The persistent cosmic web and its filamentary structure- Theory, implementation (I) and illustrations (II)

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The persistent cosmic web and its filamentary structure – I. Theory and implementation

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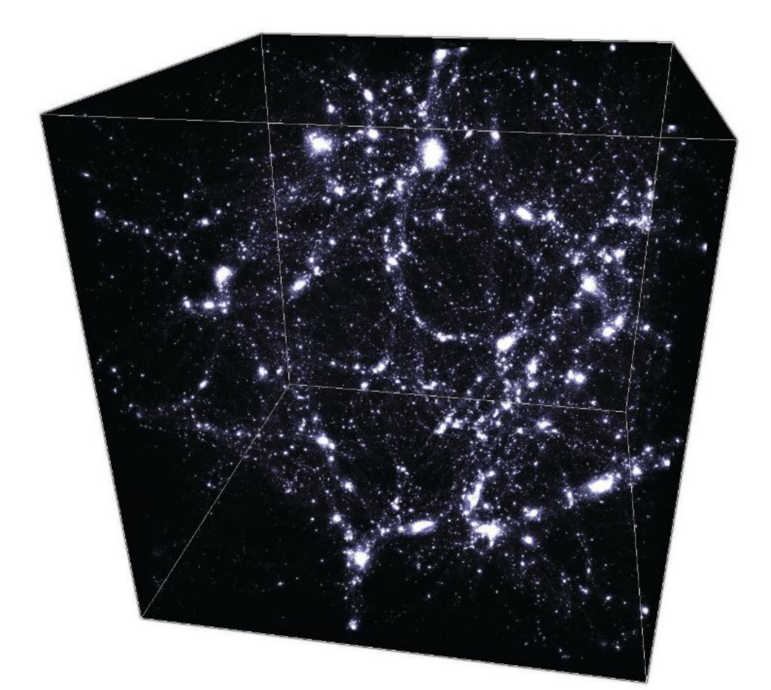
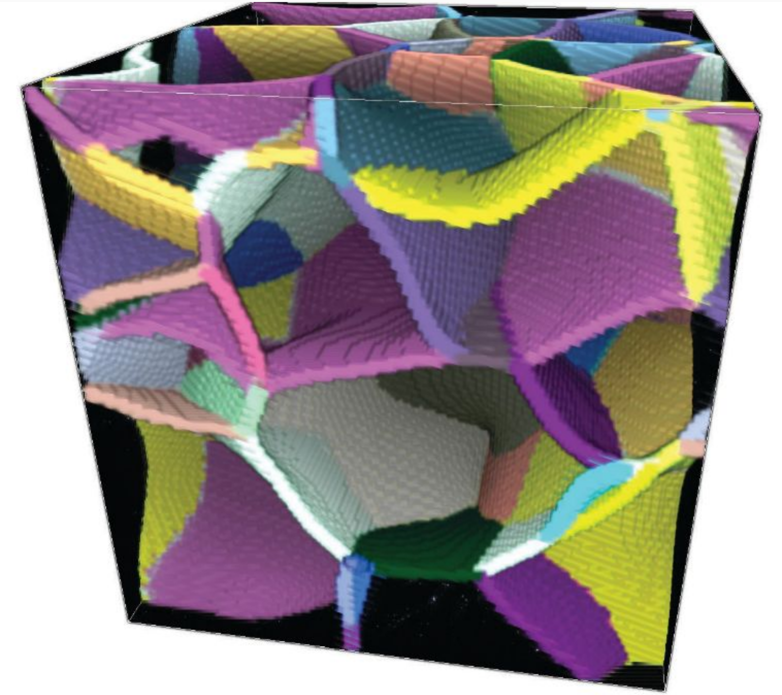
ABSTRACT

We present DisPerSE, a novel approach to the coherent multiscale identification of all types of astrophysical structures, in particular the filaments, in the large-scale distribution of the matter in the Universe. This method and the corresponding piece of software allows for a genuinely scale-free and parameter-free identification of the voids, walls, filaments, clusters and their

