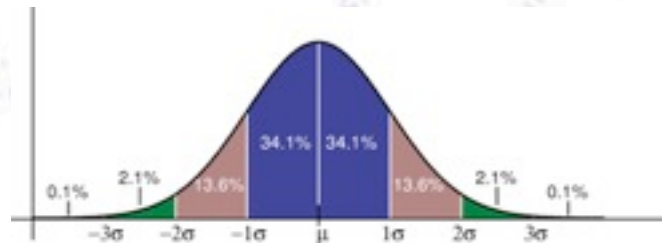


Applied Statistics

Binomial, Poisson, and Gaussian



Troels C. Petersen (NBI)



“Statistics is merely a quantization of common sense”

Binomial, Poisson, Gaussian

Given N trials each with p chance of success, how many successes (n) in total?

This distribution is...

Binomial

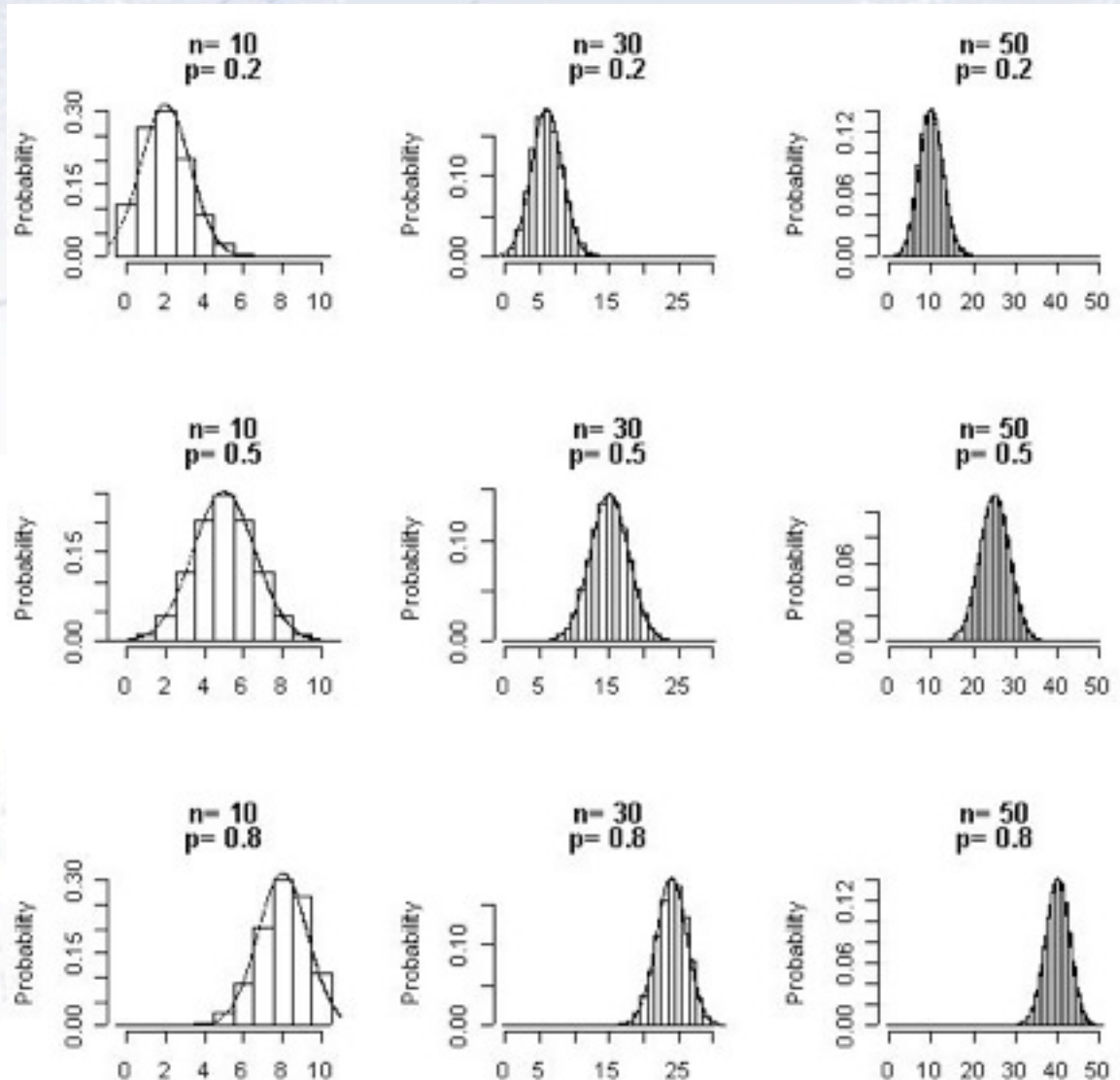
$$f(n; N, p) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n},$$

