and confidence intervals of the distribution can be estimated by distribution of the x_k

Poisson Bootstrap Application

- n observations with $n \sim \mathcal{P}_{50}(n)$
- x_i random uniform numbers in $[0, 1]$
- $x_{obs} = 22.01$
- Produce one million bootstrap samples with $n_i \sim \mathcal{P}_1(n)$
- Confidence intervals are estimated by integrals



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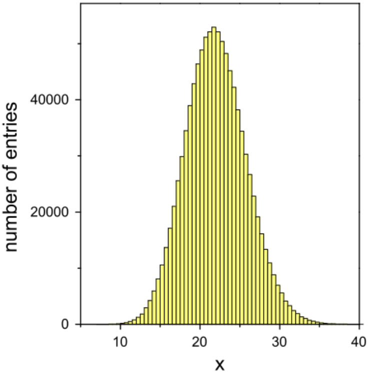

α	0.01	0.05	0.10	0.1585	0.8415	0.90	0.95	0.99
CL	13.8	16.0	17.2	18.2	25.8	26.9	28.5	31.4
CL*	14.4	16.5	17.6	18.5	26.2	27.3	28.9	32.1

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