DisPerSe: Discrete Persistent Structure Extractor





Morse theory

- Topological vs Geometrical Properties
- Morse functions:

$$f : I\!R^d \mapsto I\!R$$

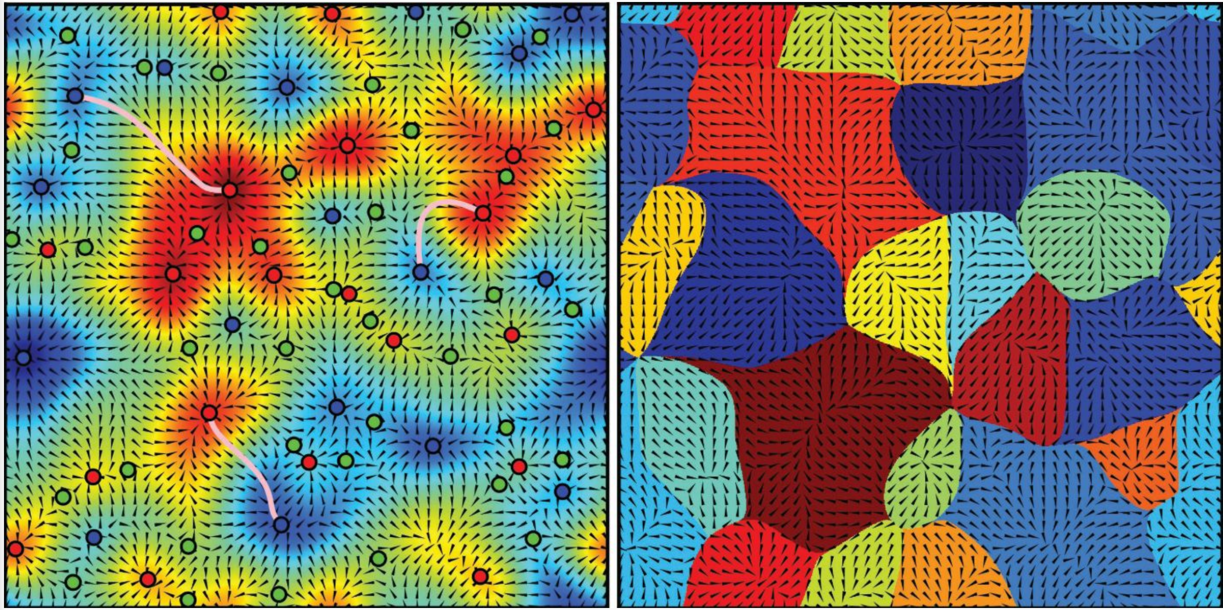
$$\nabla_x f(p) = 0\,,\ \det \mathcal{H}_f(p) \neq 0$$

- Integral/Field lines:

$$L(t) \in I\!R^d\ ,\quad \frac{dL(t)}{dt} = \nabla_x f$$

Manifolds and the Morse Complex

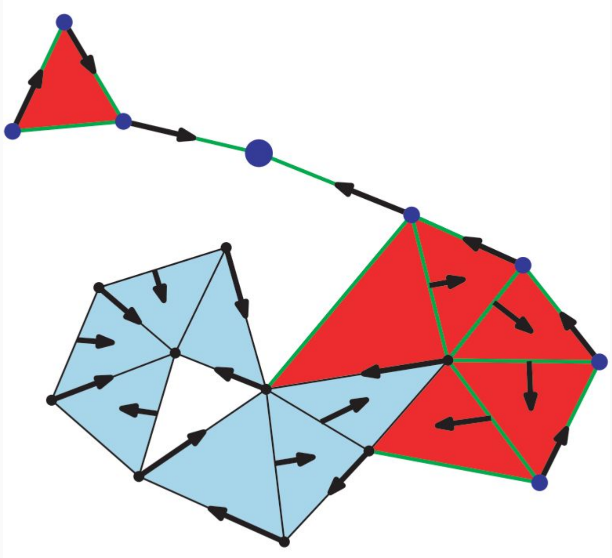
- Ascending and Descending manifolds
- Order of the manifolds defined by the critical point being considered.
- Set of all manifolds: the Morse complex



