Information spreading and development of cultural centers

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The historical interplay between societies is governed by many factors, including in particular the spreading of languages, religion, and other symbolic traits. Cultural development, in turn, is coupled to the emergence and maintenance of information spreading. Strong centralized cultures exist due to attention from their members, whose faithfulness in turn relies on the supply of information. Here we discuss a culture evolution model on a planar geometry that takes into account aspects of the feedback between information spreading and its maintenance. Features of the model are highlighted by comparing it to cultural spreading in ancient and medieval Europe, where it suggests in particular that long-lived centers should be located in geographically remote regions.

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I. INTRODUCTION

The expansion and decline of social structures depend on information spreading in the form of languages, religion, or other cultural inventions [1]. In recent years many mathematical models have been proposed for social interactions and dynamics, trying to explain social structures. However, a main feature of most models of culture dissemination is an adaptation toward local or global consensus [2,3], an equilibration that is also found in voter models [4], social impact theory [5], majority rules [6], the Sznajds model [7], the Deffuant model [8], and bounded confidence models [9].

The multitude of models that emphasize consensus dynamics contrasts a reality where consensus is often broken by the emergence of new cultures, languages, or opinions. One driving force for heterogeneity is the need for attention, where individuals not only aim at mutual understanding but at the same time also fight for individual attention. This attention battle is more than the random fluctuations of agents [10] or the rejection of other opinions [11]: The battle may, for example, involve positive-feedback mechanisms, as suggested in Ref. [12]. In the present paper we take the possibility for a new culture to emerge into account in addition to the local alignment rules. Lacking a simple realistic mechanism for the creation of new cultures, we here simply parametrize this emergence in terms of a rate p_{new} for the initiation of new cultures.

Another common theme of models dealing with the spread of information is that two different pieces of information are treated on an equal footing. This is in general an incorrect assumption, as the importance of two bits of information in general is asymmetric. One sorting principle is to use the information age as a sorting criteria, reflecting the fact that the value of information typically decays with time [13]. Previous studies [14,15] demonstrated that such a sorting principle has major consequences for the spatiotemporal dynamics of information. Importantly, newer information overriding older information has been observed in the spreading of linguistic features [16,17]. Furthermore, a simple model of the diffusion of information from a cultural stronghold with age sorting is shown to be compatible with the observed pattern of word distribution in Japan [18].

When individuals sort information based on its age, an existing cultural center will continuously need to generate new information to maintain their sphere of influence. Accordingly, we here characterize the strength of a cultural center by the rate with which it is able to generate fashions p_{repeat}. We will take this rate as a characteristic of a cultural center and it keeps generating new fashions until the center is eliminated by information generated from competing cultural centers.

A main feature of the well-known Axelrod model [2,3] for social alignment is conservativeness in communication, implemented by having individuals with many types of opinions and a preference for communication between individuals that share many traits. This preference makes people more open for communication toward sources where they earlier obtained information. We here parametrize such conservativeness into a single parameter p that counts the chance that a given site or agent changes preference about who to obtain information from.

Overall our model aims to discuss the information flow association with emerging and collapsing cultural centers, each influencing their surroundings by an ongoing generation of announcements that maintain their sphere of influence. The details of the model are presented in the following section.

II. MODEL

Our model considers many rumors, fashions, viewpoints, stories, or ideas (denoted fashions in the following) competing on a two-dimensional square lattice of sites that at any given moment can be occupied by one fashion only. We imagine each site as an agent, which in fact could be a whole group of people that by definition share the same taste. Each site listens to their immediate four neighboring sites with a history-dependent frequency. When communicating, they accept fashions only when they are newer than the current local fashion. History, or conservativeness, is quantified in terms of a preference in listening toward the direction where the last new idea came

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from. This is parametrized by the probability p ($p \leqslant 0.25$) to listen to one of the other three directions.

The fashions in the system come from cultural centers. Contrary to our previous model where only one cultural stronghold is placed in the system *a priori* [18], we assume here that a new cultural center can emerge at any site with a small probability p_{new}. Such a site is recognized as a cultural center as long as it has its own fashion and is not invaded by fashions from other sites. An existing cultural center in addition broadcasts itself repeatedly by initiating a new fashion with a rate p_{repeat}.

