Applied Statistics

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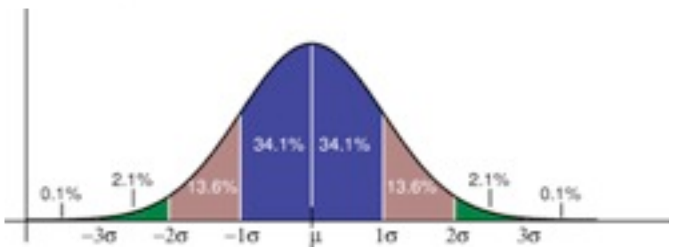

"Statistics is merely a quantization of common sense"

Error propagation

Imagine that y is a function of x, and that we wish to find the error on y from x. Making a Taylor expansion of the function y gives:

$$y(\bar{x}) \approx y(\bar{\mu}) + \sum_{i=1}^{n} \frac{\delta y}{\delta x_i}(x_i - \mu_i)$$

In order to get the uncertainty of y as a function of the variables x_i we calculate:

$$\begin{aligned} E[y(\bar{x})] &\approx y(\bar{\mu}) \\ E[y^2(\bar{x})] &\approx y^2(\bar{\mu}) + \sum_{i,j=1}^{n} \left[\frac{\delta y}{\delta x_i}\frac{\delta y}{\delta x_j}\right]_{\bar{x}=\bar{\mu}} V_{ij} \end{aligned}$$

Error propagation formula

$$\sigma_y^2 = \sum_{i,j=1}^{n} \left[\frac{\delta y}{\delta x_i} \frac{\delta y}{\delta x_j} \right]_{\bar{x}=\bar{\mu}} V_{ij}$$

If there are no correlations, only the diagonal (individual errors) enter:

$$\sigma_y^2 = \sum_{i=1}^{n} \left[\frac{\delta y}{\delta x_i} \right]_{\bar{x}=\bar{\mu}}^2 \sigma_i^2$$

Specific error propagation formula
Addition

$$x = au + bv$$

$$\sigma_x^2 = a^2\sigma_u^2 + b^2\sigma_v^2 + 2ab\sigma_{uv}^2$$

Specific error propagation formula
Multiplication

$$x = auv$$

$$\sigma_x^2 = (av\sigma_u)^2 + (au\sigma_v)^2 + 2a^2uv\sigma_{uv}^2$$

$$\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} + 2\frac{\sigma_{uv}^2}{uv}$$

Advanced example of error propagation (Higgs particle mass):