8 C 1

Nuclear Physics B46 (1972) 319–331 North-Holland Publishing Company

STATIC QUANTITIES IN WEINBERG'S MODEL OF WEAK AND ELECTROMAGNETIC INTERACTIONS

W A BARDEEN *, R GASTMANS ** and B LAUTRUP

CERN, Geneva

Received 17 May 1972

Abstract Within Weinberg's model of weak and electromagnetic interactions, we calculate the static quantities of the charged intermediate bosons We also prove that the neutrino charge remains zero in second order, and discuss its charge radius Finally, an unambiguous calculation of the muon $g-2$ is presented All calculations are done using the n dimensional regularization procedure of 't Hooft and Veltman Our results support the claim that Weinberg's model is renormalizable

1 INTRODUCTION

Quite some time ago, Weinberg [1] unified weak and electromagnetic interactions by proposing a spontaneously broken gauge theory in which a triplet of gauge fields couples to electronic isospin and a singlet field to electronic hypercharge The Higgs-Kibble phenomenon [2] then produces the masses of the leptons and the bosons and the couplings among these particles

The work of 't Hooft [3] and Lee [4] revived interest in this model, as they were able to show that various similar models were renormalizable When Weinberg [5] finally claimed that his model would lead to a finite theory of electromagnetic and weak physical processes, it became clear that one would have to perform actual calculations of such processes to establish the validity of this conjecture

Besides the leptons, the photon, and the charged intermediate bosons, the model also introduces a neutral vector and a neutral scalar boson, plus a whole series of couplings among these particles As an unfortunate consequence of all this, the calculation of some experimentally relevant process, say μ decay to second order, requires the evaluation of about 20 Feynman diagrams, and before undertaking such a gigantic task, one would like to have some assurance that the result is likely to be finite

* Alfred P Sloan Research Fellow, on leave from the Department of Physics, Stanford University

** Aangesteld Navorser, N F W O , Belgium

For this reason, we looked at the static properties of the particles in the theory, as the claculation of such quantities is considerably simpler Among these are the anomalous magnetic moment of the charged vector boson, W, and its anomalous quadrupole moment Also, the self-charge of the neutrino and the muon magnetic moment were examined Only this last quantity has a certain experimental interest

One problem arising when doing such calculations is the question of how to treat originally divergent Feynman integrals by means of a suitable regularization procedure In the conventional ξ limiting procedure [6], one replaces the manifestly unitary vector boson propagator by

$$-i\,\frac{g_{\mu\nu}-\dfrac{k_\mu k_\nu(1-\xi)}{M^2-\xi k^2-i\epsilon}}{k^2-M^2+i\epsilon}$$

This procedure does not respect the Ward identities, however, and, furthermore, complicates the algebra considerably since it introduces additional terms in the γW vertices

Recently, a new regularization scheme has been proposed by 't Hooft and Veltman [7] They calculate Feynman amplitudes as a function of the dimensionality, n, of space-time Because of the basic simplicity of these amplitudes, an analytic continuation to complex n is feasible Divergences in the calculation now show up as poles in the amplitudes for real values of n

The great advantages of this method are (i) the fact that Ward identities are preserved in the normal parity case, (ii) that the integrand is not changed, (iii) that unitarity is explicit in the limit $n \to 4$, and (iv) that all formal manipulations, like shifting of variables and symmetric integration, are allowed

One disadvantage of this approach appears when one attempts to give a definition to γ_5 consistent with all Ward identities This is particularly serious for the abnormal parity spinor loops A resolution of this problem has recently been proposed by one of us [8], which uses a modification of the n dimensional technique For the purposes of this paper it is possible to use a definition of γ_5 within the n dimensional scheme which is consistent with all the relevant Ward identities

All calculations were performed using directly the Feynman rules derived by the Weinberg Lagrangian, where the vector meson propagator has the unitary form, but the bad asymptotic behaviour

In sect 2 we present the calculation of the static quantities associated with the vector bosons, in sect 3, we examine the self-charge and the charge radius of neutrino, whereas in sect 4, we discuss the muon anomaly Finally, we present the full Weinberg Lagrangian with all possible counter-terms in appendix A, and expose the relevant rules for calculating in n dimensions in appendix B

