



The Efficiency of Geometric Samplers for Exoplanet Transit Timing Variation Models

Noah W. Tuchow, Eric B. Ford, Theodore Papamarkou and Alexey Lindo

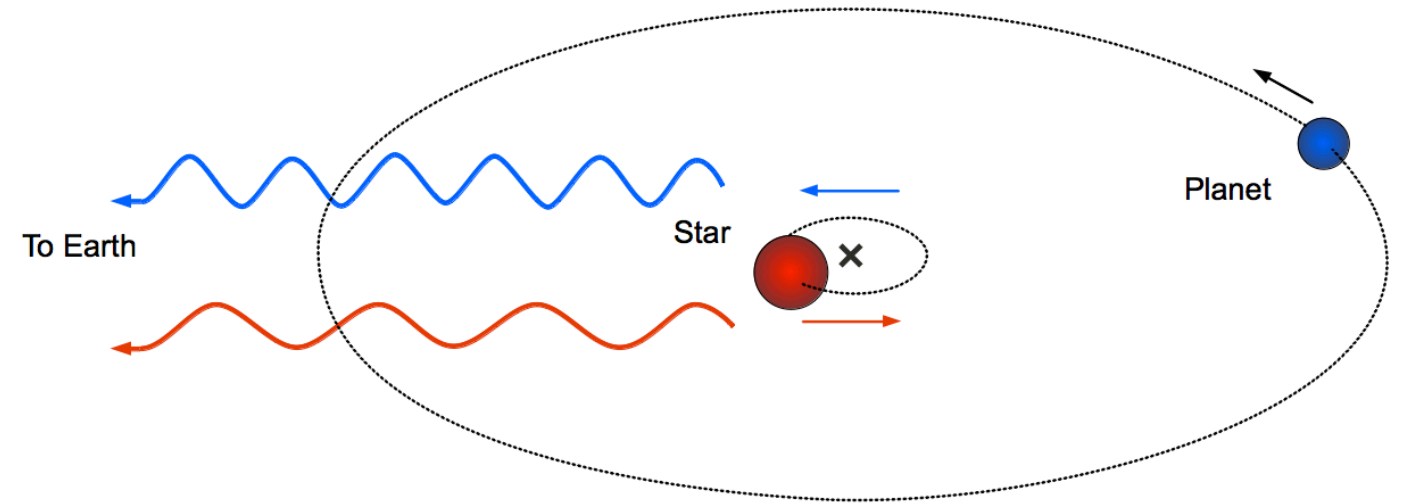
How can efficient sampling help to determine the composition of exoplanets?

- ▶ Detection of exoplanets
- ▶ Creative sampling
- ▶ How to evaluate the efficiency

EXOPLANET DETECTION

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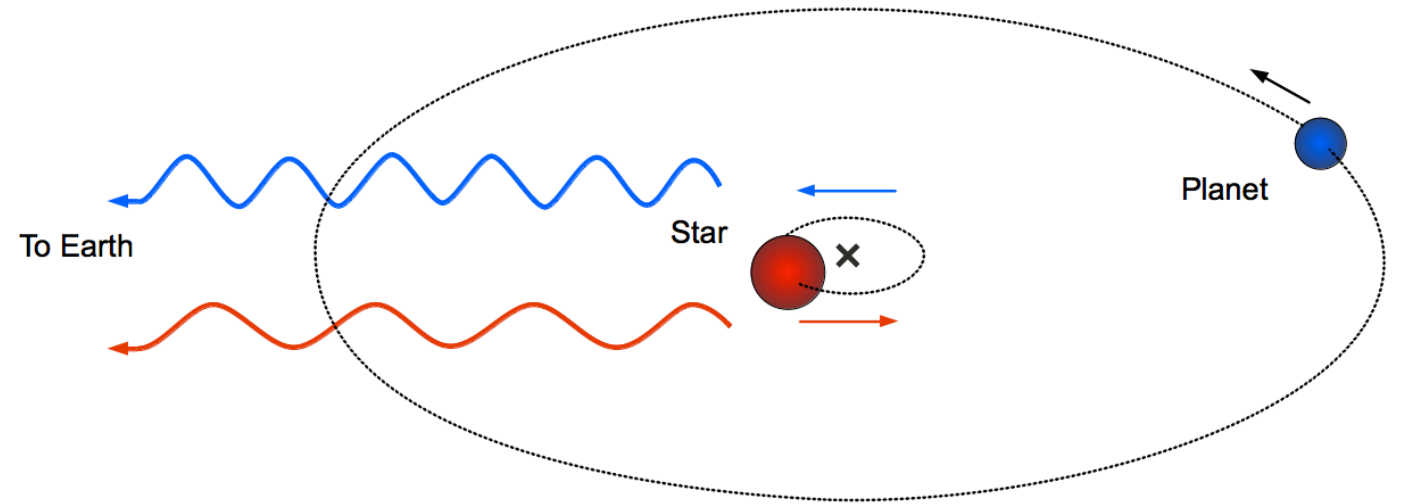
- ▶ Radial velocity \rightarrow mass