We perform a Monte Carlo simulation of the model with a parallel (synchronous) update of all agents. In the model a fashion $I(s,a)$ is characterized by its center s (the site at which the idea started) and its age a (how long ago the fashion originated at the center). At any time t each site i has its current fashion $I_i(t) = I(s_i,a_i)$ and its preferred direction $d_i(t)$ from which this fashion was obtained.

The time step from time t to $t+1$ consists of the following procedures.

(i) Emergence of new cultural centers. With a probability p_{new}, a site i is randomly chosen out of all N sites in the system to become a new cultural center. It starts its own new fashion, i.e., $I_i(t)$ is set to $I(i,0)$.

(ii) Repeated broadcast by existing cultural centers. Each cultural center i (i.e., $s_i = i$) will start to spread a new fashion with probability p_{repeat}, namely, $I_i(t)$ is set to be $I(i,0)$. Putting it differently, every cultural center can rebroadcast the same fashion as a new one, making it more appealing.

(iii) Spreading of ideas. For each site i in the system, the preferred site $d_i(t)$ is chosen with probability $1-3p$ ($p \leqslant 0.25$) or, alternately, one chooses one of the other neighboring sites with probability p. The age a_k of fashion $I_k(t) = I(s_k,a_k)$ at the chosen site k is compared with the age a_i of the fashion at the site i. If $a_k < a_i$, the site i accepts the fashion from the site k, namely, set $I_i(t+1) = I_k(t) = I(s_k,a_k)$ and update its preferred direction to $d_i(t+1) = k$. Otherwise the site i keeps its fashion unchanged, i.e., $I_i(t+1) = I_i(t) = I(s_i,a_i)$, and keeps its preferred direction $d_i(t+1) = d_i(t)$. If a site i is a cultural center, the acceptance of competing idea destroys its ability as a cultural center and hence stops repeatedly broadcasting new fashions.

(iv) Update of time. The ages of all fashions on all sites are increased by one.

We simulate the model on an $L \times L$ square lattice under periodic boundary conditions. We will also consider the model on a map of Europe, where the closed boundary conditions are imposed toward the sea regions. The initial condition is set so that no one has an opinion or preferred directions (all the sites are weighted equally). We investigate properties of the system only after the number of cultural centers has reached the steady-state value.

III. RESULTS

A. Dynamics

The dynamics of the model is depicted in Fig. 1, which presents the time evolution on the European map. In the top panels the respective cultures are presented in different colors. The middle panels show the current distance of each site from its respective cultural center, defined as the place where its current fashion was introduced. The consecutive images illustrate the dynamics of the system, with meanderings of borders as well as the emergence of new cultural centers and the disappearance of others. The bottom panels show the information pathways (river landscape), which are based on the preferred direction for each site $d_i(t)$. The arrows define the path to the cultural center to which every agent belongs. The intensity of the points in the river landscape indicates how many times information was transmitted through every node, i.e., every time the idea is copied the number of transitions on all the nodes on the path (up to the origin) are increased by one. It shows a clear river basin structure centered around the respective cultural centers, much like what was obtained for the word spreading model of Ref. [18].

