# Optimization by Simulated Annealing

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#### Inspiration – Annealing and Statistical mechanics

- Slowly decreasing temperature
  → Stronger alloy
- Minimize a cost function by slowly reducing the 'temperature'
- Proposed in "Optimization by Simulated Annealing", 1983



Annealing http://www.turingfinance.com/simulated-annealing-for-portfolio-optimization/

#### **Statistical Mechanics**

Anti-aligned spins  $\rightarrow$  higher energy

Probability of finding system in certain state is proportional to its Boltzmann factor

$$P_i = \frac{1}{Z} \exp\left(-\frac{E_i}{k_{\rm B}T}\right)$$



Magnetization in a metal https://www.youtube.com/watch?v=PWUTBnvGegg

## The Algorithm

Define cost and temperature functions *E* and *T* 

Propose a candidate change to system

Accept change with probability

$$P_{\text{transition}} = \begin{cases} 1, & \text{if } E_{\text{new}} < E_{\text{old}} \\ \exp(-\Delta E/T), & \text{if } E_{\text{new}} \ge E_{\text{old}} \end{cases}$$

Lower temperature and repeat



From: https://en.wikipedia.org/wiki/Simulated\_annealing

## Applications

Placement of chips on module

Minimize signal propagation time without creating too congested regions

Swapping two chips

Constraints on distance for low temperatures

Initial position T											T = 100	00									
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613	31	3.2.	33			36		38	39	40	1175	25	33	15	83	33			76	44	
824	41	42	43		***	***	<u>47</u>	48	49	50	13/0	48	57	56	84	79	88	13	96	50	87
826	51	52	53		55	56	57	事業	59	6.0	1388	6	39	90	97	94	41	58	89	27	***
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From: Optimization by Simulated Annealing, S. Kirkpatrick, C. D. Gelatt, M. P. Vecchi

### **Travelling Salesman**

Given a list of cities, what is the shortest route that visits each city once before returning home?

Brute force solution has time complexity  $\mathcal{O}(n!)$ 



From: http://examples.gurobi.com/traveling-salesman-problem/

#### **Travelling Salesman**



## **Travelling Salesman**

Initial random route

60 cities randomly distributed

2500 temperatures

60 swaps at each temperature

Near optimal final route