2 STATIC QUANTITIES OF VECTOR BOSONS

Let $\epsilon_\mu, \epsilon_\alpha, \epsilon_\beta$ be the polarization four-vectors of the photon, the outgoing and the incoming W, and $q, p+Q, p-Q$ their four-momenta Obviously, $2Q = q$ Then, the most general CP invariant vertex, when all particles are one the mass shell, can be written in the form

$$M_{\mu\alpha\beta} = ie\ \{A\,[2p_\mu g_{\alpha\beta} + 4(Q_\alpha g_{\beta\mu} - Q_\beta g_{\alpha\mu})] + 2(\kappa - 1)(Q_\alpha g_{\beta\mu} - Q_\beta g_{\alpha\mu}) + 4(\Delta Q/M_W^2) p_\mu Q_\alpha Q_\beta\}$$

Here, A is a real constant, κ the anomalous magnetic moment of the W, and ΔQ its anomalous quadrupole moment

In Weinberg's model, the lowest order electromagnetic vertex of the W is obtained by setting $A = 1$, $\kappa = 1$, and $\Delta Q = 0$

$$M^0_{\mu\alpha\beta} = ie\,[2p_\mu g_{\alpha\beta} + 4(Q_\alpha g_{\beta\mu} - Q_\beta g_{\alpha\mu})]$$

The static quantity of the W which is the easiest to calculate, is no doubt its anomalous quadrupole moment There are five graphs which contribute to ΔQ, and we list them in fig 1 Since we work in the limit $Q^2 = 0$, we have no contribution from the two longitudinal parts of the W propagator dotted simultaneously into the photon vertex This reduces the superficial degree of divergence to a logarithmic one As there is no counter-term in the Lagrangian to subtract out a divergent part in ΔQ (see appendix A), this quantity has to be finite, if the theory is renormalizable

As pointed out in the introduction, we use 't Hooft and Veltman's n dimensional technique [7] to evaluate the Feynman diagrams The only point where we differ, is that we take as definition of γ_5 in n dimensions a matrix which anticommutes with all other γ matrices This definition is perfectly consistent for normal parity loops, and thus for calculating static quantities

The anomalous quadrupole moment is indeed finite, and we give the contributions from the different graphs

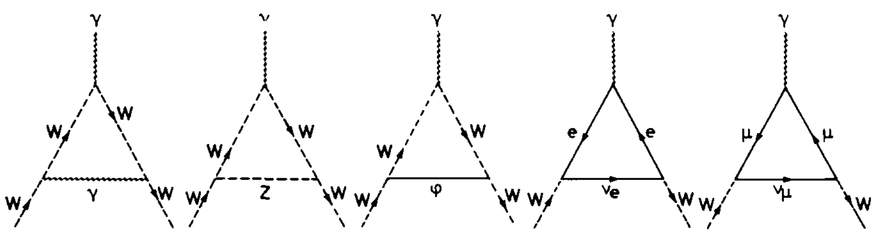

Fig 1 Feynman diagrams contributing to the anomalous quadrupole moment of the W

$$\Delta Q^{\gamma} = \frac{\alpha}{\pi}\,\frac{1}{9}\,, \qquad \Delta Q^{Z} = \frac{GM_{W}^{2}}{2\pi^{2}\sqrt{2}}\,\frac{1}{3R}\int_{0}^{1} dx\, \frac{x^{3}(1-x)(8+R)}{x^{2}+R(1-x)}\,,$$

$$\Delta Q^{\ell} = \frac{GM_{W}^{2}}{2\pi^{2}\sqrt{2}}\,\frac{4}{9}\,, \qquad \Delta Q^{\varphi} = \frac{GM_{W}^{2}}{2\pi^{2}\sqrt{2}}\,\frac{1}{3}\int_{0}^{1} dx\, \frac{x^{3}(1-x)}{x^{2}+\mu^{2}(1-x)}\,,$$

where the superscripts γ, Z, ℓ, or φ refer to the graphs in which a photon, a neutral vector boson, a lepton, or a scalar boson is exchanged, as indicated in fig 1

We have introduced the Fermi constant, G, and the quantities $R = (M_Z/M_W)^2$ and $\mu^2 = (m_\varphi/M_W)^2$ Furthermore, the expression for ΔQ^{ℓ}, the sum of electron and muon loop contributions, is valid in the limit $m_\ell/M_W \to 0$ only

It had already been pointed out by Lee [9] that ΔQ^{γ} had to be finite for vector bosons with a bare gyromagnetic ratio of 2, and this is exactly the case in Weinberg's model

It is amusing to note that the lepton contribution will be cancelled by that arising from the quark loops in a three-quartet model with fractional charges $(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ for all quartets [10], provided one again neglects the quark masses compared to M_W