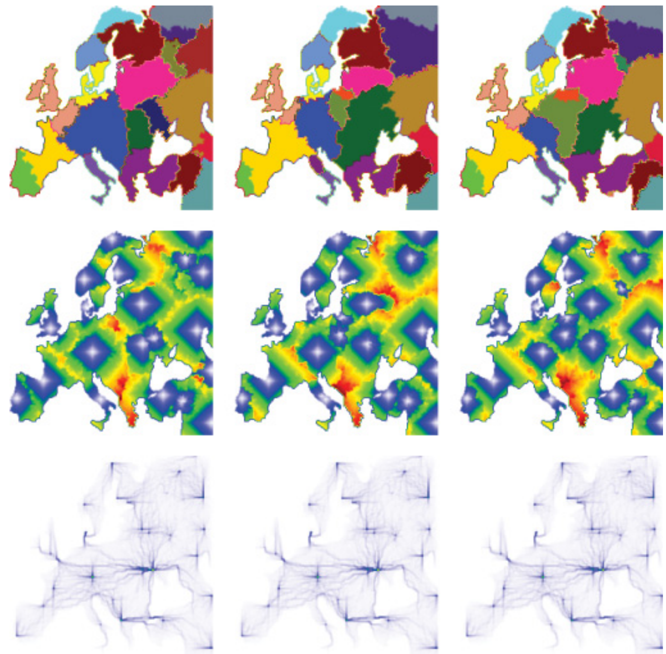

FIG. 1. (Color online) System snapshots at various times $t = \{47,48,49\} \times 10^3$ (from left to right), i.e., consecutive columns present subsequent states of the system separated by 1000 Monte Carlo steps. The different panels show the spatial structure of cultures (top panels), the distance to the cultural center (middle panels), and information pathways (bottom panels). Cultural centers are located at the white and blue stars in the middle panels. The parameters are $p = 0.01$, $p_{\text{new}} = 0.004$, and $p_{\text{repeat}} = 0.015$.

B. Analysis of parameters

The role of the three parameters p, p_{new}, and p_{repeat} is summarized in Fig. 2, representing the behavior of the 100×100 system with periodic boundary conditions. In Fig. 2 the average age $\langle \text{age} \rangle$ (left column) and average size $\langle \text{size} \rangle$ (right column) of the cultural centers are presented as a function of p_{new} and p_{repeat}, with $p = 0.25$ (top panels) and $p = 0.01$ (bottom panels), respectively. Note that $p = 0.25$ examines the case where there is no conservativeness in the dynamics. As p becomes smaller, the directions to existing cultural centers are preferred and it becomes difficult for a new cultural center to

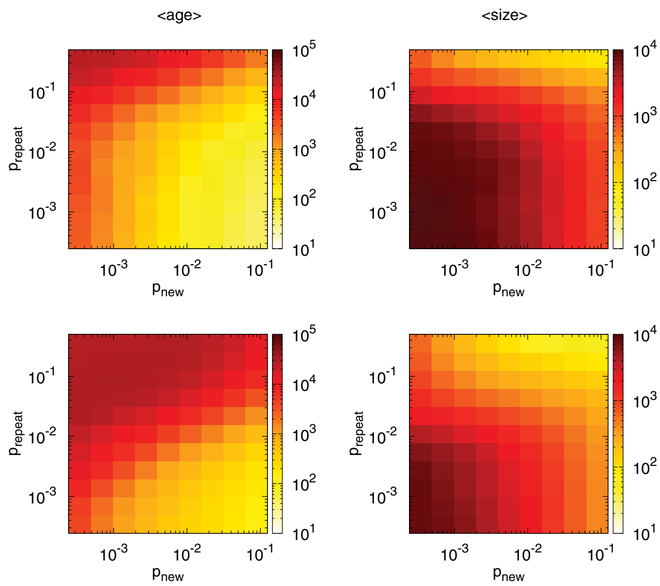

FIG. 2. (Color online) Phase diagram showing the average age ⟨age⟩ (left column) and average size ⟨size⟩ (right column) of cultural centers as a function of the probability of the introduction of new cultures $p_{\rm new}$ and the probability of resending new signals (rumors) $p_{\rm repeat}$. The system size is 100×100. The top row corresponds to $p = 0.25$, while the bottom presents results for $p = 0.01$.

emerge, as can be seen in the longer-lived cultural centers for smaller p.

For a given p, cultures live longer for smaller $p_{\rm new}$ and larger $p_{\rm repeat}$ because small $p_{\rm new}$ decreases the emergence of new cultural centers, while large $p_{\rm repeat}$ ensures stability of the cultural center, i.e., constant broadcasting of new signals is vital for the maintenance of cultural centers. In the opposite limit of large $p_{\rm new}$ and small $p_{\rm repeat}$, the ongoing strong competition between various ideas results in short-lived cultural centers.

The biggest cultural centers are observed for small $p_{\rm new}$ and small $p_{\rm repeat}$ (see the right column of Fig. 2). Such a combination of parameters reduces the competition among cultural centers and guarantees that each culture has enough time to spread over the whole system, resulting in one dominating culture of the system size. The opposite limit of large $p_{\rm new}$ and large $p_{\rm repeat}$, in contrast, means the frequent emergence of new cultural centers that survive relatively well, resulting in the coexistence of many small cultural centers.

We checked the effect of the system size by comparing the results with a 200×200 system and confirmed that the data in Fig. 2 collapse onto the data from the bigger system very well as long as the data with the same p, $p_{\rm repeat}$, and $p_{\rm new}/N$ are compared. The only exceptions are observed when the average size of the fashion reaches the system size (data not shown).