Simplices, simplicial complexes and the discrete gradient

- *k*-simplex:

$$\sigma_k : \mathrm{Conv}(S)\ , S = \{p_0, \ldots, p_k\}$$



- Facets and co-facets.
- Simplicial Complexes.
- Discrete gradient (pairs):

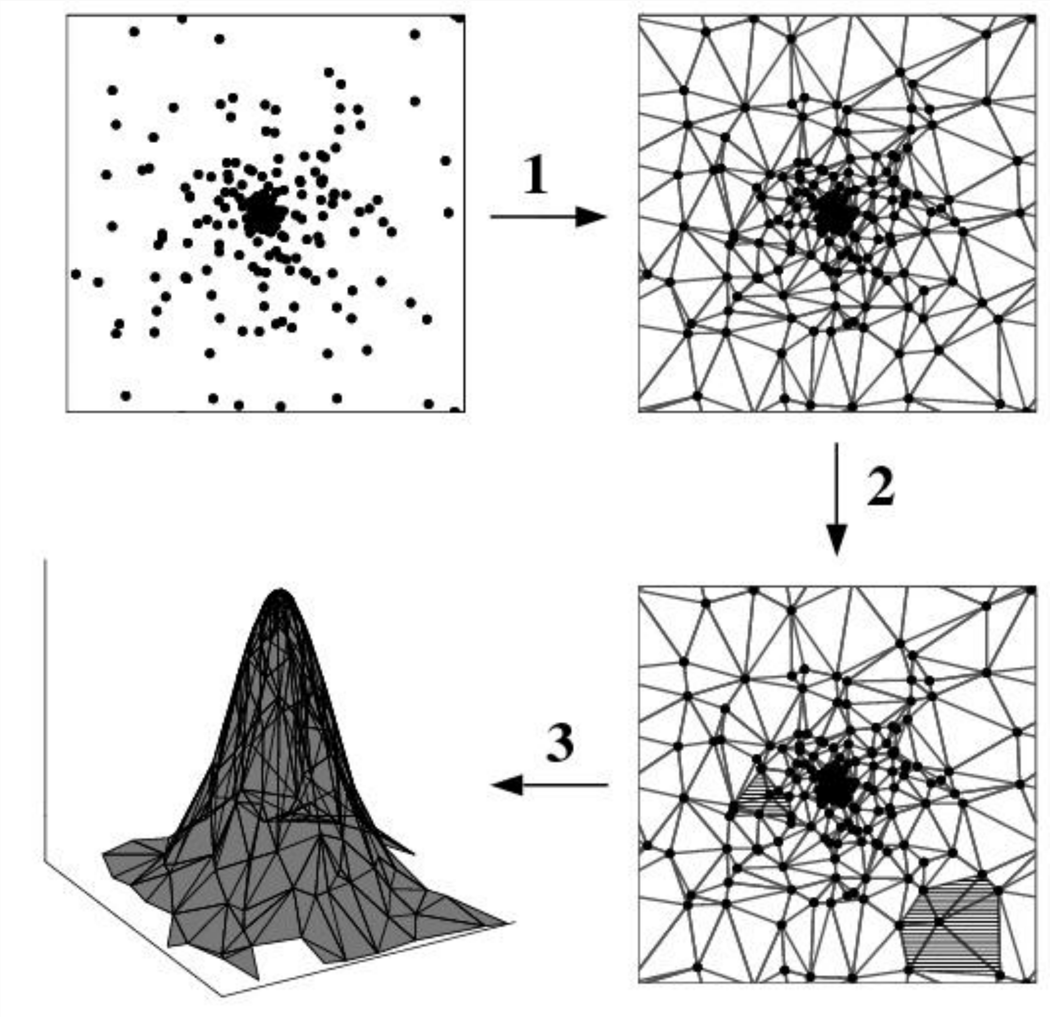
$$[\sigma_k, \alpha_{k+1}], [\sigma_k, \beta_{k-1}]$$