The other static quantity of the W, which has to be finite, is its dynamic anoma-

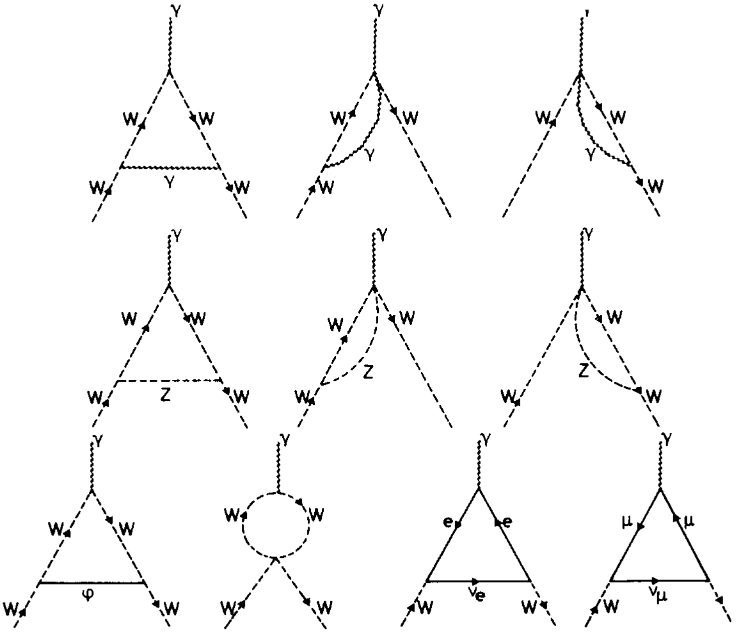

Fig 2 Feynman diagrams contributing to the dynamic anomalous magnetic moment of the W

lous magnetic moment, κ This time, we have to calculate the ten diagrams of fig 2

It turns out that κ is indeed finite, although the superficial degree of divergence of some graphs is quadratic It is also interesting to see that the contributions from the photon graphs and the Z graphs are both divergent, but that the W loop graph exactly cancels these divergences

The different contributions now are

$$\kappa^{\gamma W} = \frac{\alpha}{\pi}\frac{5}{3} ,$$

$$\kappa^{ZW} = \frac{GM_W^2}{2\pi^2\sqrt{2}}\frac{2}{R}\int_0^1 dx\, x\, \frac{8x^3-8x^2+8x+R(x^3-5x^2-2x)+\frac{1}{2}R^2(-x^2+5x-4)}{x^2+R(1-x)} ,$$

$$\kappa^{\ell} = -\frac{GM_W^2}{2\pi^2\sqrt{2}}\frac{1}{3} ,$$

$$\kappa^{\varphi} = \frac{GM_W^2}{2\pi^2\sqrt{2}}\int_0^1 dx\, x^2\, \frac{x^2-x+2-\frac{1}{2}\mu^2(x-1)}{x^2+\mu^2(1-x)}$$

We have made the same approximation, $m_\ell/M_W \to 0$, as in the ΔQ^ℓ case, and, here too, addition of the quark quartets will cancel the lepton contribution

The calculation of κ is very involved, and this is where we fully appreciated the fact that the 't Hooft-Veltman regularization scheme does not make the algebra more complicated The number of terms one would have to handle in a ξ limiting procedure, e g , makes such a calculation not only quite tedious, but also rather obscure

3 THE SELF-CHARGE OF THE NEUTRINO

Because of CP and γ_5 invariance, the neutrino only has one electromagnetic form factor, $F(q^2)$, on mass shell, and the current matrix element can be written as

$$M_\mu = ieF(q^2)\bar{u}\gamma_\mu(1+i\gamma_5)u$$

Since there are no counter-terms in the Lagrangian to reduce the neutrino charge to zero, we must have in all orders that $F(0) = 0$ In lowest order, there are two graphs (see fig 3) which might give the neutrino a charge

Using the n dimensional regularization procedure, it is not difficult to show that the sum of the two diagrams, in the limit $q \to 0$, can be written as a total derivative Evaluating this derivative then leads to a neutrino charge which remains zero

