Parameter Estimation of Hidden Markov Models

Segmentation K-means algorithm

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The Article

The Segmental *K*-Means Algorithm for Estimating Parameters of Hidden Markov Models

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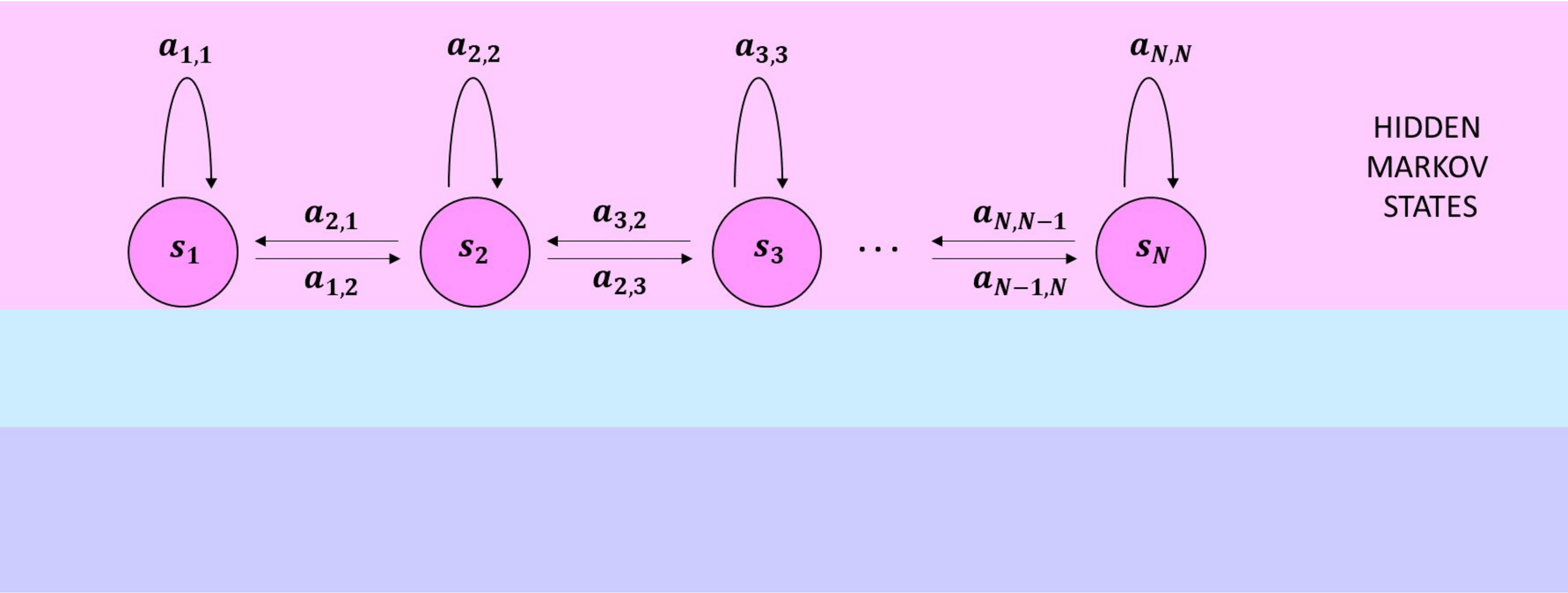
Abstract—Statistical analysis techniques using hidden Markov models have found widespread use in many problem areas. This correspondence discusses and documents a parameter estimation algorithm for data sequence modeling involving hidden Markov models. The algorithm which we call the segmental *K*-means method uses the state-optimized joint likelihood for the observation data and the underlying Markovian state sequence as the objective function for estimation. We prove the convergence of the algorithm and compare it with the traditional Baum–Welch reestimation method. We also point out the increased flexibility this algorithm offers in the general speech modeling framework.