EXOPLANET DETECTION

- ▶ Radial velocity → mass
- ▶ Transit → radius

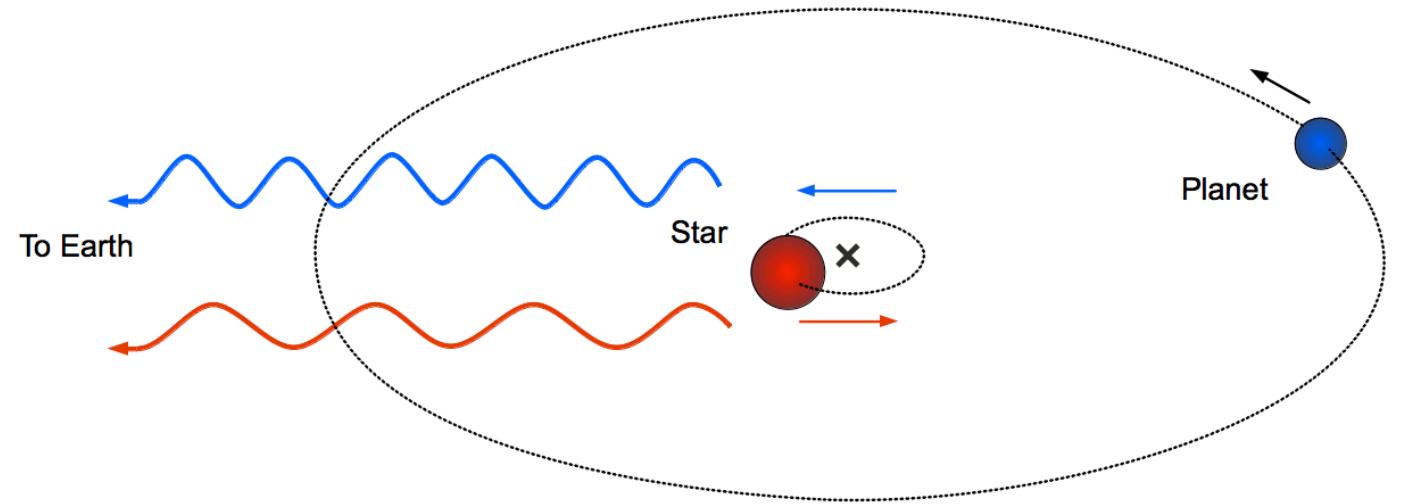
Often not combinable



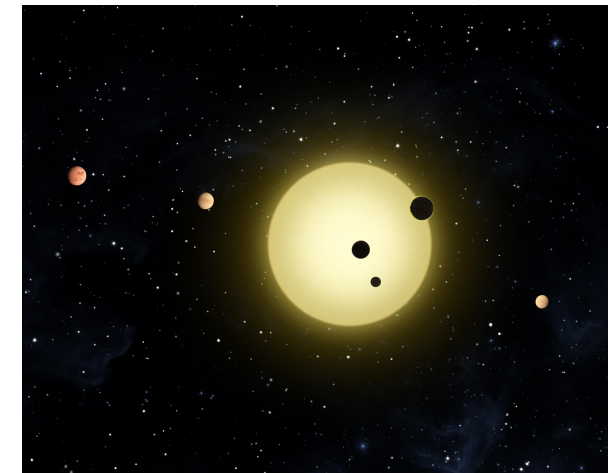
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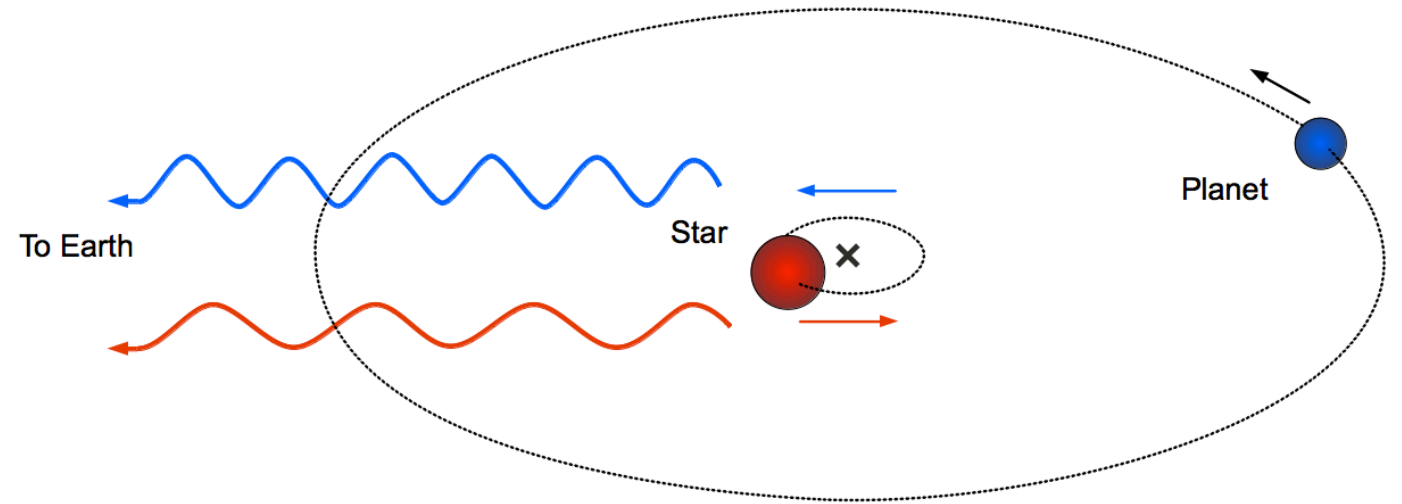
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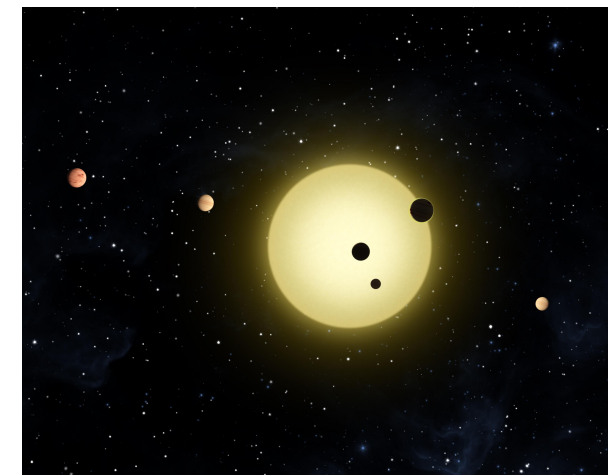