One could ask whether the neutrino charge radius, defined as

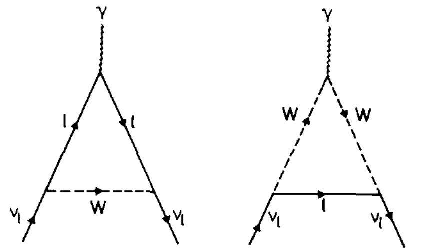

Fig 3 Feynman diagrams contributing to the self-charge of the neutrino

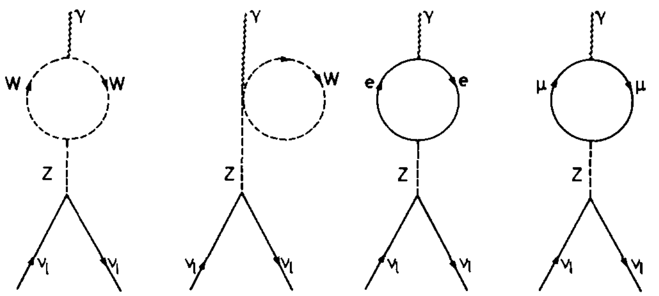

Fig 4 Feynman diagrams which also contribute to the charge radius of the neutrino, but not to its self-charge, because of gauge invariance

$$\langle r^2 \rangle = 6 \left. \frac{\partial F(q^2)}{\partial q^2} \right|_{q^2=0} ,$$

takes on a well defined value in this model

We remark that the charge radius of the neutrino is not a static quantity, since one cannot measure it with an external electromagnetic field If, however, one wants to measure the form factor with virtual photons, in elastic eν scattering, say, then one also has to consider the competing processes like two Z or two W exchange and radiative corrections to single Z exchange Indeed, in Weinberg's model, all particles which couple to the photon also couple to the Z

In order for the theory to be consistent, only the total scattering S-matrix element has to be finite, and not necessarily $F'(0)$ Indeed, we find that $F'(0)$ is divergent and can, therefore, not be a physical quantity in this theory It is clear that the calculation of elastic eν to fourth order will be another crucial test of the model

4 THE MUON ANOMALY

Besides the well-known $\alpha/2\pi$ term for the anomalous magnetic moment of the

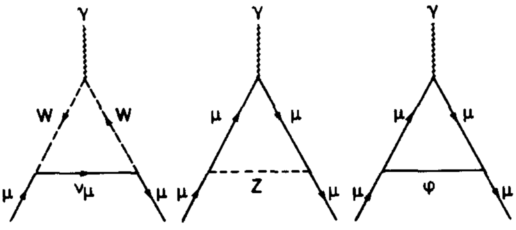

Fig 5 Feynman diagrams contributing to the anomaly of the muon

muon, Weinberg's model also predicts contributions in second order from the graphs of fig 5

Here, we are faced with an ambiguity in the regularization scheme As explained in appendix B, the vector algebra and the Diracology has to be done in n dimensions Unfortunately, no generalization of γ_5 to n dimensions exists which preserves the Ward identities for the axial current in n dimensions

't Hooft and Veltman [7] suggest a γ_5 which anti-commutes with the first four γ matrices and commutes with the others This definition of γ_5 does not preserve all the Ward identities associated with the spinor line As a result, the longitudinal parts of the W and Z propagators give additional anomalous contributions, which are finite

If, however, one retains a γ_5 which anti-commutes with all γ matrices, then no such anomalies occur, and agreement is found with other calculations of the muon anomaly [11–13] Indeed, the different contributions turn out to be

$$a_\mu^\nu = \frac{Gm_\mu^2}{8\pi^2\sqrt{2}}\,\tfrac{10}{3}\,, \qquad a_\mu^Z = -\frac{\alpha}{\pi}\,\tfrac{1}{3}\left(\frac{m_\mu}{M_Z}\right)^2 + \frac{Gm_\mu^2}{8\pi^2\sqrt{2}}\,\tfrac{4}{3}\left(1-\frac{2}{R}\right),$$

$$a_\mu^\varphi = \frac{Gm_\mu^2}{8\pi^2\sqrt{2}}\,2\int_0^1 \mathrm{d}x\,\frac{x^2(2-x)}{x^2+r(1-x)}\,,$$

corresponding to the graphs of fig 5 Here, $r = (m_\varphi/m_\mu)^2$ is a free parameter which could be of order unity, in which case a_μ^φ becomes of the same order of magnitude as a_μ^ν and a_μ^Z It is evident, from the mere size of these contributions ($\sim 10^{-9}$), that the agreement pure QED calculations of the muon anomaly and experiment is not upset by these weak effects Conversely, no limit on m_φ can be deduced from the present or planned experiments on the muon anomaly