Hidden Markov Model



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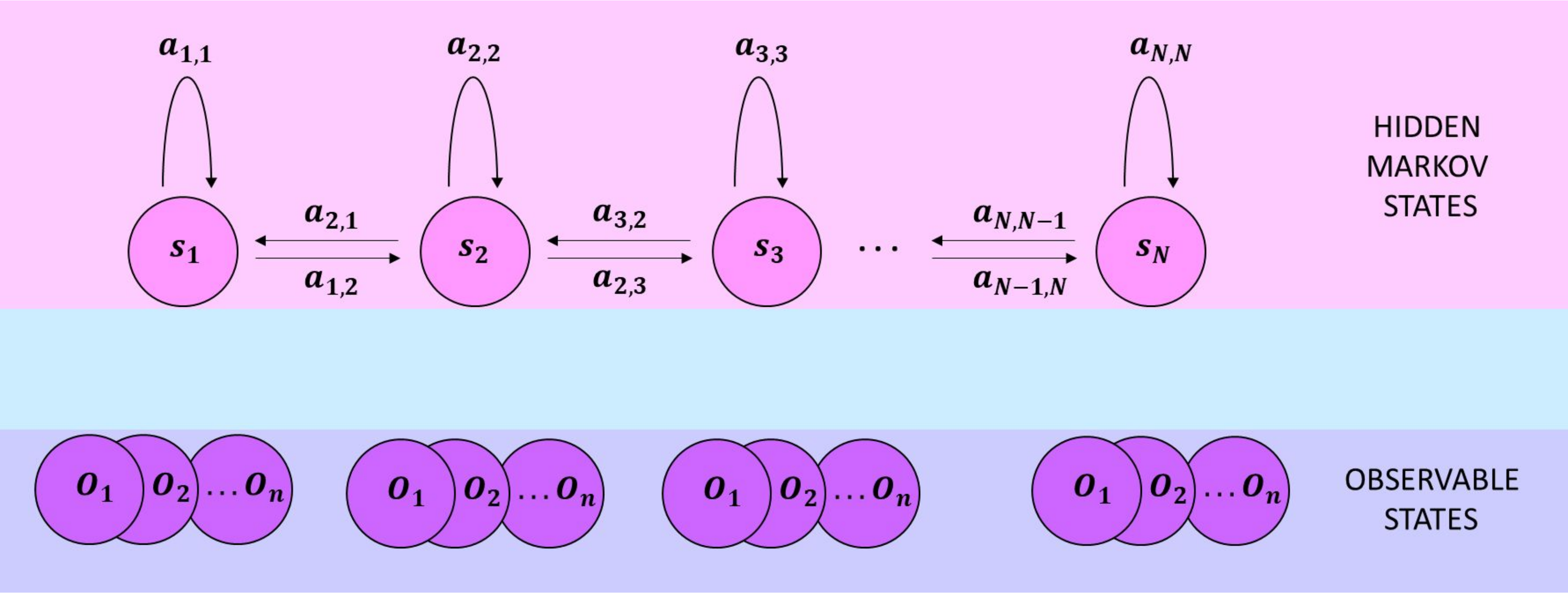

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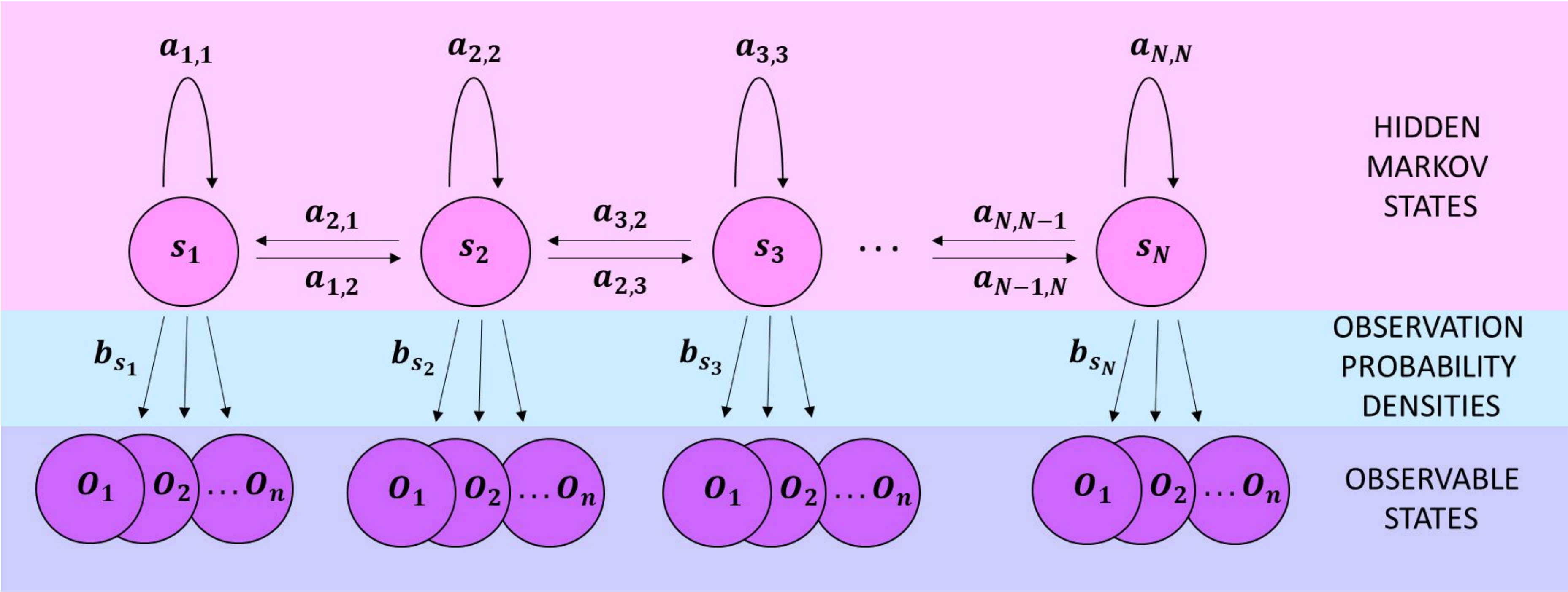