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Planetary properties



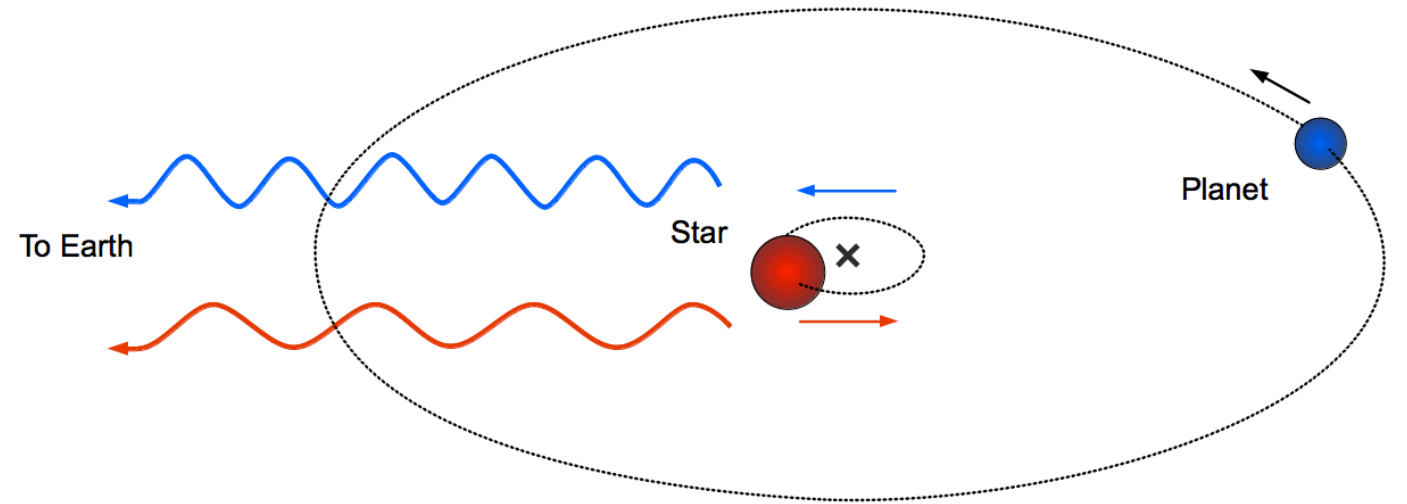
TTV



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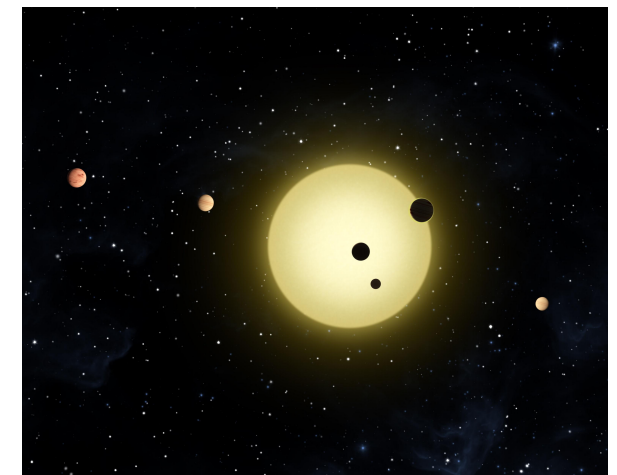
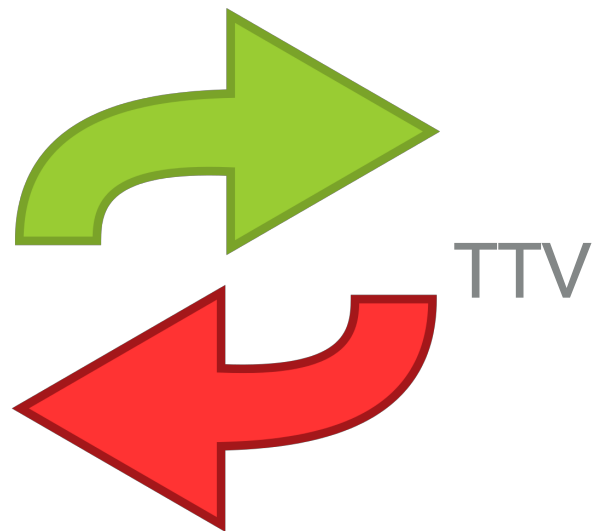
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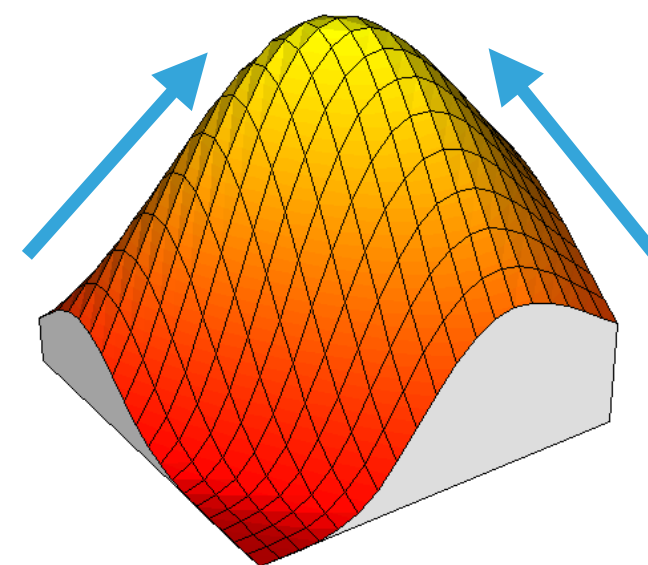
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CREATIVE SAMPLER METHODS