Discrete Morse theory

Smooth Morse theory	Discrete Morse theory
• Manifolds and points	• Simplicial Complexes and Simplices
• Gradient and Critical points	• Discrete gradient and critical simplices
• Morse functions	• Discrete Morse functions
• Integral/Field lines	• V-paths
• *k*-ascending/descending manifolds and the Morse Complex	• Discrete *k*-ascending/descending manifolds and the Discrete Morse complex (DMC)

Delaunay Tessellation Field Estimator (DTFE)

A mathematical tool for reconstructing a volume-covering and continuous density or intensity field from a discrete point set.



Topological Persistence

- Excursion Set: Set of points which satisfies,

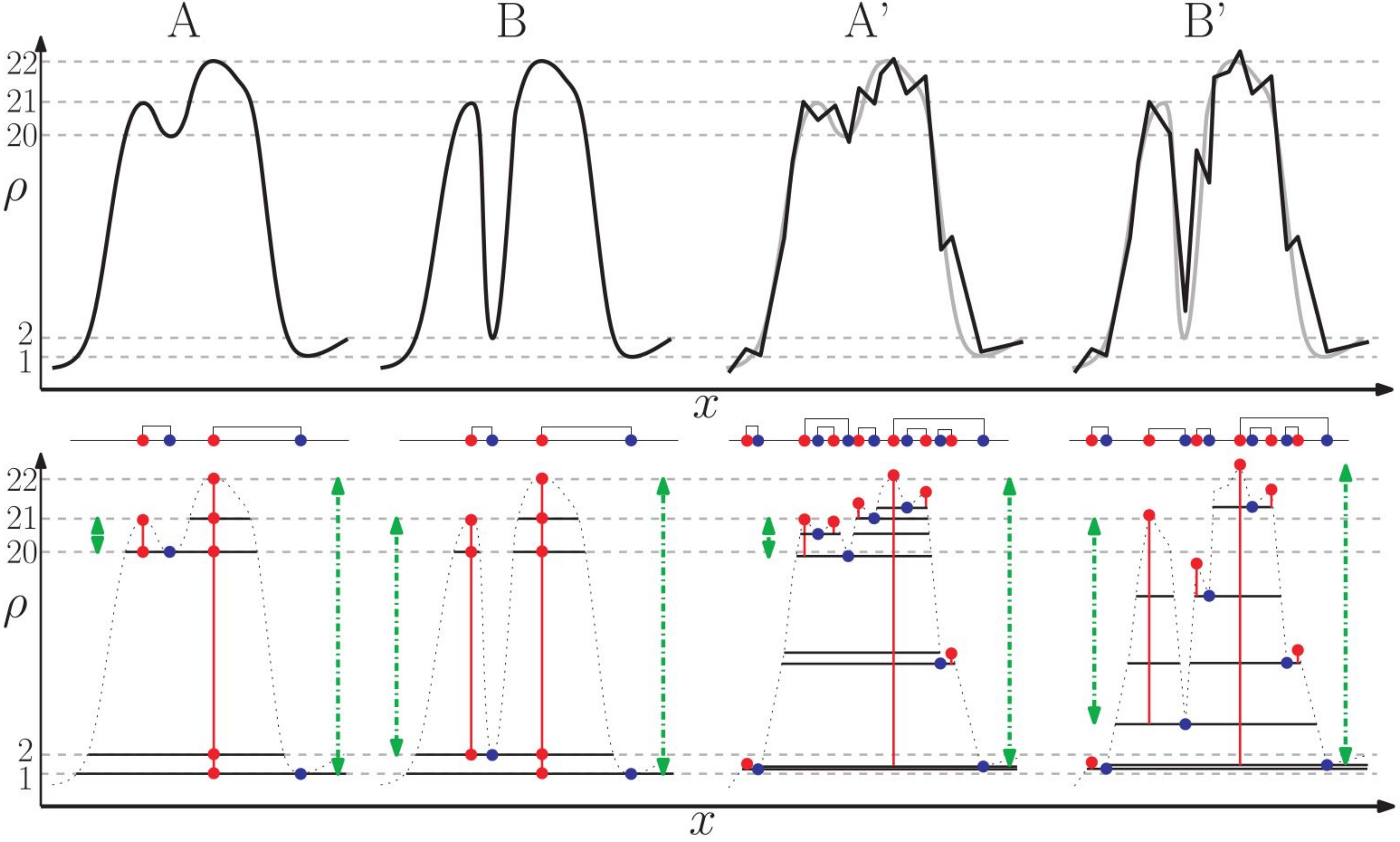
$$(x_1, \ldots, x_n) | \rho(x_1, \ldots, x_n) \geq \rho_0$$