The Problem in a Nutshell

- Structure of the hidden markov model is known.
- How do we estimate the parameters of the model based on the observed sequence?

Estimating Parameters - The Usual Method (Baum-Welch)

$$P(s|A,\pi) = \pi_{s_0} \prod_{t=1}^{T} a_{(s_{(t-1)},s_t)}$$

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May encounter numerical difficulties

Estimating Parameters - “New” Method

$$f(x|\lambda) = \sum_{s} \pi_{s_0} \prod_{t=1}^{T} a_{(s_{(t-1)}, s_t)} b_{s_t}(x_t)$$

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Encounters problems with similar likelihoods

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Algorithm in three steps:

1. Initialization of parameters in λ

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$$\bar{\lambda} = \arg\max_{\lambda}\{\max_{s}[f(x, s|\lambda)]\}$$

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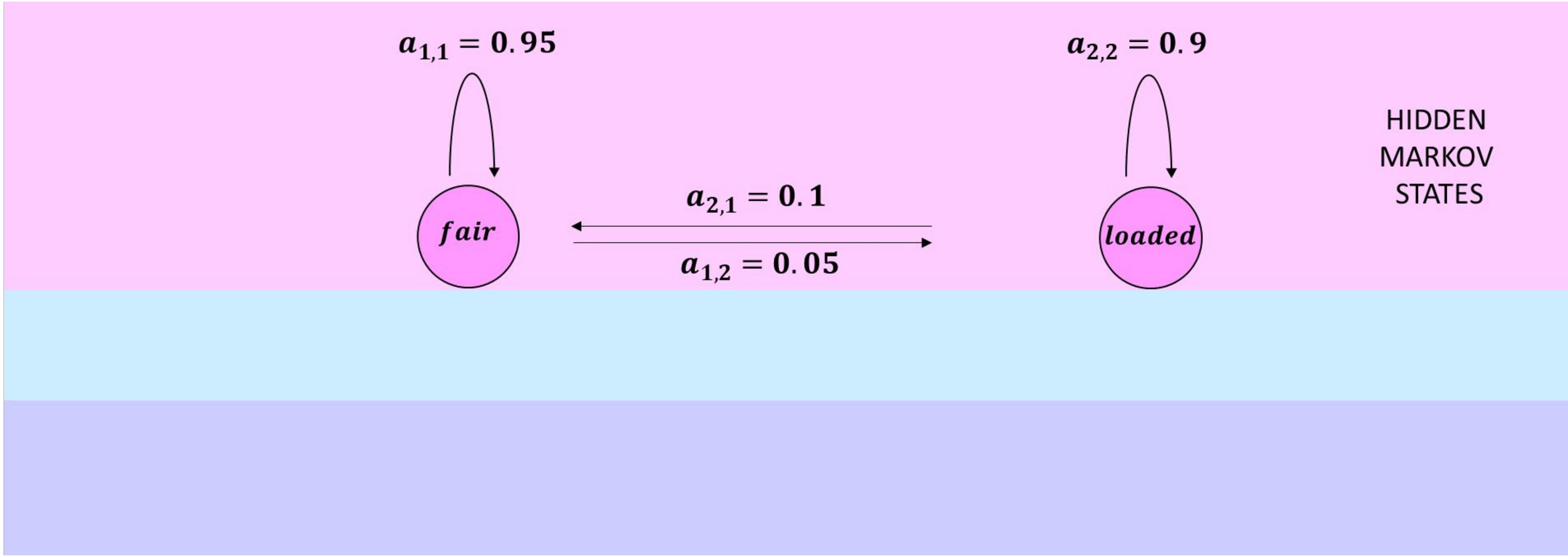
Example calculation: The Occasionally Dishonest Casino