C. Competition between cultural centers

The interesting aspect of our model is the ongoing replacement of old cultural centers by new ones, a dynamics primarily governed by the emergence of new cultural centers. To quantify this we examine where new cultural centers tend to emerge when there are already established cultural centers

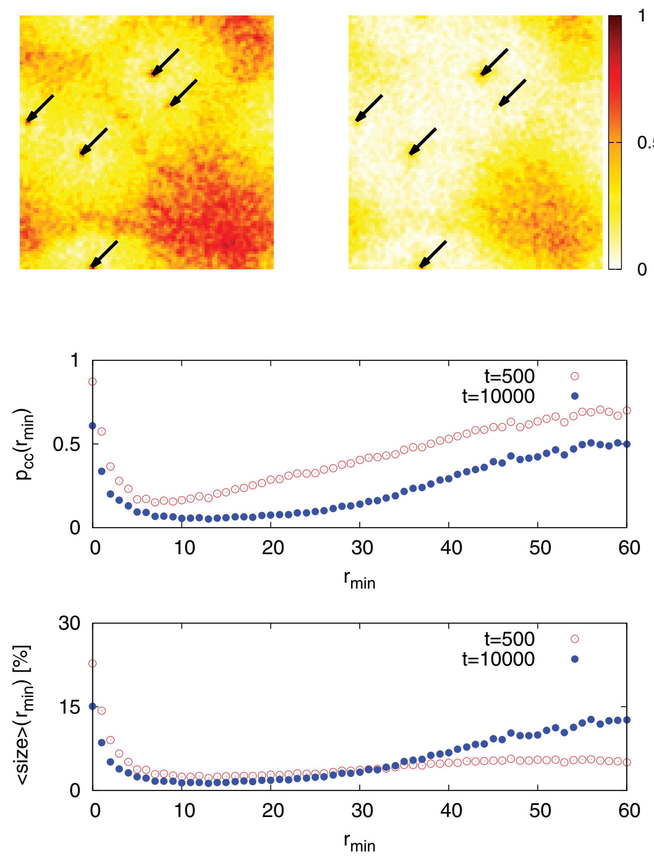

FIG. 3. (Color online) Probability of surviving as a cultural center $p_{\rm cc}$ (middle row) and average size ⟨size⟩ (bottom row) as a function of the distance to the nearest cultural center $r_{\rm min}$. Different lines correspond to times $t = 500$ and 10 000. The average size is presented as the ratio of the whole system size. The top row shows the probability of surviving as a cultural center as a function of the position (x,y) at $t = 500$ (left map) and $t = 10\,000$ (right map). The arrows in the maps indicate the positions of cultural centers in the initial configuration. The simulation parameters are $p = 0.01$, $p_{\rm new} = 2^{-10}$, and $p_{\rm repeat} = 2^{-6}$. The system size is 100×100.

in the system. We rerun history multiple times using a given snapshot of a 100×100 system as the initial configuration. For this initial configuration we insert a new cultural center at a site at (x,y) at time zero and run the simulation to see how long the new center survives, i.e., until it is overwritten by fashions from neighboring centers. This procedure is repeated 25 times for each site to estimate the survival probability as a function of time t.

The top panels of Fig. 3 show the two-dimensional map of the survival probability after 500 time steps (left panel) and 10 000 time steps (right panel) as a function of the position (x,y), where the new cultural center was inserted. In the plot, the positions of the cultural centers that exist in the initial configuration are marked by arrows. We see that new cultural centers are successful when either emerging very close to the existing cultural centers or exploring remote regions.

In order to see this tendency more clearly, the middle panel presents the survival probability $p_{\rm cc}$ as a function of the distance to the closest cultural center $r_{\rm min}$. Similarly, the bottom panel of Fig. 3 shows the average size of the culture as a function of $r_{\min}$. We can see that both the survival probability and average size are a nonmonotonic functions of the distance to the closest cultural center. The insertion points located very close to the existing cultural centers lead to maximal chances of surviving p_{cc} and the largest size of cultures $\langle\text{size}\rangle$. Both p_{cc} and $\langle\text{size}\rangle$ drop quickly with distance to the existing cultural center $r_{\min}$ and then show a slow recovery with the distance $r_{\min}$. Namely, new centers either explore a strategy of acquiring the existing network by taking over a previous center or have to explore the weaknesses of boundary regions to build their own network of influence. The difficulty in building rather than taking over an empire is also reflected in the smaller size of new cultures emerging in distant regions compared to new cultures built on deposing existing rulerships. For larger times, both the survival probability p_{cc} and average size $\langle\text{size}\rangle$ decrease for all places (Fig. 3, middle and bottom panels) since competition with existing cultural centers makes for a finite extinction rate.

