Turbulent, Magnetized, MRI-driven Accretion Disks: Beyond $\alpha$-disk Models

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Image Credit: NASA
Outstanding Problems in Accretion Physics

- Growth of massive black holes
- Radiative efficiency of accretion
- Black hole variability
A Well-Defined But Hard Problem

GOAL:
* Global structure of an accretion disk
* Dynamical properties
* Long-term interaction with compact object

We know the equations for a magnetized fluid in a gravitational field,
Why can we not just SOLVE them?

HUGE dynamical range…
Timescales and Lengthscales in Accretion Disks

1Gyr
1yr
1sec

0.1AU 1AU
1000AU

Mean Field Models
BH Growth
BLR
Inside Horizon
Numerical Simulations
Viscous Orbital Saturation
BH Variability
Disk Spectrum

$10^7 M_\odot$
Angular Momentum Transport in Accretion Disks

- Central problem in accretion disk theory
- Turbulence provides effective viscosity (Shakura & Sunyaev, ‘73)
- VERY hard to make Keplerian disks turbulent (Hawley et al, ‘99)
- Convection? Not efficient (Stone & Balbus, ‘96)

Magnetorotational Instability (MRI)

* Velikhov & Chandrasekhar, early 60’s
  Balbus & Hawley, early 90’s

* Mechanism to disrupt laminar flows

* Numerical simulations confirm development of MHD turbulence
Magnetorotational Instability

- Magnetic Field

\[ B_z \]

\[ \phi \]

\[ r \]

\[ z \]
Magnetorotational Instability

Magnetic Field

$B_z$, $\phi$, $r$, $z$
Magnetorotational Instability

Magnetic Field
Magnetorotational Instability

\[ B_z \delta B_\phi \]

Magnetic Field
Magnetorotational Instability

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\[ B_z \delta B_\phi \]

Magnetic Field
\[ B_z \delta B_\phi \]

Magnetic Field

\[ \omega^2 = k^2 v_A^2 \]

\[ v_A^2 = \frac{B^2}{4\pi\rho} \]
Magnetorotational Instability

To the black hole

Differential rotation

Magnetic Field

\[ \frac{d\Omega}{dr} \]

\[ B_z \]

\[ \phi \]

\[ r \]
Magnetorotational Instability

Differential rotation

Magnetic Field
Magnetorotational Instability

\[ B_z \delta B_\phi \]

Differential rotation

Magnetic Field
Magnetorotational Instability

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Differential rotation

Magnetic Field

Magnetorotational Instability

\[ k^2 v^2_{Az} < - \frac{d\Omega^2}{d\ln r} \]

\[ v_A^2 = \frac{B^2}{4\pi\rho} \]
MRI and MHD Turbulence

Differential Rotation

+ Weak Magnetic Field

Magnetorotational Instability

\[ k^2 v_A^2 < -2\Omega^2 \frac{d\ln \Omega}{d\ln r} \]

Stability criterion involving shear and a characteristic scale!

MRI can generate and sustain MHD turbulence
The Ultimate Accretion Disk Model

We want to model these processes!

“Stress Power”

Mode interactions
Parasitic Instabilities

large scales

Parker
MRI
Dissipation

small scales

k
The Mean Field Approach

* Continuity Equation
\[ \frac{D \rho}{Dt} = 0 \]

* Momentum Equation
\[ \frac{D v}{Dt} = f(v, \rho, P, B) \]

* Induction Equation
\[ \frac{D B}{Dt} = f(B, v) \]

* Energy Equation
\[ \frac{D \varepsilon}{Dt} = f(\rho, P) \]

We want to understand the MEAN properties of the disk
Average over small scales!!!
We can derive an equation for angular momentum conservation

\[
\frac{\partial \bar{l}}{\partial t} + \nabla \cdot (\bar{l} \mathbf{v}) = -\frac{1}{r} \frac{\partial}{\partial r} r^2 \left[ \langle \rho \delta v_r \delta v_\phi \rangle - \frac{\langle \delta B_r \delta B_\phi \rangle}{4\pi} \right]
\]

which involves 2nd order correlations…

We can derive equations for 2nd order correlations…

… but they involve 3rd order correlations…

We can derive equations for 3rd order correlations…

This is the CLOSURE problem in MHD turbulence!!
Standard $\alpha$-disks as a Closure Scheme

\[ \frac{\partial \bar{l}}{\partial t} + \nabla \cdot (\bar{l} \mathbf{v}) = -\frac{1}{r} \frac{\partial}{\partial r} r^2 (\bar{R}_{r\phi} - \bar{M}_{r\phi}) \]

Angular Momentum

Reynolds Stress
\[ \bar{R}_{r\phi} = \left\langle \rho \delta v_r \delta v_\phi \right\rangle \]

Maxwell Stress
\[ \bar{M}_{r\phi} = \left\langle \delta B_r \delta B_\phi \right\rangle / 4\pi \]