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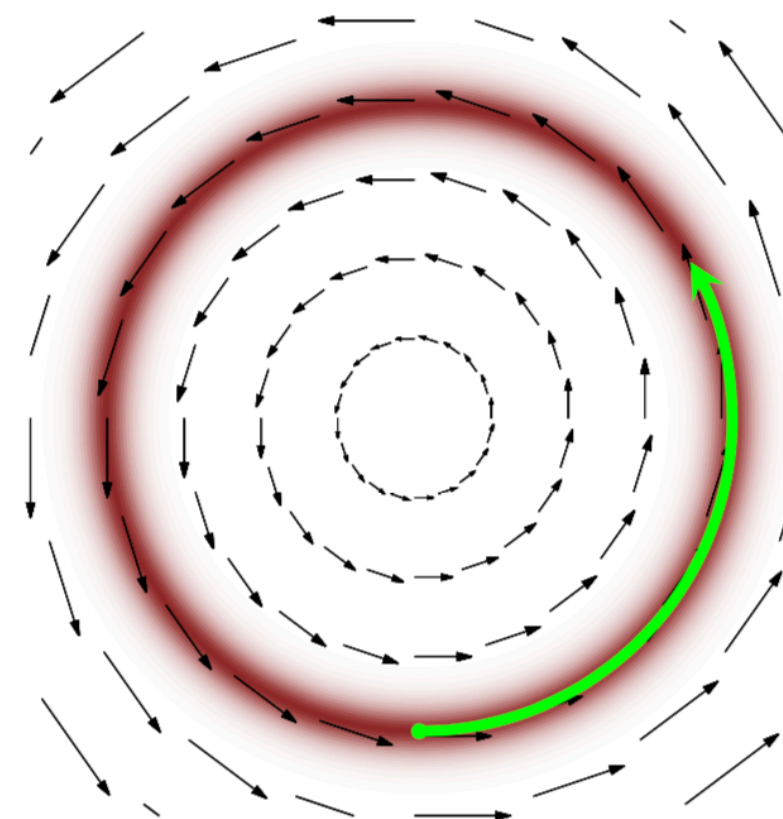
- ▶ **MALA:** Uses the gradient of posterior distribution
- ▶ **DEMCMC** and **AIMCMC:** Walkers communicate
- ▶ **SMMALA** and **GAMC:** Uses the Hessian
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Sampler should explore the *typical set*:
the band around the mode in which
almost all random draws fall