- Filtration: Sequence of N+1 subcomplexes such that,

$$\emptyset = K^0 \subseteq K^1 \subseteq \ldots \subseteq K^N = K,$$
$$K^{i+1} = K^i \cup \delta^i$$

- Persistence:

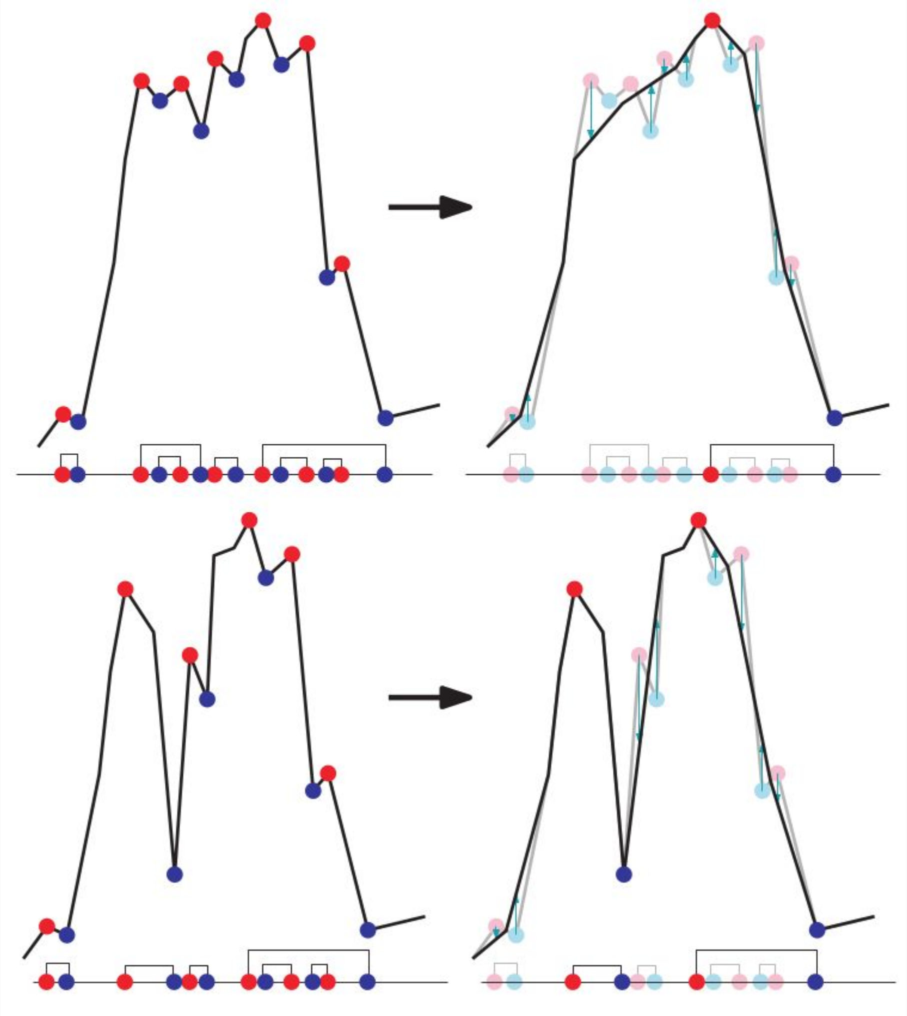
$$\text{Smooth} : \rho(P_a) - \rho(P_b) \ , \ \text{Discrete} : \rho_D(\sigma_a) - \rho_D(\sigma_b)$$