Since the ξ limiting procedure is not an invariant one with respect to the gauge transformations of the charged gauge fields, the agreement we find should not be taken too seriously It is clear, however, that the regularization procedure must preserve all the relevant Ward identities, and, for the cases considered here, our regularization scheme works The treatment we propose is justified by the existence of a fully consistent regularization procedure, which coincides with the above scheme for these calculations [8]

5 CONCLUSIONS

By introducing a γ_5 which anticommutes with all the other matrices in n dimensions we modified the 't Hooft-Veltman regularization scheme while preserving all the vector Ward identities We were then able to calculate the static quantities of the particles in Weinberg's model

We found that the anomalous quadrupole moment and the dynamic $g-2$ of the charged vector bosons were finite, and we calculated their values We then showed that the neutrino charge remains zero in second order, and explained why the neutrino charge radius is not a physical quantity in Weinberg's model Using our regularization scheme, we gave a consistent calculation of the weak contributions to the anomaly of the muon, which confirms certain results given in the literature

All those calculations support the conjecture that Weinberg's model of weak and electromagnetic interactions of leptons, vector and scalar bosons is indeed renormalizable

APPENDIX A

If the Weinberg theory is to describe a renormalizable theory of weak interactions then physical quantities must become finite through the use of counter-terms generated by the original Lagrangian The local gauge symmetry of the Weinberg Lagrangian severely restricts the form of these counter-terms

At the one-loop level, many physical quantities must be well defined and finite, even though power counting suggests that counter-terms may be necessary The quantities computed in the text using the 't Hooft-Veltman regularization scheme are of this type In this appendix, we exhibit the most general structure of counter-terms possible for the Weinberg Lagrangian used for the computations in the text

The Weinberg Lagrangian obtained from ref [1] is given in eq (A1) We have included all renormalization constants to generate the counter-terms We have

$$
\begin{aligned}
L = &-\tfrac{1}{4}Z_1(\partial_\mu A_\nu - \partial_\nu A_\mu + gA_\mu \times A_\nu)^2 - \tfrac{1}{4}Z_2(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\
&+ Z_3\bar{L}i\gamma^\mu(\partial_\mu - \tfrac{1}{2}ig\boldsymbol{\tau}\cdot A_\mu - \tfrac{1}{2}ig'B_\mu)L + Z_4\bar{R}i\gamma^\mu(\partial_\mu - ig'B_\mu)R \\
&+ Z_5|(\partial_\mu - \tfrac{1}{2}ig\boldsymbol{\tau}\cdot A_\mu + \tfrac{1}{2}ig'B_\mu)\varphi|^2 \\
&- G_e(\bar{L}\varphi R + \bar{R}\varphi L) - \mu_0^2\bar{\varphi}\varphi - h_0(\bar{\varphi}\varphi)^2
\end{aligned}
\qquad (A\,1)
$$

The renormalization parameters $Z_1, \quad ,Z_5$ and the constants G_e, μ_0^2, h_0 generate the counter-terms while g and g' are finite quantities whose precise definition depends upon how the finite normalization of $Z_1, \quad ,Z_5$ is chosen

In this Lagrangian, the local gauge symmetry is evident However, the calculations in the text were made using a Lagrangian obtained from (A1) by removing the redundant degrees of freedom of the fields This new Lagrangian is manisfestly unitary and is obtained from (A1) by the following substitutions

$$A_\mu^1 \pm iA_\mu^2 = \sqrt{2}\, W_\mu^\mp , \quad A_\mu^3 = W_\mu^3 = (g^2+g'^2)^{-\frac{1}{2}}(gZ_\mu - g'A_\mu) ,$$
$$B_\mu = (g^2+g'^2)^{-\frac{1}{2}}(g'Z_\mu + gA_\mu) , \quad L = \tfrac{1}{2}(1+i\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix},$$
$$R = \tfrac{1}{2}(1-i\gamma_5)(e) , \quad \varphi = \frac{1}{\sqrt{2}}(\lambda+\varphi)\begin{pmatrix}0\\1\end{pmatrix} \tag{A 2}$$