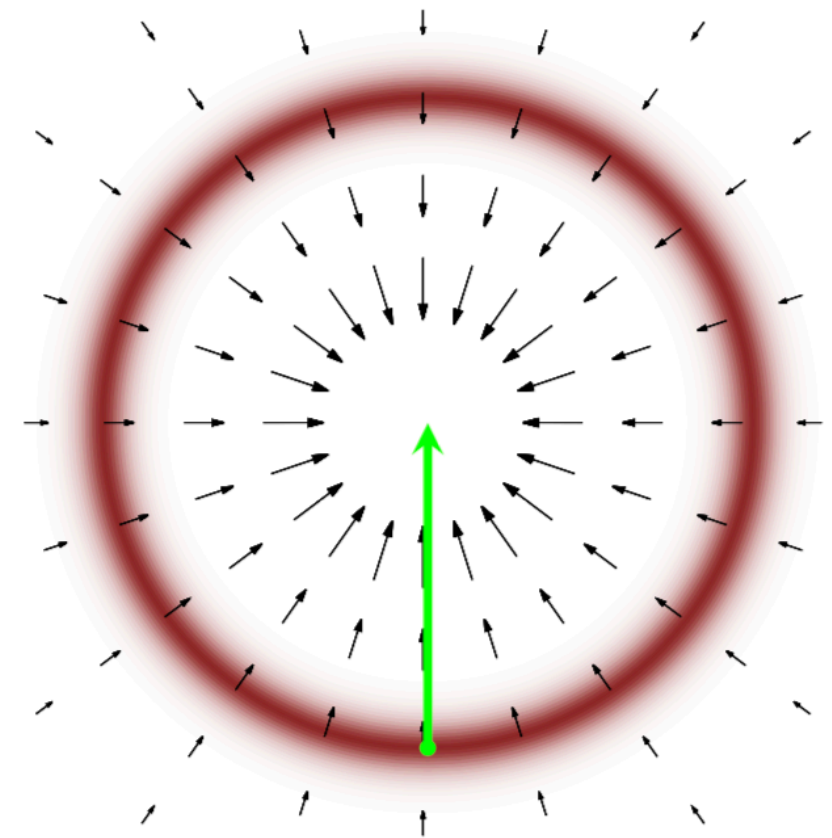


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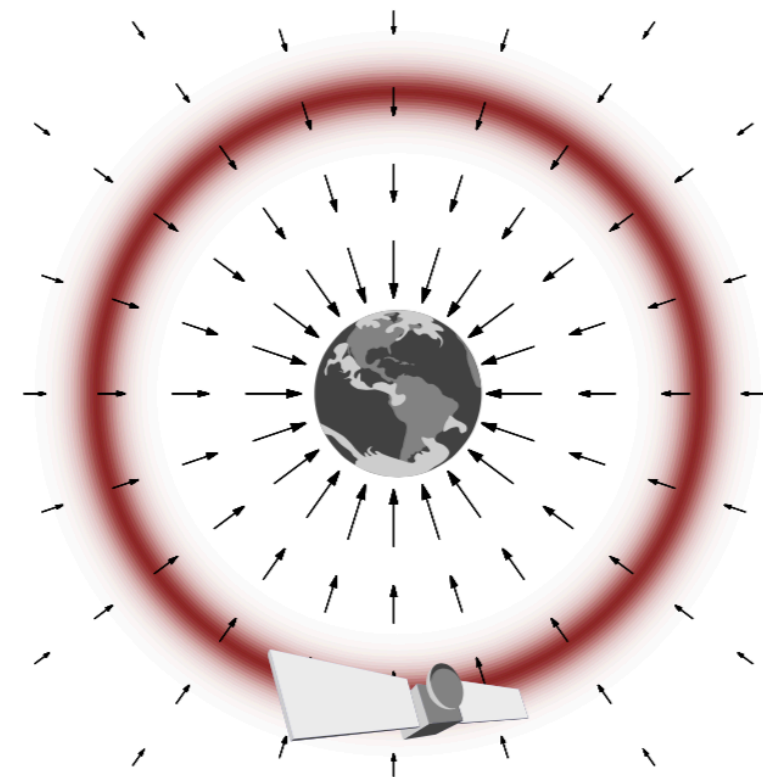
However, the gradient is always directed inwards



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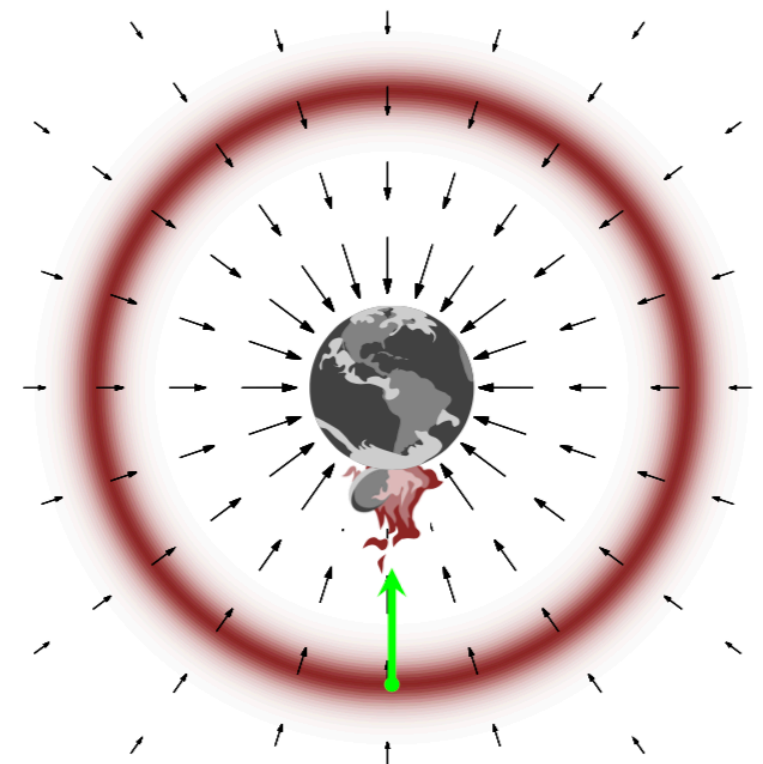
