Applied Statistics

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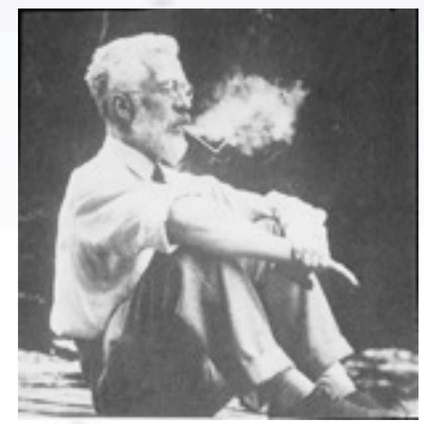


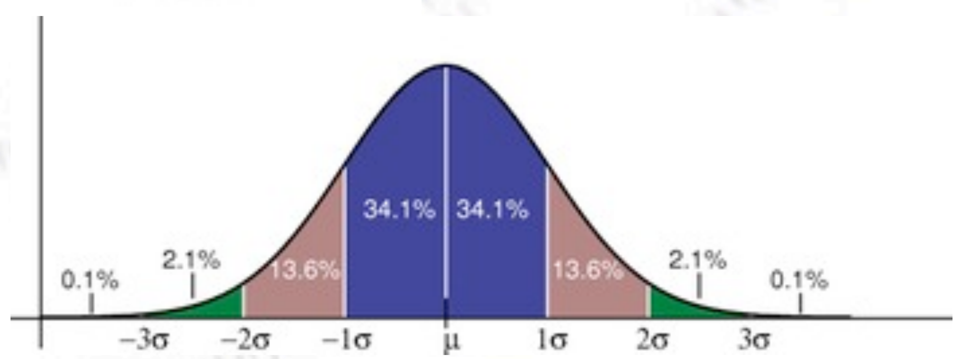

"Statistics is merely a quantization of common sense"

Likelihood function

I shall stick to the principle of likelihood...

[Plato, in Timaeus]

Likelihood function

Given a set of measurements, x, and parameter(s) a, the likelihood function is defined as:

$$L(x_1, \ldots, x_N; a) = \prod P(x_i, a)$$

The **principle of maximum likelihood** for parameter estimation consist of maximizing the likelihood of parameter(s) (here a) given some data (here x).

The likelihood function plays a central role in statistics, as it can be shown to be:
• Consistent (converges to the right value!)
• Asymptotically normal (converges with Gaussian errors).
• Efficient (reaches the Cramer-Rao lower bound for large N).

To some extend, this means that the likelihood function is “optimal”, that is, if it can be applied in practice.

Likelihood vs. ChiSquare

For computational reasons, it is often much easier to minimize the logarithm of the likelihood function:

$$\left.\frac{d \ln L}{da}\right|_{a=\hat{a}} = 0.$$

In problems with Gaussian errors, it turns out that the **likelihood function** boils down to the **ChiSquare** with a constant offset and a factor -2 in difference.

In practice, the likelihood comes in two versions:

- Binned likelihood (using Poisson).
- Unbinned likelihood (using PDF).

The "trouble" with the likelihood is, that it is unlike the ChiSquare, there is NO simple way to obtain a probability of obtaining certain likelihood value!

Also, the unbinned likelihood only works, when you actually know the PDF in question. That is not always the case (e.g. data of project 1).