With these definitions, the Lagrangian (A 1) may be written in terms of the free and interaction Lagrangians given in (A 3) and (A 4)

$$\begin{aligned} L_0 = &-\tfrac{1}{2}|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+|^2 + M_W^2|W_\mu^+|^2 - \tfrac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \tfrac{1}{2}M_Z^2(Z_\mu)^2 \\ &- \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \tfrac{1}{2}(\partial_\mu\varphi)^2 - \tfrac{1}{2}\mu^2\varphi^2 \\ &+ \bar{e}(i\gamma\ \partial - m)e + \bar{\nu}i\gamma\ \partial\tfrac{1}{2}(1+i\gamma_5)\nu , \end{aligned} \tag{A 3}$$

$$\begin{aligned} L_I = &Z_1 g\ \{W_\mu^3 W_\nu^- i\overleftrightarrow{\partial^\mu} W^{+\nu} + W_\mu^- W^{+\nu} i\overleftrightarrow{\partial^\mu} W_\nu^3 + W_\mu^+ W^{3\nu} i\overleftrightarrow{\partial^\mu} W_\nu^-\} \\ &+ Z_1 g^2\ \{\tfrac{1}{2}(W_\mu^+ W^{-\mu})^2 - \tfrac{1}{2}(W_\mu^+)^2(W_\nu^-)^2 + (W_\mu^3)^2(W_\nu^+ W^{-\nu}) - (W_\mu^3 W^{-\mu})(W_\nu^3 W^{+\nu})\} \\ &+ Z_3 \frac{g}{2\sqrt{2}}\ \{\bar{\nu}\gamma^\mu(1+i\gamma_5)e W_\mu^+ + \bar{e}\gamma^\mu(1+i\gamma_5)\nu W_\mu^-\} \\ &+ Z_3 \tfrac{1}{4}g(g^2+g'^2)^{\frac{1}{2}}\ \bar{\nu}\gamma^\mu(1+i\gamma_5)\nu Z_\mu \\ &+ \bar{e}\gamma^\mu Z_\mu\ \{\tfrac{1}{4}Z_3 \frac{g'^2-g^2}{(g^2+g'^2)^{\frac{1}{2}}}(1+i\gamma_5) + \tfrac{1}{2}Z_4 \frac{g'^2}{(g^2+g'^2)^{\frac{1}{2}}}(1-i\gamma_5)\}\ e \\ &- \bar{e}\gamma^\mu A_\mu\ \{\tfrac{1}{2}Z_3 \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}}(1+i\gamma_5) + \tfrac{1}{2}Z_4 \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}}(1-i\gamma_5)\}\ e \\ &- \frac{G_e}{\sqrt{2}}\bar{e}e\varphi + Z_5\tfrac{1}{4}g^2|W_\mu^+|^2\varphi(2\lambda+\varphi) \\ &+ Z_5\tfrac{1}{8}(g^2+g'^2)(Z_\mu)^2\,\varphi(2\lambda+\varphi) - h_0\lambda\varphi^3 - \tfrac{1}{4}h_0\varphi^4 \end{aligned}$$

$$-\tfrac{1}{2}(Z_1-1)|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu|^2 + |W^+_\mu|^2\{-M_W^2+\tfrac{1}{4}Z_5 g^2\lambda^2\}$$
$$-\tfrac{1}{4}(Z_1-1)(\partial_\mu W^3_\nu - \partial_\nu W^3_\mu)^2 - \tfrac{1}{4}(Z_2-1)(\partial_\mu B_\nu - \partial_\nu B_\mu)^2$$
$$+\tfrac{1}{2}(Z_\mu)^2\{-M_Z^2 + Z_5\tfrac{1}{4}(g^2+g'^2)\lambda^2\}$$
$$+(Z_3-1)\bar{e}i\gamma\ \partial\tfrac{1}{2}(1+i\gamma_5)e + (Z_4-1)\bar{e}i\gamma\ \partial\tfrac{1}{2}(1-i\gamma_5)e$$
$$+(Z_3-1)\bar{\nu}i\gamma\ \partial\tfrac{1}{2}(1+i\gamma_5)\nu + \left(m - \frac{G_e\lambda}{\sqrt{2}}\right)\bar{e}e$$
$$+(Z_5-1)\tfrac{1}{2}(\partial_\mu\varphi)^2 + \tfrac{1}{2}(\mu^2-\mu_0^2-3h_0\lambda^2)\varphi^2$$