A
B
A'
B'
22
21
20
ρ
2
1
x
22
21
20
ρ
2
1
x

Topological Simplification

- Low persistence means short-lived.
- Key idea: low persistence features are possibly noise.
- Eliminate these pairs to remove spurious detection.



Filtering Poisson noise

- Persistence ratio :

$$r\left(q_k\right) = \rho_{\mathrm{D}}\left(\sigma_{k+1}\right) / \rho_{\mathrm{D}}\left(\sigma_k\right).$$

- Significance of a persistence pair:

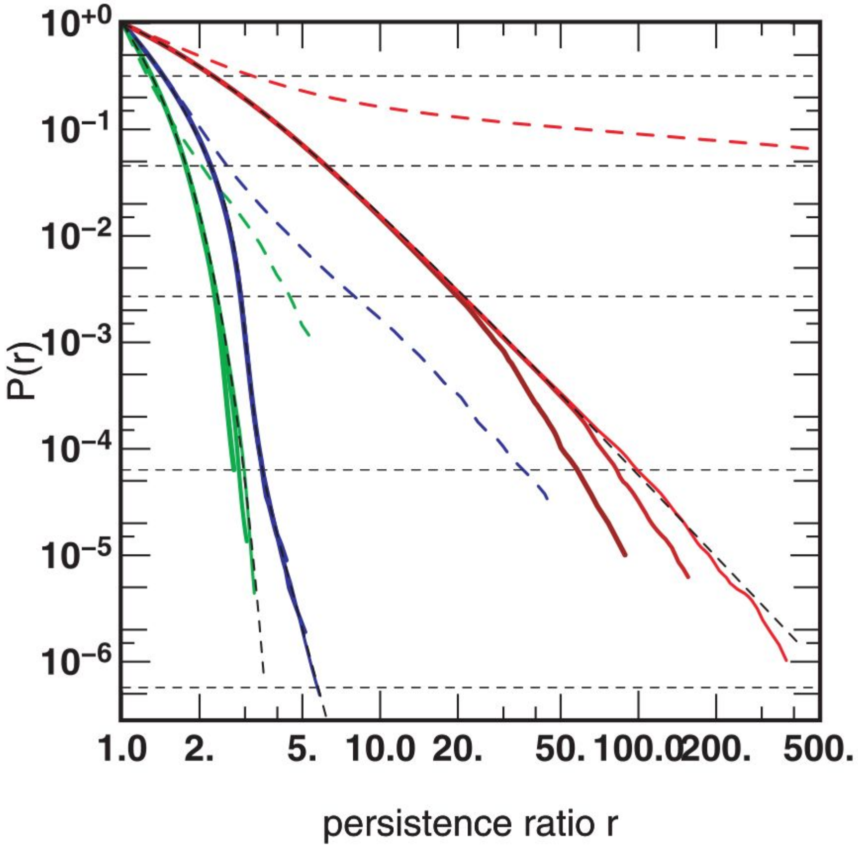
$$S\left(q_k\right) = S_k\left(r\left(q_k\right)\right) = \mathrm{Erf}^{-1}\left(\frac{P_k\left(r\left(q_k\right)\right) + 1}{2}\right)$$