Mean = Np

Variance = $Np(1-p)$

This means, that the error on a fraction $f = n/N$ is:

$$\sigma(f) = \text{Sqrt}(f \cdot (1-f) / N)$$



Binomial, Poisson, Gaussian

Given N
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$f(n; N, p) =$

Mean =]
Variance

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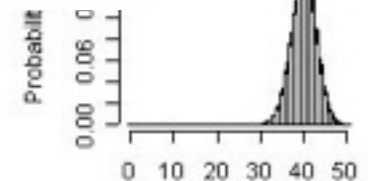
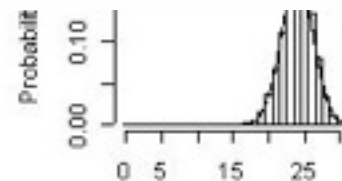
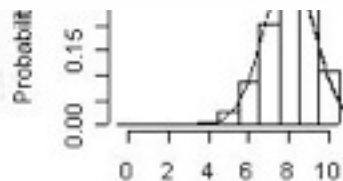
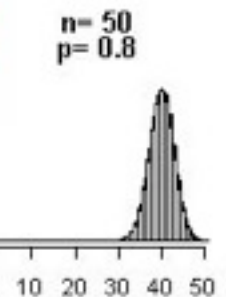
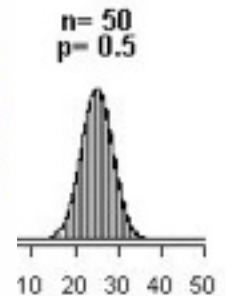
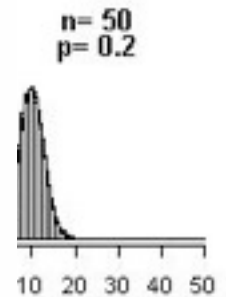
$$\sigma(f) = \text{Sqrt}(f*(1-f)/N)$$

Binomial

Mean $\mu = n \times \pi$

Variance $n\pi(1-\pi)$

Std Dev $\sqrt{n\pi(1-\pi)}$



Binomial, Poisson, Gaussian

You count 100 cars, and see 15 red cars. What is your estimate of the fraction (i.e. probability) of red cars and its uncertainty?

- a) 0.150 ± 0.030
- b) 0.150 ± 0.026
- c) 0.150 ± 0.036
- d) 0.125 ± 0.030
- e) 0.150 ± 0.010

(From previous page: $\sigma(f) = \text{Sqrt}(f*(1-f)/N)$)

A friend tells you, that 8% of the cars on Blegdamsvej are red. What is the chance of that? Could he be right?

Binomial, Poisson, Gaussian

If $N \rightarrow \infty$ and $p \rightarrow 0$, but $Np \rightarrow \lambda$ then a Binomial approaches a Poisson:

$$f(n; \nu) = \frac{\nu^n}{n!} e^{-\nu}$$

In reality, the approximation is already quite good at e.g. $N=50$ and $p=0.1$.