In this formulation, g, g' and λ are constants, while $Z_1, \quad, Z_5, G_e$ and h_0 are renormalization constants The physical masses, $M_W^2, M_Z^2\ m^2$ and μ^2 are not free parameters, but must be determined from the zeros of the appropriate self-energy functions The μ_0^2 is also not independent, but is determined by the condition that cancels the φ meson tadpoles This condition reads

$$\begin{aligned} 0 &= -\mu_0^2\lambda \quad Z_5 h_0\lambda^3 \\ &- \langle 0|\{3h_0\lambda\varphi^2 + h_0\varphi^3 - \tfrac{1}{4}Z_5(g^2+g'^2)(Z_\mu)^2(\lambda+\varphi) \\ &- \tfrac{1}{2}Z_5 g^2|W^+_\mu|^2(\lambda+\varphi) + \frac{G_e}{\sqrt{2}}\bar{e}e\}|0\rangle \end{aligned} \quad \text{(A 5)}$$

Additional (finite or infinite) wave function renormalizations are required in order to define properly normalized S-matrix elements *

APPENDIX B

In this appendix we briefly outline the use we made of the 't Hooft-Veltman regularization scheme [7]

Suppose that, instead of working in the four dimensions of ordinary space-time, one were to calculate in $n \geqslant 4$ dimensions To evaluate Feynman diagrams in that case, one would have to consider integrals of the type

$$I(n,m) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{[k^2 - L + i\epsilon]^m} \quad \text{(B 1)}$$

For $n < 2m$, the integral exists and is given by

$$I(n,m) = \frac{i^{1-2m}}{(2\sqrt{\pi})^n} L^{\frac{1}{2}n-m} \frac{\Gamma(m-\frac{1}{2}n)}{\Gamma(m)} \quad \text{(B 2)}$$

* We thank T Appelquist and G t' Hooft for discussions on this point

For the cases in which the integral does not exist, the right-hand side of eq (B 2) is taken as the definition of the integral

One can then show that all formal manipulations, like symmetric integration, partial integration, and shifting of integration variables are allowed, provided one consistently uses the relation $g_{\mu\nu}g^{\mu\nu} = n$

Divergences in integrals will now show up as poles along the real axis, e g , a logarithmic integral has poles at $n = 4 + 2m$, a quadratic one at $n = 2 + 2m$ where m is a non-negative integer These singularities arise from the Γ function

In practical calculations of Feynman amplitudes, one now proceeds as follows

(i) perform all the Dirac algebra keeping in mind that $g_{\mu\nu}g^{\mu\nu} = n$,

(ii) symmetrize the integrand using rules like

$$k_\mu k_\nu \to \frac{1}{n} k^2 g_{\mu\nu} ,$$

(iii) perform integrals over the loop momenta using the definition of eq (B 2),

(iv) take the limit $n \to 4$ If the Feynman amplitude is finite, there will be no singularity at $n = 4$

As an example, let us consider the calculation of ΔQ^γ, arising from the first diagram in fig 1 This electromagnetic vertex is given by

$$M_{\mu\alpha\beta} = e^3 \int \frac{d^n k}{(2\pi)^n} V_{\lambda\sigma\alpha} P^{\tau\sigma}(k+Q) W_{\mu\rho\tau} P^{\rho\gamma}(k-Q) X_{\nu\beta\gamma}$$
$$\times \frac{g^{\nu\lambda}}{(p-k)^2} \frac{1}{[(k+Q)^2 - M_W^2]} \frac{1}{[(k-Q)^2 - M_W^2]} ,$$

where

$$P_{\sigma\tau}(l) = g_{\sigma\tau} - l_\sigma l_\tau / M_W^2 ,$$

$$V_{\lambda\sigma\alpha} = (k + p + 2Q)_\lambda g_{\alpha\sigma} - 2(k+Q)_\alpha g_{\lambda\sigma} - (2p + Q - k)_\sigma g_{\alpha\lambda} ,$$

$$W_{\mu\rho\tau} = 2k_\mu g_{\rho\tau} - (k - 3Q)_\tau g_{\rho\mu} - (k + 3Q)_\rho g_{\tau\mu} ,$$

$$X_{\nu\beta\gamma} = (k + p - 2Q)_\nu g_{\beta\gamma} - 2(k - Q)_\beta g_{\gamma\nu} - (2p - Q - k)_\gamma g_{\beta\nu}$$

We are only interested in that part of $M_{\mu\alpha\beta}$ which is proportional to $p_\mu Q_\alpha Q_\beta$, and then only in the limit $Q^2 = 0$

The term in $1/M_W^4$ from the longitudinal parts of the W propagators does not contribute, as it is proportional to Q^2