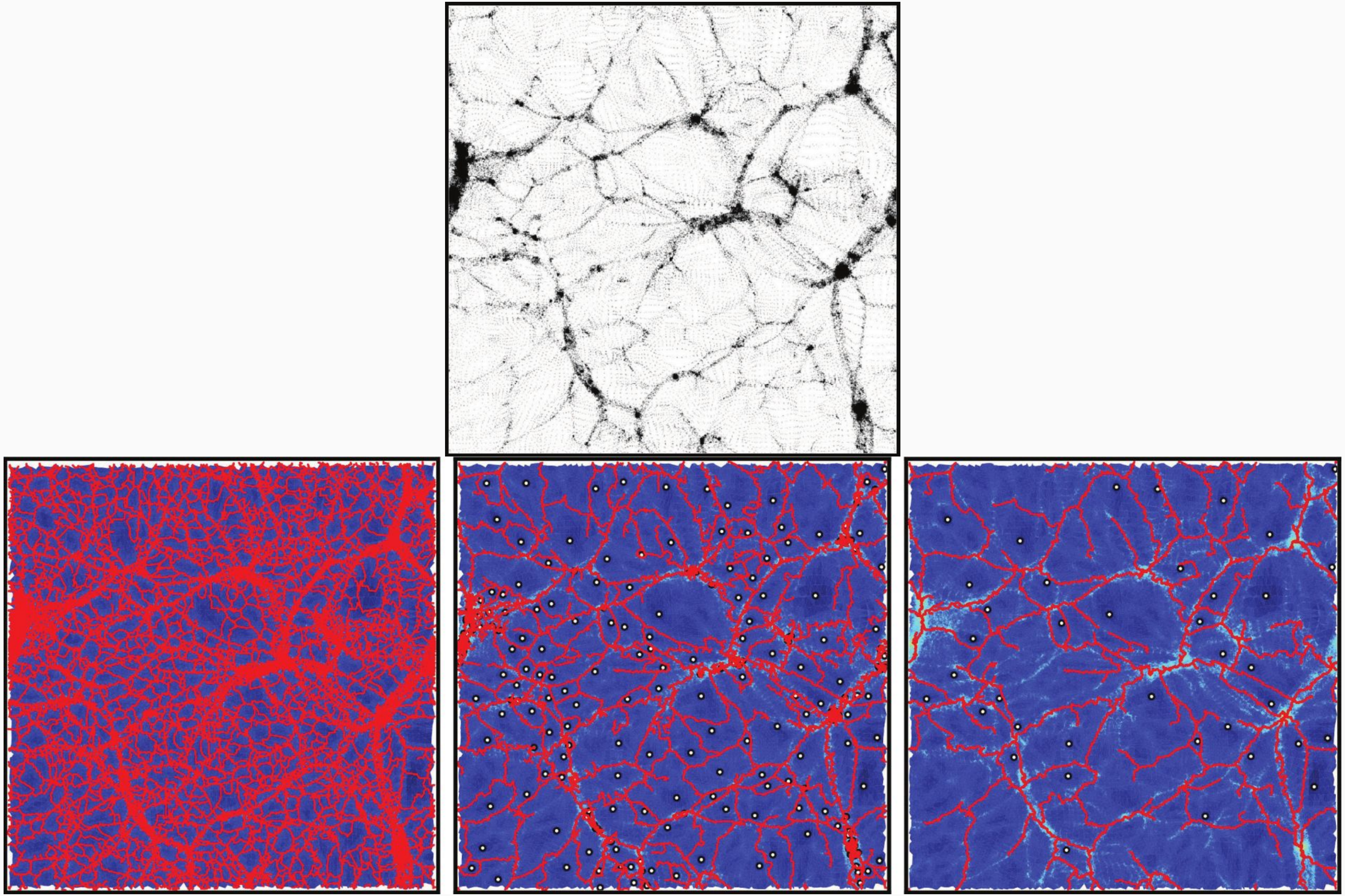
- Bias towards higher densities and the nature of DTFE: sparser regions have bigger Voronoi cells; minima are consequently rarer.