$\alpha$–prescription (Shakura & Sunyaev)
\[ \bar{R}_{r\phi} - \bar{M}_{r\phi} = \alpha P \left( -\frac{d \ln \Omega}{d \ln r} \right) \]
Problems with the Standard Model

- Angular momentum diffuses …
  *Causality Problems* (Popham & Narayan, 1992)

- MHD turbulence assumed but …
  *NO explicit MRI*

- Angular momentum is always transported …
  *NO Stability Criterion*

**Approach**
- Model for *MRI-driven angular momentum transport*
What do we know about MRI-turbulence?

- Maxwell
- Reynolds
- Saturation
- Non-linear
- MRI growth
The equations for the stresses will look like

\[
\frac{\partial \overline{R_{r\phi}}}{\partial t} + (\overline{v \cdot \nabla}) \overline{R_{r\phi}} = \text{linear} + \text{non-linear}
\]

We know the linear terms exactly
(2nd order correlations)

We must model the non-linear terms
(3rd order correlations)
A Model for Turbulent MRI-driven Stresses

\[
(\partial_t + \mathbf{v} \cdot \nabla) \bar{R}_{r\phi} = 2 \bar{R}_{\phi\phi} - \frac{\kappa^2}{2} \bar{R}_{rr} - \bar{W}_{rr} + \bar{W}_{\phi\phi}
\]

\[
(\partial_t + \mathbf{v} \cdot \nabla) \bar{W}_{rr} = q \bar{W}_{r\phi} + 2 \bar{W}_{\phi r} + \zeta^2 k_{\text{max}}^2 (\bar{R}_{r\phi} - \bar{M}_{r\phi})
\]

\[
(\partial_t + \mathbf{v} \cdot \nabla) \bar{W}_{\phi\phi} = -\frac{\kappa^2}{2} \bar{W}_{r\phi} - \zeta^2 k_{\text{max}}^2 (\bar{R}_{r\phi} - \bar{M}_{r\phi})
\]

\[
(\partial_t + \mathbf{v} \cdot \nabla) \bar{M}_{r\phi} = -q \bar{M}_{rr} + \bar{W}_{rr} - \bar{W}_{\phi\phi}
\]

**Epicyclic freq.**
\[
k^2 = 2(2 - q)
\]

**Local shear**
\[
q = -\frac{d \ln \Omega}{d \ln r}
\]

**\( \bar{k}_{ij} = \zeta \ k_{\text{max}} \)**

\[
k_{\text{max}}^2 = q - \frac{q^2}{4}
\]

The ratio between the different stress components MUST be only a function of the local shear \( q \)!!!!!
The correlation $W$ is not only important in the linear regime but also in the turbulent state!

 Chan, Pessah, & Psaltis, 2006

**$W$ in the Turbulent State**

![Graph showing stresses over orbital periods](image)
The parameter $\zeta$ controls the ratio between Reynolds & Maxwell stresses.

$$\bar{k}_{ij} = \zeta k_{\text{max}}$$

We have a model that produces initial exponential growth AND leads to stresses with ratios in agreement with the saturated regime.
A Model for the Saturation of the MRI

We have a model for MRI pumping. We need a model for the saturation.

Dimensional analysis:

\[
\overline{M_0} \sim \rho_0 H^\delta \Omega_0^\delta \overline{V_{Az}}^{(2-\delta)}
\]

\[
\overline{M_0} = \xi \rho_0 H \Omega_0 \overline{V_{Az}}
\]

A physically motivated model for angular momentum transport in Keplerian disks with only two parameters
The parameter $\xi$ controls the level at which the magnetic energy saturates.

$$\overline{M_0} = \xi \rho_0 H \Omega_0 v_{Az}$$

A physically motivated model for angular momentum transport Keplerian disks that incorporates the MRI!
A Local Model for Angular Momentum Transport in Turbulent Magnetized Disks

A model for MRI-driven angular momentum transport in agreement with numerical simulations
Global Models for Accretion Onto Stars

\[ \Omega(R) \]

\[ \Omega_* \]

\[ R_* \]

\[ R \]

\[ \alpha \text{-prescription postulates negative stresses in the Boundary layer} \]

\[ \bar{T}_{r\phi} = \alpha P \left( -\frac{d \ln \Omega}{d \ln r} \right) < 0 \]

Keplerian disk

\[ \Omega(R) \sim R^{-3/2} \]

MRI-stable <-> MRI-unstable

Advection \sim R
It is usually argued that stresses vanish at the inner disk.

Simulations show stresses *DO NOT* vanish inside ISCO.

Implications:

* Inner disk structure
* Emission inside ISCO
* Spin measurements
* Spin evolution
We now have a model for a key process in accretion physics.

Global models will shed new light into turbulent magnetized accretion onto stars and compact objects.

We need good theoretical *global* disk models to take full advantage of Beyond Einstein missions such as *Con-X*. 