Physical analogy: planet orbiting a star



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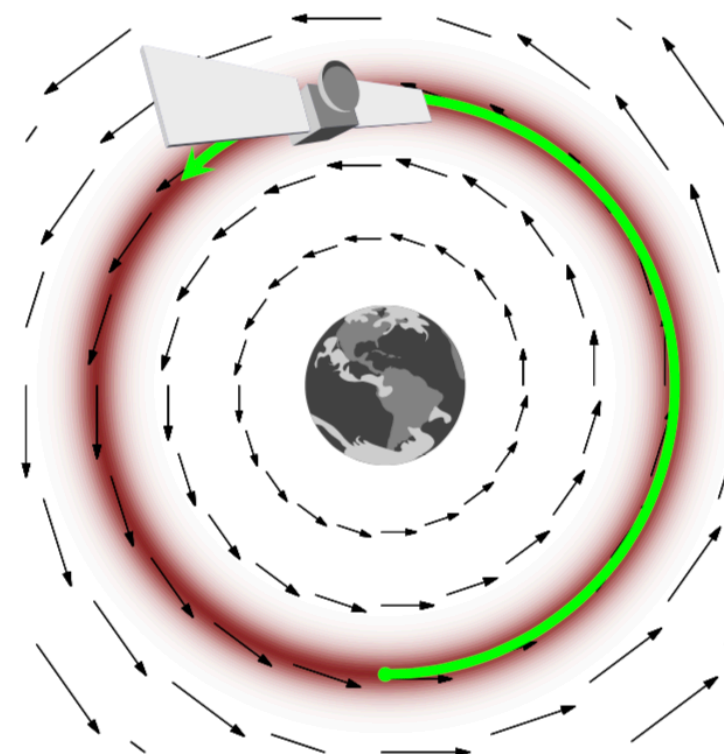
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Physical analogy: planet orbiting a star

Need *momentum* to maintain a stable orbit.