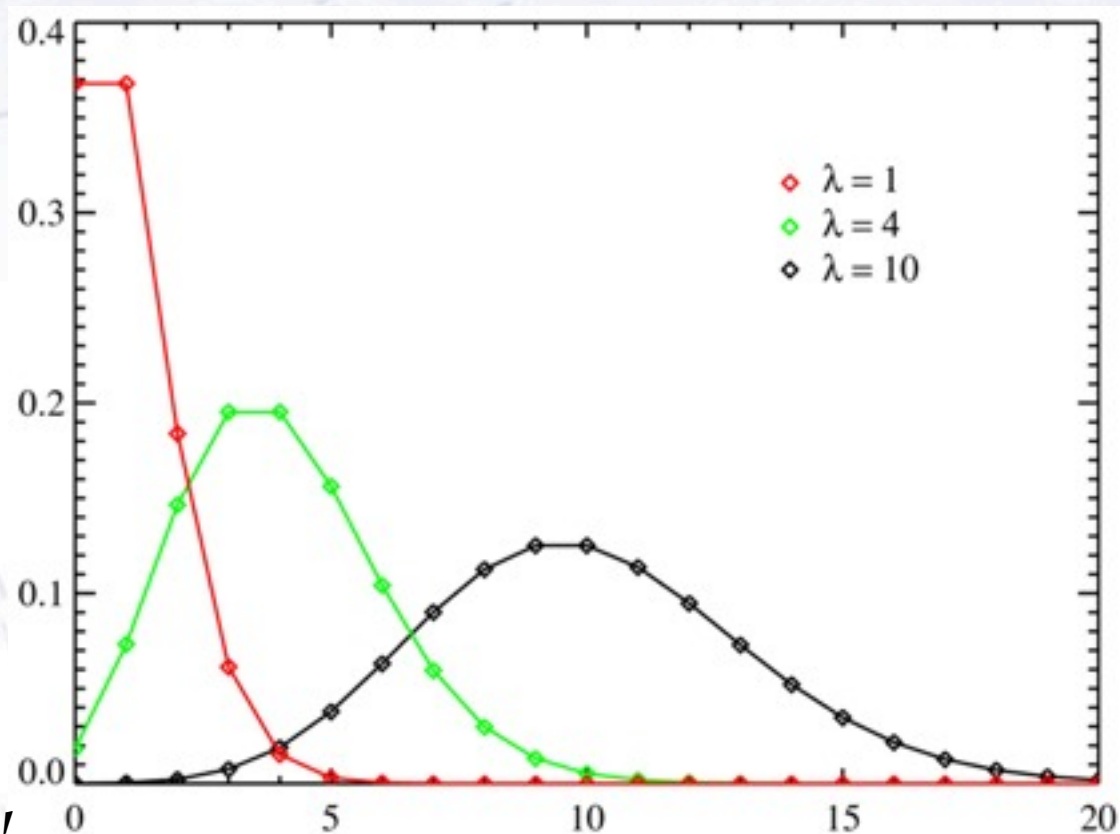
The Poisson distribution only has one parameter, namely λ .

Mean = λ

Variance = λ

So the error on a number is...

...the square root of that number!



Binomial, Poisson, Gaussian

The error on a

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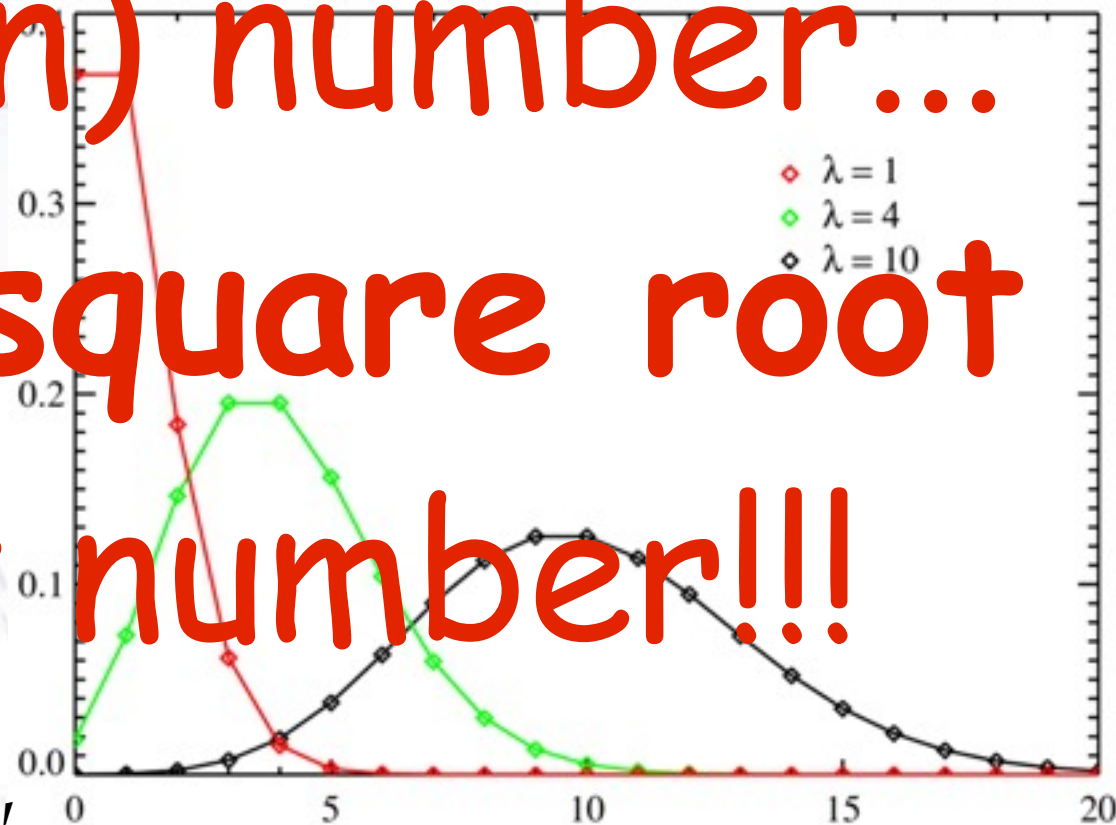
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The error on a
(Poisson) number...
is the square root
of that number!!!