The term in $1/M_W^2$ leads to the expression

$$k_\mu (k+Q)_\alpha (k-Q)_\beta [2M_W^2 + 4p \cdot k - 4k^2] ,$$

when all dummy indices are summed over Similarly, one finds for the $g_{\sigma\tau} g_{\rho\gamma}$ term

$$(8n - 14)k_\mu(k+Q)_\alpha(k-Q)_\beta + 32p_\mu(k_\alpha Q_\beta - k_\beta Q_\alpha) + 64k_\mu Q_\alpha Q_\beta$$

Since we only have to consider terms up to second power in Q, we can replace

$$\frac{1}{[(k-Q)^2 - M_W^2][(k+Q)^2 - M_W^2]} \to \frac{1}{[k^2 - M_W^2]^2} + \frac{4(kQ)^2}{[k^2 - M_W^2]^4}$$

Combining denominators with the Feynman trick, one finds for the over-all denominator

$$D = (k - p(1-x))^2 - M_W^2 x^2 \equiv l^2 - M_W^2 x^2$$

Making the shift, performing symmetric integration, and using eq (B 2) we have

$$M_{\mu\alpha\beta} \to \frac{ie^3}{(2\sqrt{\pi})^n} p_\mu Q_\alpha Q_\beta \int_0^1 \mathrm{d}x \, \{ [M_W x]^{n-6} x^3 (1-x)(8n-14)\Gamma(3-\tfrac{1}{2}n)$$

$$- \tfrac{1}{3}[M_W x]^{n-6} x^3 (1-x)(8n-14)(1-\tfrac{1}{2}n)(2-\tfrac{1}{2}n)\Gamma(1-\tfrac{1}{2}n)$$

$$- \frac{1}{M_W^2}[2M_W^2(M_W x)^{n-6} x^3(1-x)\Gamma(3-\tfrac{1}{2}n)$$

$$- \tfrac{2}{3}M_W^2(M_W x)^{n-6} x^3 (1-x)(1-\tfrac{1}{2}n)(2-\tfrac{1}{2}n)\Gamma(1-\tfrac{1}{2}n)$$

$$+ 2(M_W x)^{n-4} x^2(1-x)\Gamma(2-\tfrac{1}{2}n)$$

$$- 2(M_W x)^{n-4} x^2 (1-x)(1-\tfrac{1}{2}n)\Gamma(1-\tfrac{1}{2}n)]\}$$

The Γ functions which have poles for $n = 4$ can be shown to cancel, and the whole expression becomes finite for $n = 4$

$$M_{\mu\alpha\beta} \to \frac{ie^3}{16\pi^2}(p_\mu Q_\alpha Q_\beta / M_W^2) \int_0^1 \mathrm{d}x \;\; \{18x(1-x) - 6x(1-x)$$

$$- [2x(1-x) - \tfrac{2}{3}x(1-x)]\} = \frac{ie^3}{16\pi^2}\frac{p_\mu Q_\alpha Q_\beta}{M_W^2}\tfrac{32}{18},$$

from which follows

$$\Delta Q^\gamma = \frac{\alpha}{\pi}\tfrac{1}{9}$$

REFERENCES

[1] S Weinberg, Phys Rev Letters 19 (1967) 1264,
A similar model was discussed by A Salam in Elementary Particle Theory, Proceedings of the Eighth Nobel Symposium, 1968, ed N Svartholm (Wiley, New York, 1968)

[2] P W Higgs, Phys Letters 12 (1964) 132, Phys Rev Letters 13 (1964) 508, Phys Rev 145 (1966) 1156,
F Englert and R Brout, Phys Rev Letters 13 (1964) 321,
G S Guralnik, C R Hagen and T W B Kibble, Phys Rev Letters 13 (1964) 585,
T W B Kibble, Phys Rev 155 (1967) 1554

[3] G 't Hooft, Nucl Phys B35 (1971) 167

[4] B W Lee, Phys Rev , D5 (1972) 823

[5] S Weinberg, Phys Rev Letters 27 (1971) 1688

[6] T D Lee and C N Yang, Phys Rev 128 (1962) 885

[7] G 't Hooft and M Veltman, Nucl Phys B44 (1972) 189

[8] W A Bardeen, to be published

[9] T D Lee, Phys Rev 128 (1962) 899

[10] C Bouchiat, J Iliopoulos and Ph Meyer, Phys Letters 38B (1972) 519

[11] S J Brodsky and J D Sullivan, Phys Rev 156 (1967) 1644

[12] I Bars and M. Yoshimura, Berkeley preprint (1972)

[13] B E Lautrup, A Peterman and E de Rafael, Phys Reports 3 (1972) 193