D. Analysis of the one-dimensional model

The reported features of the present culture spreading model can be understood qualitatively by considering a simplified one-dimensional model of the random walk of the boundary between two cultural centers. Suppose that there is a cultural center C_0 at site 0 and another cultural center C_1 at site R. We consider the motion of the leftmost site r that belongs to C_0. Moreover, we assume that no additional cultural center appears, i.e., $p_{\text{new}} = 0$. In this limit, the age a_r of the fashion at site r, which belongs to C_0, and the age a_{r+1} of the fashion at site $r' = r + 1$, which belongs to C_1, can be approximated as

$$a_r \approx \frac{r}{1-p} + \tau, \quad a_{r'} \approx \frac{R-r'}{1-p} + \tau', \tag{1}$$

respectively. In Eq. (1), τ and τ' are independent discrete stochastic variables, both having a probability distribution

$$P(\tau) = p_{\text{repeat}}(1 - p_{\text{repeat}})^{\tau}. \tag{2}$$

For the sake of simplicity, the stochasticity of the fashion propagation in the preferred direction and the time correlation of the age are ignored [see Eq. (1)].

We can calculate the rate at which the position r of the leftmost site that belongs to C_0 decreases (increases) by one, which happens if $a_r > a_{r+1}$ ($a_r < a_{r+1}$) when site r ($r + 1$) is influenced by site $r + 1$ (r) with probability p. The explicit form of the rates is given in the Appendix. Recall that the presented derivation is valid when $p \ll p_{\text{repeat}}$, where the approximation of the age in Eq. (1) is justified.

In Fig. 4(a) we compare the mean first passage time of the boundary starting at $r = (R - 1)/2$ to reach one of the cultural centers $T(p, p_{\text{repeat}}, R)$, which gives the typical survival time of established cultural centers separated by distance R. The simplified random-walk model of the boundary between two cultural centers agrees reasonably well with the full one-dimensional simulation results. We also checked that the level of agreement improves when $p \ll p_{\text{repeat}}$ (data not shown). We can see that the survival time is longer for larger p_{repeat} and increases exponentially for large R.

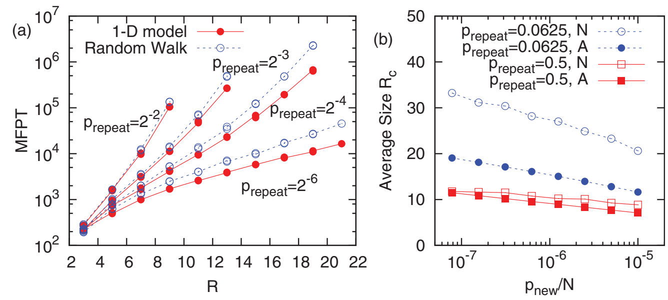

FIG. 4. (Color online) (a) Mean first passage time (MFPT) (average survival time of a cultural center) vs distance between two cultural centers R in the one-dimensional (1D) model (solid circles) and the simplified random-walk model (open circles), with $p = 0.01$. One can see good agreement for $p \leqslant p_{\text{repeat}}$. The deviation grows for $p \geqslant p_{\text{repeat}}$, but the qualitative behavior is captured in the random-walk model. (b) Average size of the cultural area in one dimension. The numerical simulation is denoted by N and the analytical estimate is denoted by A. As p_{repeat} becomes smaller the disagreement between the simulation and the analysis becomes larger.