Binomial, Poisson, Gaussian

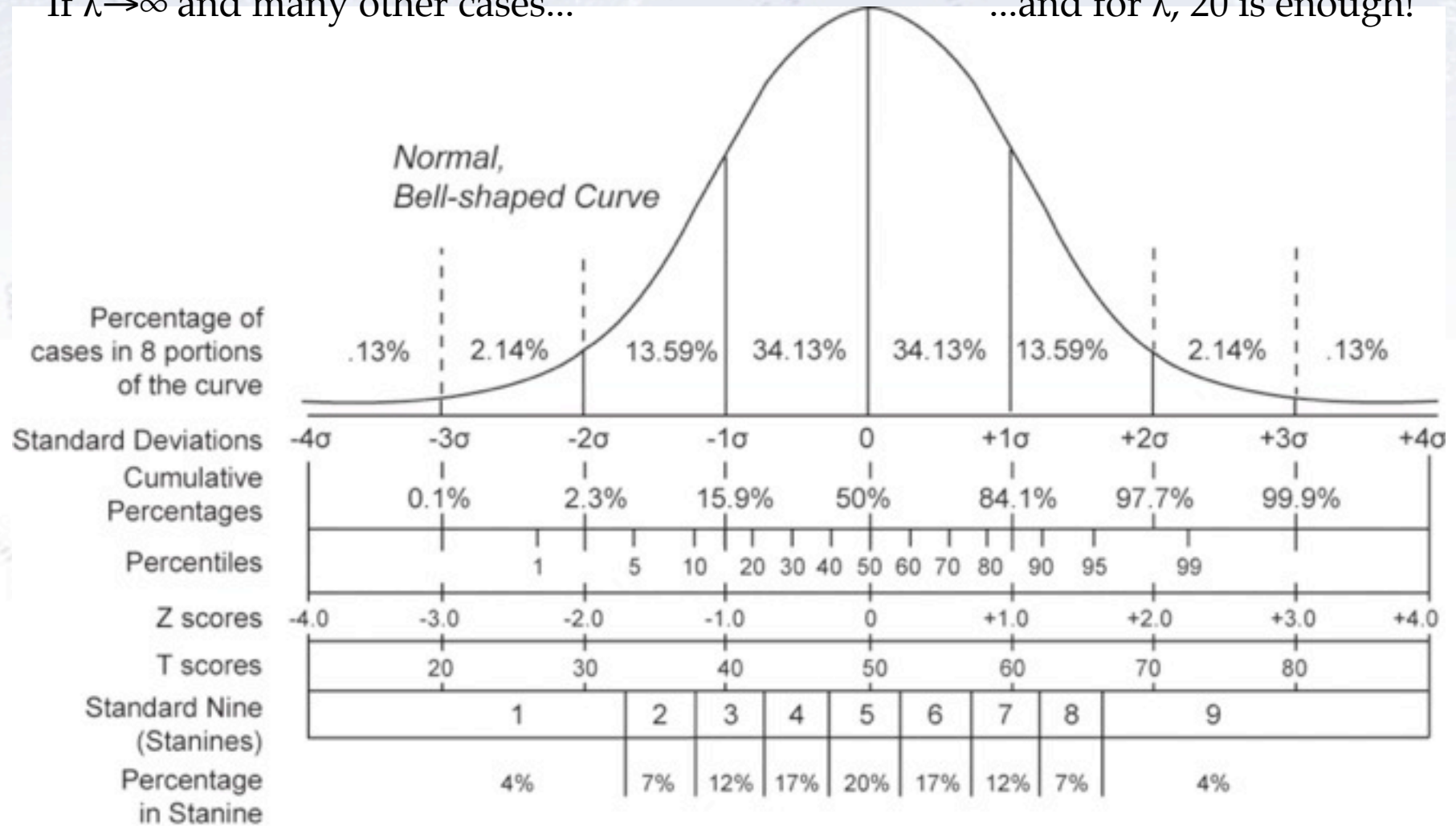
Each year approximately 6300 students start their studies at University of Copenhagen. What is roughly the uncertainty on this number?