HMC: introduce auxiliary momentum variable to system.

$$\mathcal{H} = T + V$$



SIMULATED DATA SETS

- ▶ Different TTV models: Simple Sinusoidal & TTVFaster

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- ▶ Kepler-307

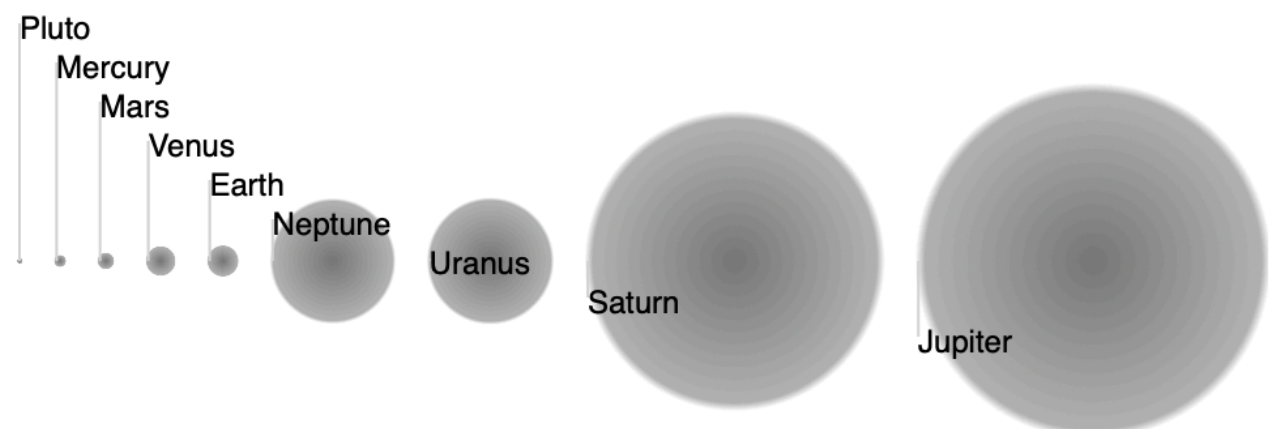
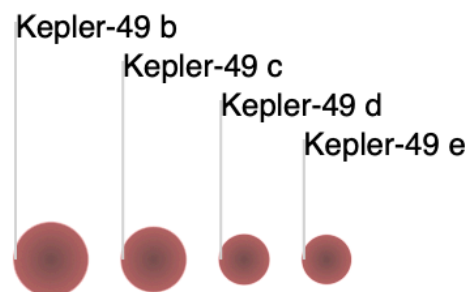
Well understood system

- ▶ Kepler-49

Two additional outer planets

- ▶ Kepler-57

Bimodality in posterior distribution

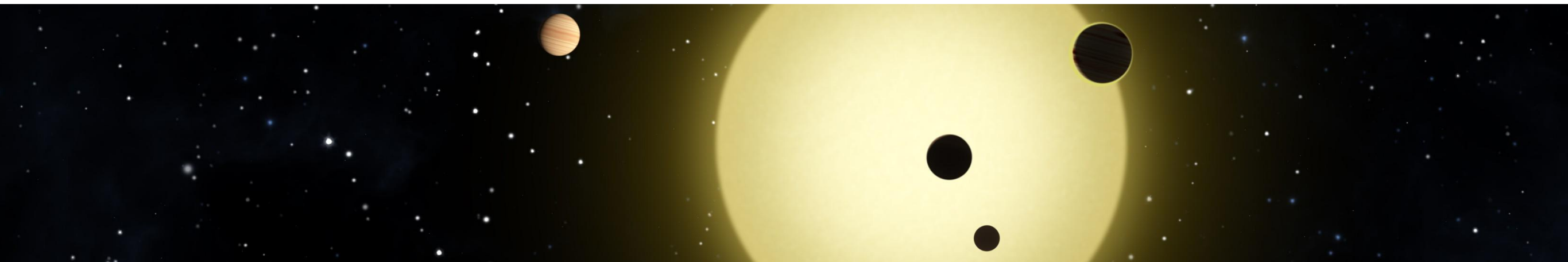


HOW TO DETERMINE THE EFFICIENCY

- ▶ Each of the samplers was first burned-in
- ▶ Then, they were ran for **10,000** iterations
- ▶ The **Effective Sample Size / total elapsed time** was evaluated
Effective Sample Size: **number of effectively independent draws from the posterior distribution.**
- ▶ The best sampler was run for 2 million iterations to compare the final results with the true parameters of the model