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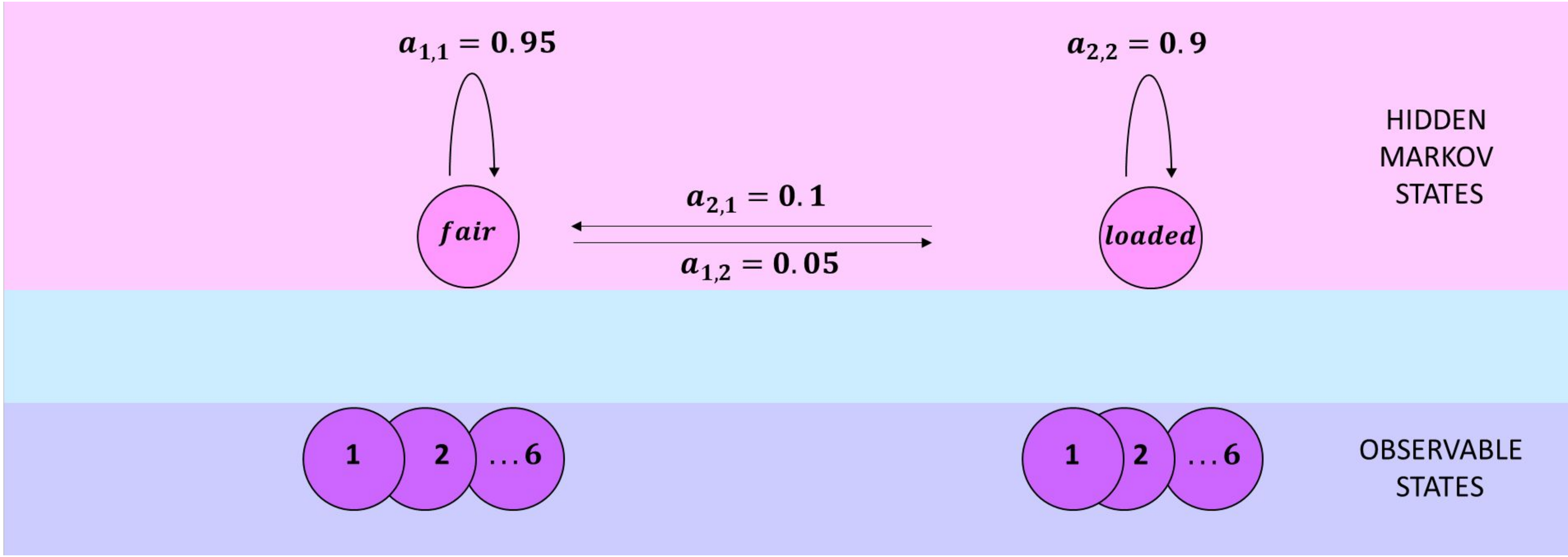