It is more intuitive to interpret these results by making a continuous approximation for space and time and deriving the Fokker-Planck equation for the probability $P(x,t)$ that the boundary is at the position $x = r - (R - 1)/2$ (therefore the cultural centers are located at $x \approx \pm R/2$) at time t. The resulting equations are (see the Appendix)

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} J(x,t), \tag{3}$$

where

$$J(x,t) = -p\frac{dU(x)}{dx}P(x,t) - \frac{\partial}{\partial x}\left[D(x)P(x,t)\right], \tag{4}$$

and

$$U(x) = |x| - \ell\left[1 - \exp\left(-\frac{|x|}{\ell}\right)\right], \tag{5}$$

$$D(x) = \frac{p}{2}\left[1 - \frac{p_{\text{repeat}}}{1 - p_{\text{repeat}}}\exp\left(-\frac{|x|}{\ell}\right)\right], \tag{6}$$

$$\ell(p, p_{\text{repeat}}) = \left[-\frac{2\ln(1 - p_{\text{repeat}})}{1-p}\right]^{-1} \approx \frac{1-p}{2p_{\text{repeat}}}. \tag{7}$$

The potential $U(x)$ has a minimum at $x = 0$ with harmonic behavior $U(x) \approx x^2/2\ell(p, p_{\text{repeat}})$ for $|x| \ll \ell(p, p_{\text{repeat}})$, while $U(x)$ grows linearly with $|x|$ for large values of the argument $|x| \gg \ell(p, p_{\text{repeat}})$. In contrast, the dependence of the diffusion coefficient $D(x)$ on position is rather weak; it can be considered as $D \approx p/2$. The probability p also defines the mobility; therefore the time scale of the random walk of the boundary is proportional to $1/p$.

Now we can estimate the typical size of the cultural area or the typical distance between centers R_c. Suppose there is a cultural center at position 0 and a new cultural center is inserted at a distance R. If a new cultural center is not inserted, the time scale when one of them will be overwritten by the other one can be estimated by the mean first passage time $T(p, p_{\text{repeat}}, R)$ starting from the stable point $x = 0$. During this period, however, a new cultural center can be inserted between 0 and R with a probability approximately equal to $p_{\text{new}}R/N$, where p_{new}/N is the insertion probability per site. Therefore, the insertion and the coarsening balance lead to

$$T(p, p_{\text{repeat}}, R_c)/2 = N/(p_{\text{new}}R_c). \quad (8)$$

The factor 1/2 on left-hand side of Eq. (8) comes from the fact that each center is competing with two other centers on both sides. A comparison of the average cultural center size from the one-dimensional simulation and the estimate based on Eq. (8) with the mean first passage time T evaluated under a continuum approximation [see Eq. (9)] are shown in Fig. 4(b). The agreement is satisfactory for large p_{repeat} but becomes worse for smaller p_{repeat}. One reasons for the disagreement is that $\ell(p, p_{\text{repeat}})$ becomes considerably large for small p_{repeat}. Consequently, the potential becomes flatter and newly inserted cultural centers have higher probability to be overwritten before the boundary reaches the central point ($x = 0$), which enhances coarsening and hence increases the average size of the cultural area.

In the two-dimensional case the size becomes proportional to R_c^2, but other parameter dependencies are expected to be qualitatively the same. The mean first passage time T is proportional to $1/p$ [see Eq. (9)], which is the time scale of the dynamics and a rapidly growing function of both p_{repeat} and R. Therefore, the average size R_c^2 estimated with Eq. (8) is expected to decrease with p, p_{repeat}, and p_{new}, which is consistent with Fig. 2.

E. Replaying history of Europe

Finally, we examine the dynamics of our model on the European map on a 200 × 200 square lattice with a land mass that consists of 24 156 sites. The only constraint that the map gives is its boundary conditions where in particular the sea is impenetrable and thus information cannot travel across seas. We use parameters where roughly 20 cultural centers coexists (with $p = 0.01$, $p_{\text{new}} = 0.004$, and $p_{\text{repeat}} = 0.015$, as also used in Fig. 1). Assuming that the fastest time it takes a rumor to cross Europe is about 10 years, the corresponding number of updates on our 200×200 lattice will be approximately 200. From this perspective the subsequent snapshots in Fig. 1 correspond to 50 years, a time scale over which changes on the European map indeed occurred throughout the last millennium. Figure 5 shows a fraction of time for which respective sites were cultural centers, illustrating that centers tend to be more stable or more often established at the tips of peninsulas or other remote regions of the continents. That is, the chance to be invaded by competing fashions and cultures diminishes in these remote edge regions.