- a) 100
- b) 300
- c) 63
- d) 80
- e) Cannot be determined from the information given.

Binomial, Poisson, Gaussian

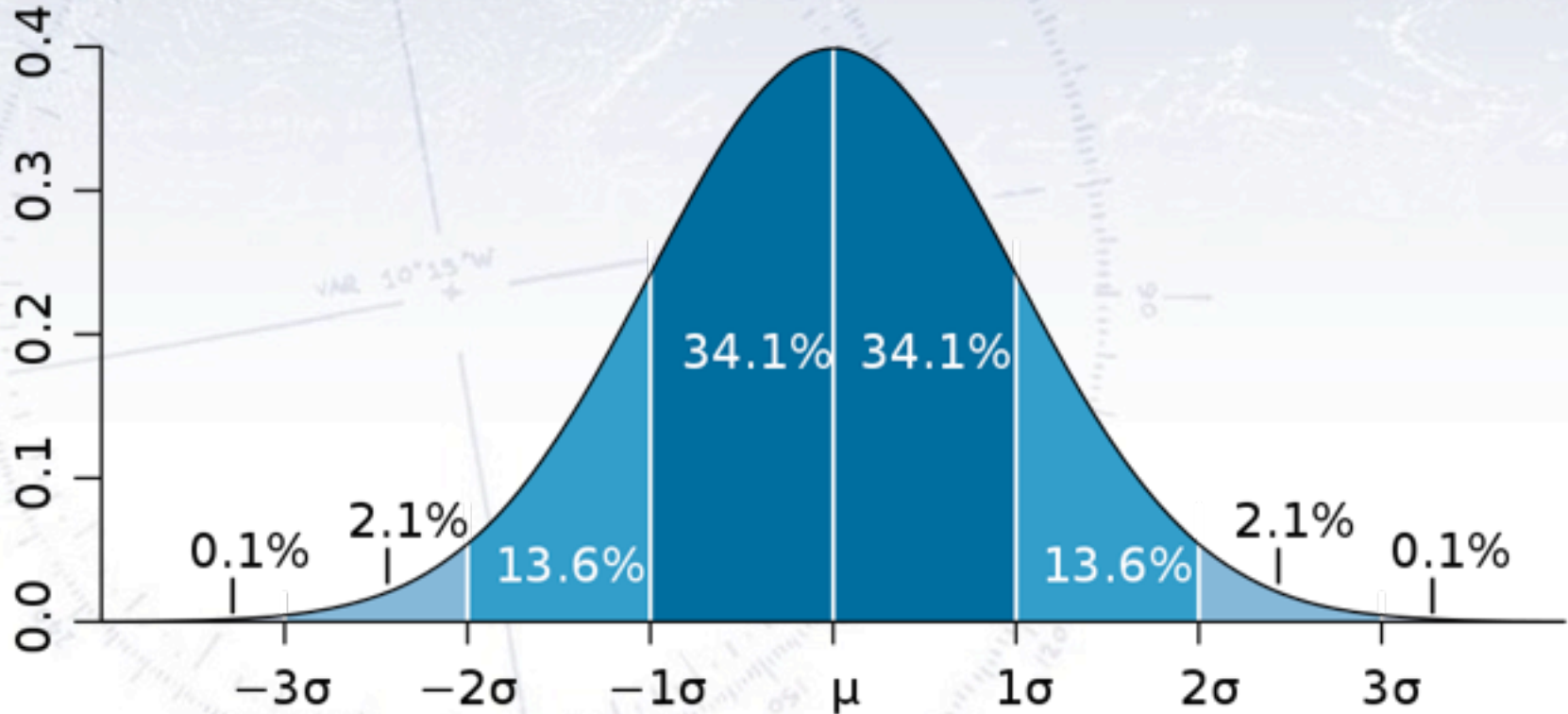
If $\lambda \rightarrow \infty$ and many other cases...

...and for λ , 20 is enough!



Binomial, Poisson, Gaussian

"If the Greeks had known it, they would have deified it."



"If the Greeks had known it, they would have deified it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The more huge the mob and the greater the apparent anarchy, the more perfect is its sway. It is the supreme Law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshaled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to be latent all along." [Karl Pearson]