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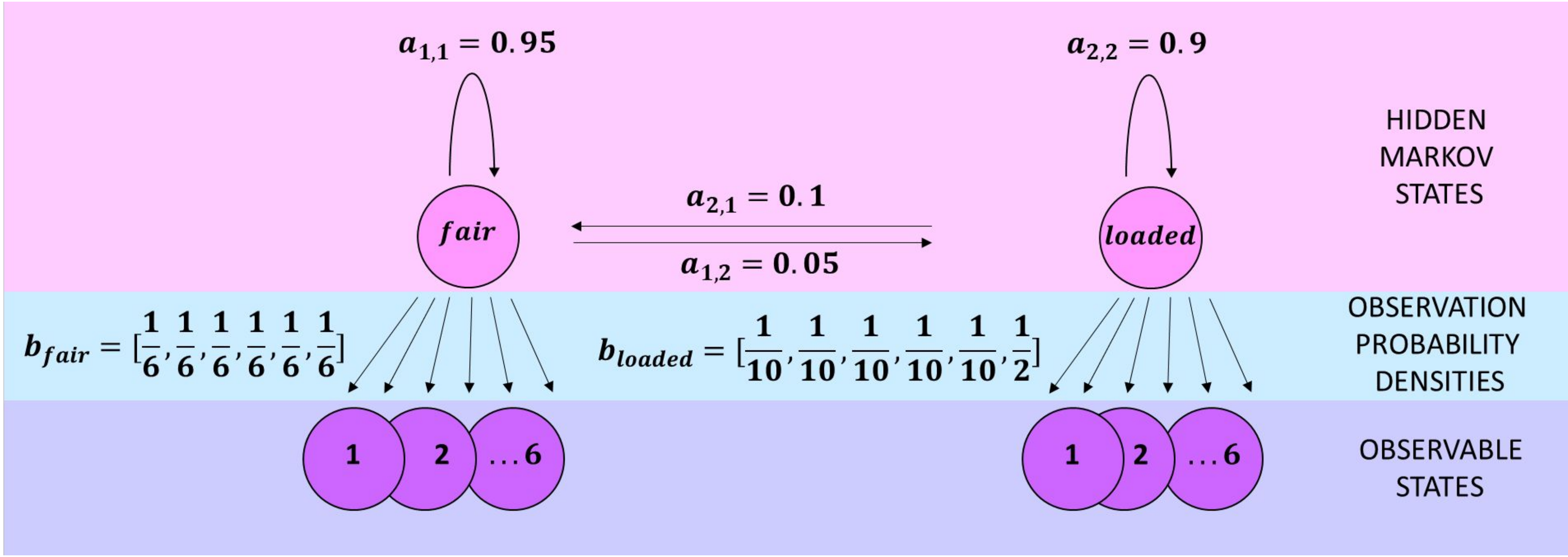

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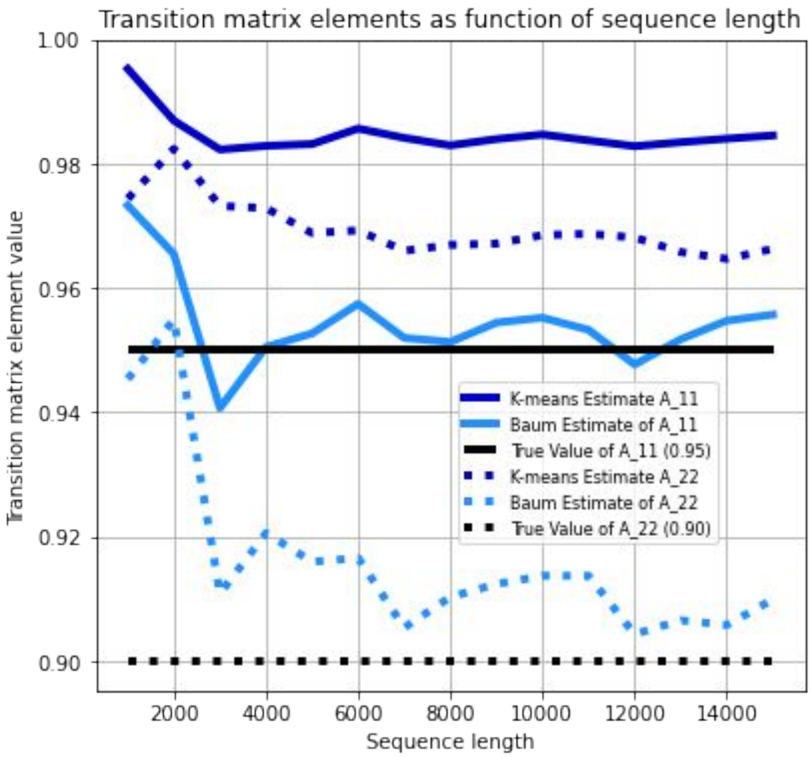

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