Additionally, we have checked the robustness of the observed patterns in Fig. 5. More precisely, we have constructed frequency histograms for the size-dependent broadcast probability p_{repeat} ($p_{\text{repeat}} \sim$ size/system size). This modification is based on a picture that new fashions appear more frequently when the size of the culture is bigger, which in consequence makes a bigger culture more stable and convincing. The presence of such a positive feedback weakens the contrast between peripheries and internal regions. Nevertheless, distant points remain harder to invade than internal points (data not shown). We also studied the effect of shortcuts that connect

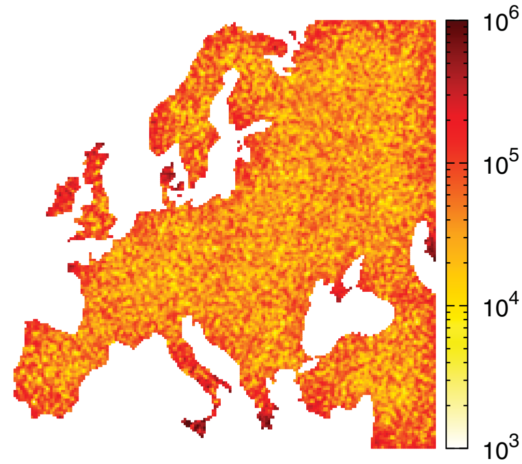

FIG. 5. (Color online) Frequency histogram presenting the fraction of time for which each point constitutes a cultural center. The parameters are $p_{\text{new}} = 0.004$ and $p_{\text{repeat}} = 0.015$, with $p = 0.01$. The simulation time $T = 10^8$ Monte Carlo steps.

two randomly chosen remote sites, having the possibility of building main roads between cities in mind. The presence of shortcuts further reduces the contrast between remote and central points (data not shown).

It should also be pointed out that the model studied does not account for geographical constraints such as rivers, mountain chains, climate, and population distribution, which are crucial for the spread of fashions and cultures in real life situations. It is assumed that the transmission of fashions is purely local and in particular that fashions do not travel overseas. As a consequence, the European simulation is more an illustration of the basic principle of the model than a valid simulation of available information highways on an ancient European landscape.

The incorporation of the mentioned constraints can significantly change the properties of the model. Rivers and roads constituted information paths in pretelegraph Europe while mountain chains provide natural communication barriers. In contrast to the geographical landscape, the role of varying the population density is more complex and less apparent. On the one hand it is natural to imagine that a large population density leads to larger creativity to start a new culture and the number of people sharing the same culture also affects the ability of the culture to convince other people. On the other hand it is likely that there is positive feedback from a cultural center to the local population, i.e., a larger population density appears in places that are close to the existing cultural centers. It would be an interesting future project to incorporate such an effect in the present model.

IV. CONCLUSION

We have explored a simple model for the emergence and decline of cultural strongholds, parametrized with rigidity in a local social network p, the probability of emergence p_{new}, and the probability at which an existing cultural center broadcast fashions p_{repeat}. The overall assumption of the model was the postulate that individuals always accept the newest (locally) available viewpoint as their own, but obviously cannot adopt a viewpoint that is not available in their social surroundings.

Social surroundings were restricted to their four nearest neighbors on a two-dimensional square lattice and further biased with their preference for listening in the direction where they last obtained useful information. As the probability of listening in other competing directions decreases ($p \to 0$) the cultural map freezes into many small, fragmented regions influenced by different cultural centers.

The acceptance of a fashion according to its age only is an important feature of our model. In some models of opinion dynamics, in contrast, a set of rules, which on average makes an agent accept a fashion shared by the majority, has been adopted (e.g., voter models [4] or majority rules models [6]). The incorporation of such rules in the present model should make it more difficult for a new cultural center to emerge and grow, though the quantitative effect depends on the exact rules.

It is also worth mentioning the relation between the present model and the Axelrod model [2], which is a model of the formation of a cultural area widely accepted by both social scientists and physicists [3]. In the Axelrod model each agent has a set of opinions represented by a vector and interacts with the neighboring agents according to the overlap of opinions: It is more likely to interact when the opinions are close, and when they interact the agent copies one of the different opinions from the neighbor to its opinion set. In this dynamics, the conservativeness is taken into account as the tendency to talk to agents that have a close opinion and the cultural area is formed as the agents align their opinions with their neighbors. In a sense, a culture spontaneously appears via interactions between agents in this model. The model can show the coexistence of multiple cultural areas, but it turned out that the coexistence is unstable against spontaneous flipping of the opinions [19] and thus several modifications of the model have been studied to realize a stable coexistence of cultures [3].