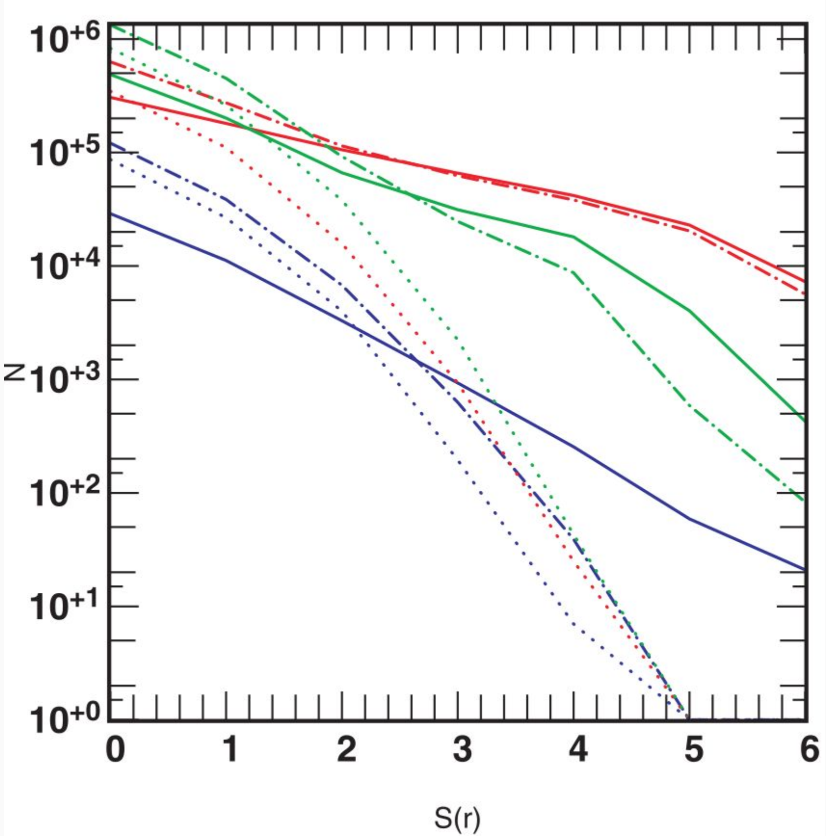
$$P_0(r) = \exp[-\alpha_0(r-1) - \alpha_1(r-1)^{\alpha_2}]$$
$$\text{with}\,\alpha \approx [3.694, 0.441, 2.538],$$

$$P_1(r) = f_1(1-t) + f_2 t$$
$$\text{with}\ f_1 = \exp[-\beta_0(r-1)], \quad f_2 = \beta_1 r^{-\beta_2},$$
$$t = (1 + \beta_3/u^{\beta_4})^{-1},$$
$$\beta \approx [2.554, 4.000, 9.000, 1.785, 14.000],$$

$$P_2(r) = [1 + \gamma_0(r-1)]^{-\gamma_1}$$
$$\text{with}\,\gamma \approx [0.449, 2.563],$$







- Choosing the appropriate significance level: why not as high as possible?
- Comparing 3 scenarios: undersampled simulation, undersampled simulation + noise and pure noise.
- 0-order persistence pairs as the special case.

Persistence based detection of structures using Discrete morse theory is a very successful technique. However, special care needs to be taken when eliminating low-persistence minima because of the scale-free nature of the DTFE.

References:

1. Sousbie T., *The persistent cosmic web and its filamentary structure- I: Theory and Implementations*, 2011, MNRAS, in press (doi:10.1111/j.1365- 2966.2011.18394.x)
2. Sousbie T., Pichon C. and Kawahara H., *The persistent cosmic web and its filamentary structure- II. Illustrations*, 2011, MNRAS, in press (doi: 10.1111/j.13652966.2011.18395.x)

Thanks!