RESULTS

- ▶ Kepler-307 HMC
- ▶ Kepler-49 GAMC
- ▶ Kepler-57 GAMC & DEMCMC

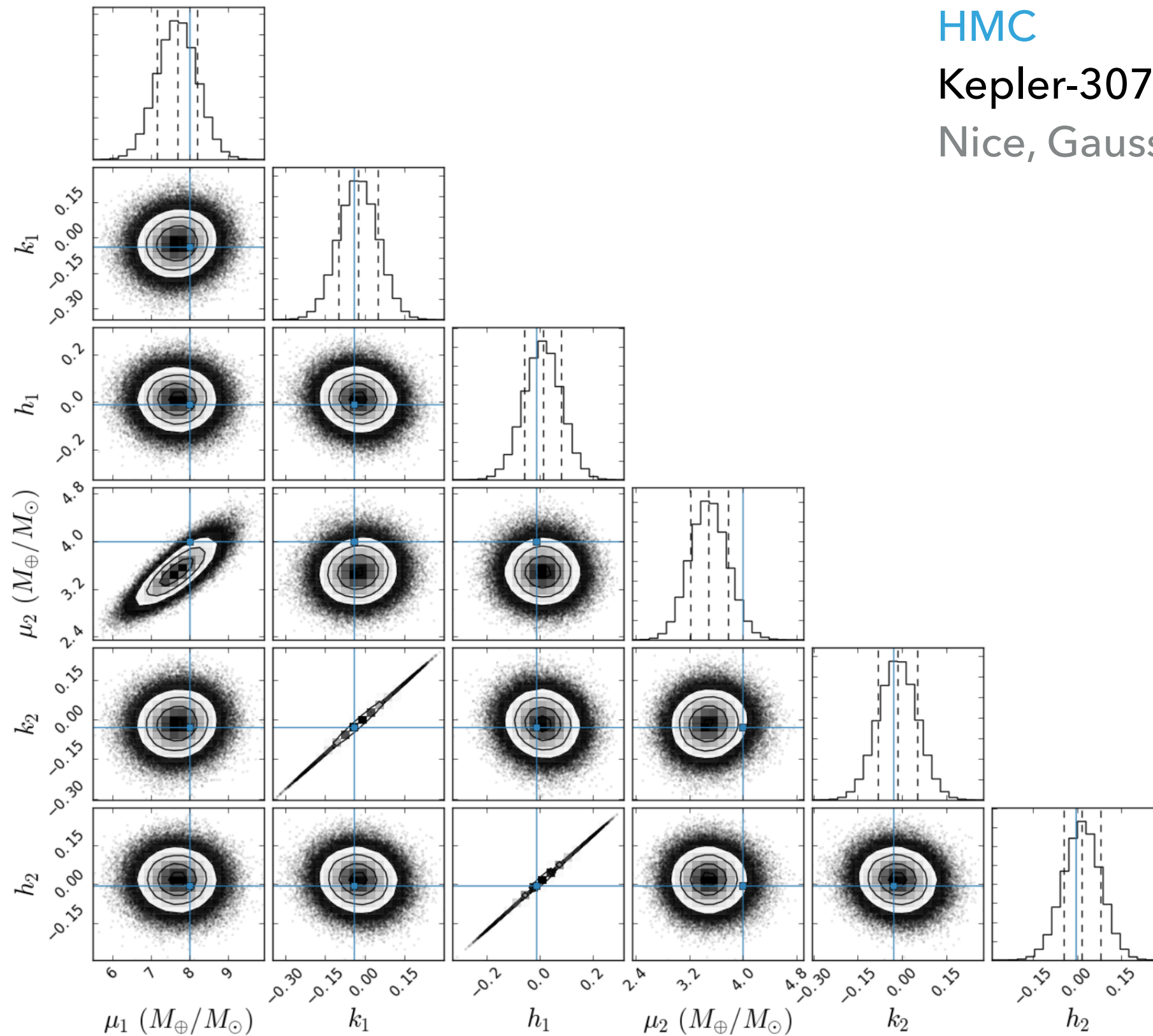


RESULTS

HMC

Kepler-307 system

Nice, Gaussian posteriors



CONCLUSIONS

- ▶ Different samplers for different scenarios
- ▶ HMC very suitable if posterior is near Gaussian
- ▶ GAMC and DEMCMC performed continuously alright
- ▶ Future research: investigate samplers performance on burn-in and with a more complicated TTV model