In contrast, in our model the cultural area is defined as the area that shares the same information source. The random appearance of a new cultural center, which can be viewed as a spontaneous change of the opinion set in the Axelrod model, is actually an important feature in keeping the multiple cultural centers against one culture from taking over the whole system. The key feature of our model that makes this possible is the importance of newer information, which gives some chance for a newcomer to win against existing cultural centers. It can be interesting to add a similar feature to the Axelrod model, i.e., give some rate to renew opinions and value newer information more to verify whether multiple cultures can coexist in that case.

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APPENDIX: ONE-DIMENSIONAL RANDOM-WALK MODEL OF THE BOUNDARY BETWEEN TWO CULTURAL CENTERS

Here we derive the simplified one-dimensional model of a random walk of the boundary between two cultural centers. As summarized in Sec. III D, two cultural centers C_0 and C_1 are located at sites 0 and R, respectively. We analyze the motion of the leftmost site r that belongs to C_0 (the site $r+1$ belongs to C_1).

The rate at which the position r decreases (increases) by one $W_{r\to r-1}$ ($W_{r\to r+1}$) is given by the probability that $a_r > a_{r+1}$ ($a_r > a_{r+1}$) when site r ($r+1$) is influenced by the unpreferred direction $r+1$ (r) at a given time step. From Eqs. (1) and (2) we get (note that $0 < r < R$)

$$W_{r\to r-1} = \begin{cases} p\frac{(1-p_{\rm repeat})^{(R-2r-1)/(1-p)+1}}{2-p_{\rm repeat}} & \text{for} \quad r \leqslant \frac{R-p}{2} \\ p\left[1 - \frac{(1-p_{\rm repeat})^{-(R-2r-1)/(1-p)}}{2-p_{\rm repeat}}\right] & \text{for} \quad r > \frac{R-p}{2} \end{cases}$$

and

$$W_{r\to r+1} = \begin{cases} p\left[1 - \frac{(1-p_{\rm repeat})^{(R-2r-1)/(1-p)}}{2-p_{\rm repeat}}\right] & \text{for} \quad r \leqslant \frac{R-p}{2} \\ p\frac{(1-p_{\rm repeat})^{-(R-2r-1)/(1-p)+1}}{2-p_{\rm repeat}} & \text{for} \quad r > \frac{R-p}{2}. \end{cases}$$

Using the above transition rates, we can write the master equation for the probability density $P_{r,t}$ that the boundary is at site r at time t:

$$\begin{aligned} P_{r,t+1} - P_{r,t} &= W_{r-1\to r} P_{r-1,t} - W_{r\to r+1} P_{r,t} \\ &\quad + W_{r+1\to r} P_{r+1,t} - W_{r\to r-1} P_{r,t}. \end{aligned}$$

Assuming that the time step and the lattice spacing are small, we obtain the Fokker-Planck equation [20]

$$\begin{aligned} \frac{\partial P(r,t)}{\partial t} &= \frac{\partial}{\partial r}\{[W(r\to r-1) - W(r\to r+1)]P(r,t)\} \\ &\quad + \frac{1}{2}\frac{\partial^2}{\partial r^2}\{[W(r\to r-1) + W(r\to r+1)]P(r,t)\}. \end{aligned}$$

Replacing r with $x = r - (R-1)/2$, we get Eqs. (3)–(7).

The mean first passage time T from $x=0$ to $\pm R/2$ is given by the closed formula [20]

$$T(p, p_{\rm repeat}, R) = -\int_{-R/2}^{0} ds\left[e^{\Phi(s)}\int_0^s \frac{e^{-\Phi(y)}}{D(y)}dy\right],$$

where

$$\Phi(x) = \int_0^x \frac{pU'(y)}{D(y)}dy.$$

Approximating $D(x)$ with $p/2$ [see Eq. (6)], we get

$$T(p, p_{\rm repeat}, R) \approx -\frac{2}{p}\int_{-R/2}^{0} ds\left[e^{2U(s)}\int_0^s e^{-2U(y)}dy\right],$$

from which Eq. (8) can be evaluated using Wolfram MATHEMATICA.

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