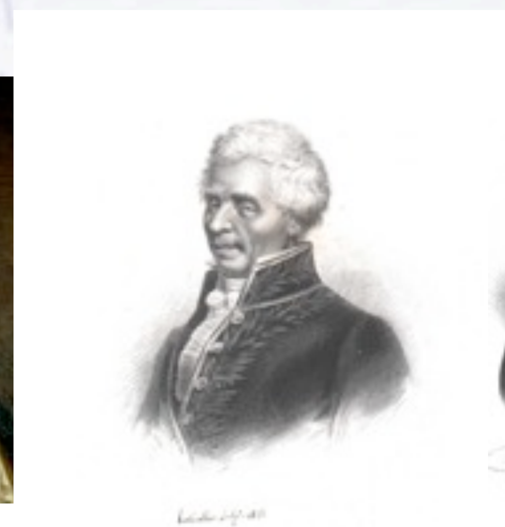
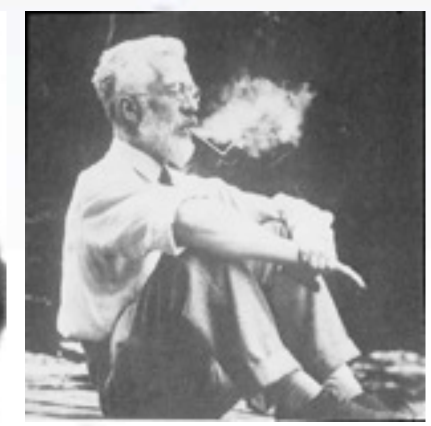
Applied Statistics

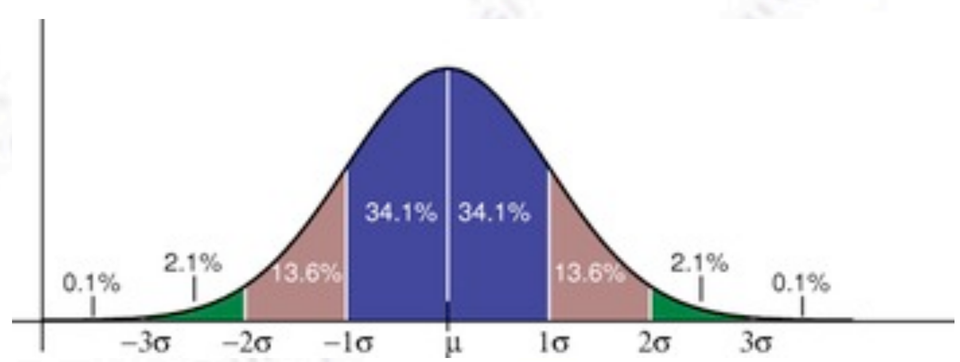
Monte Carlo Simulations







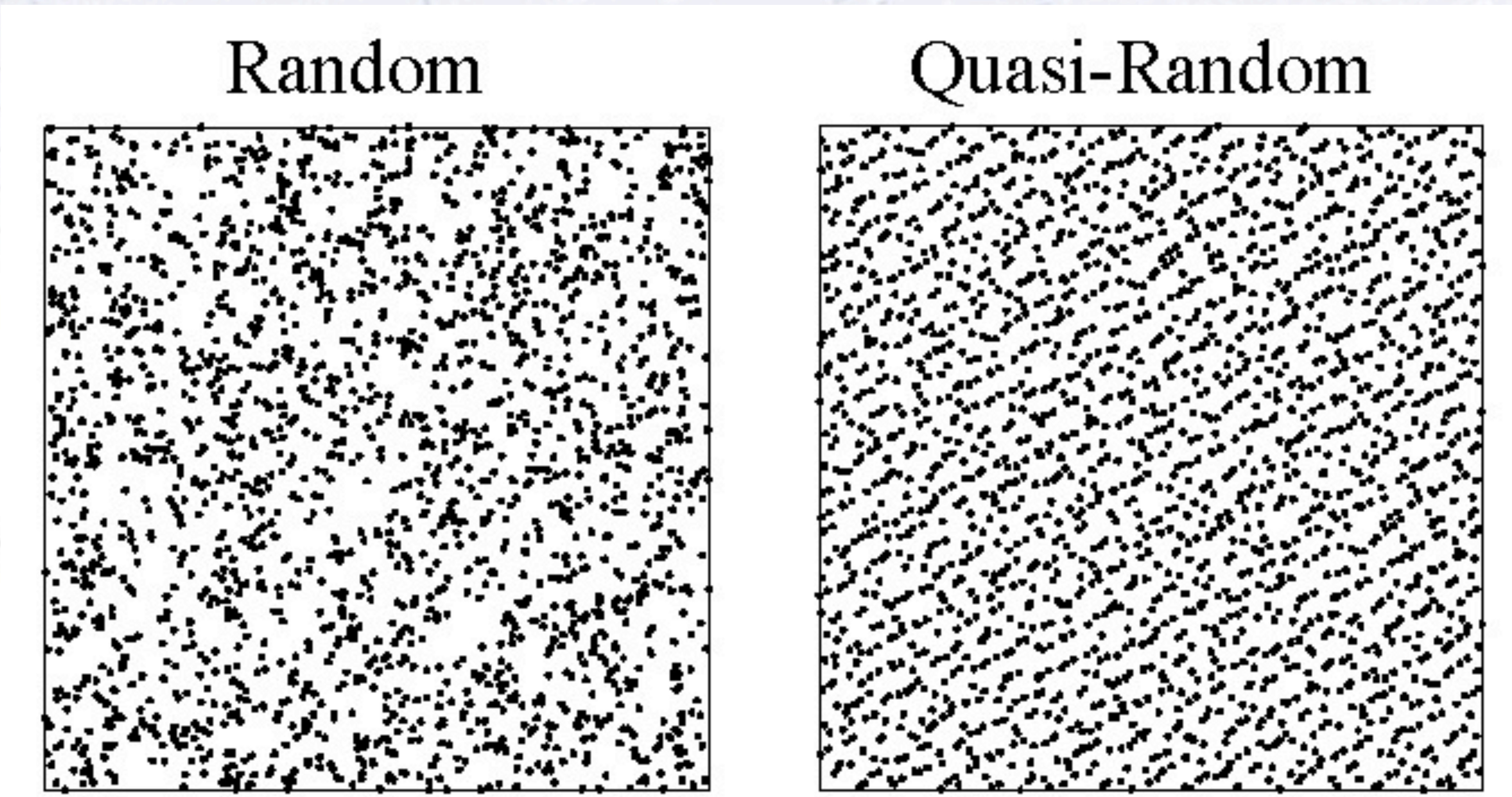

Troels C. Petersen (NBI)



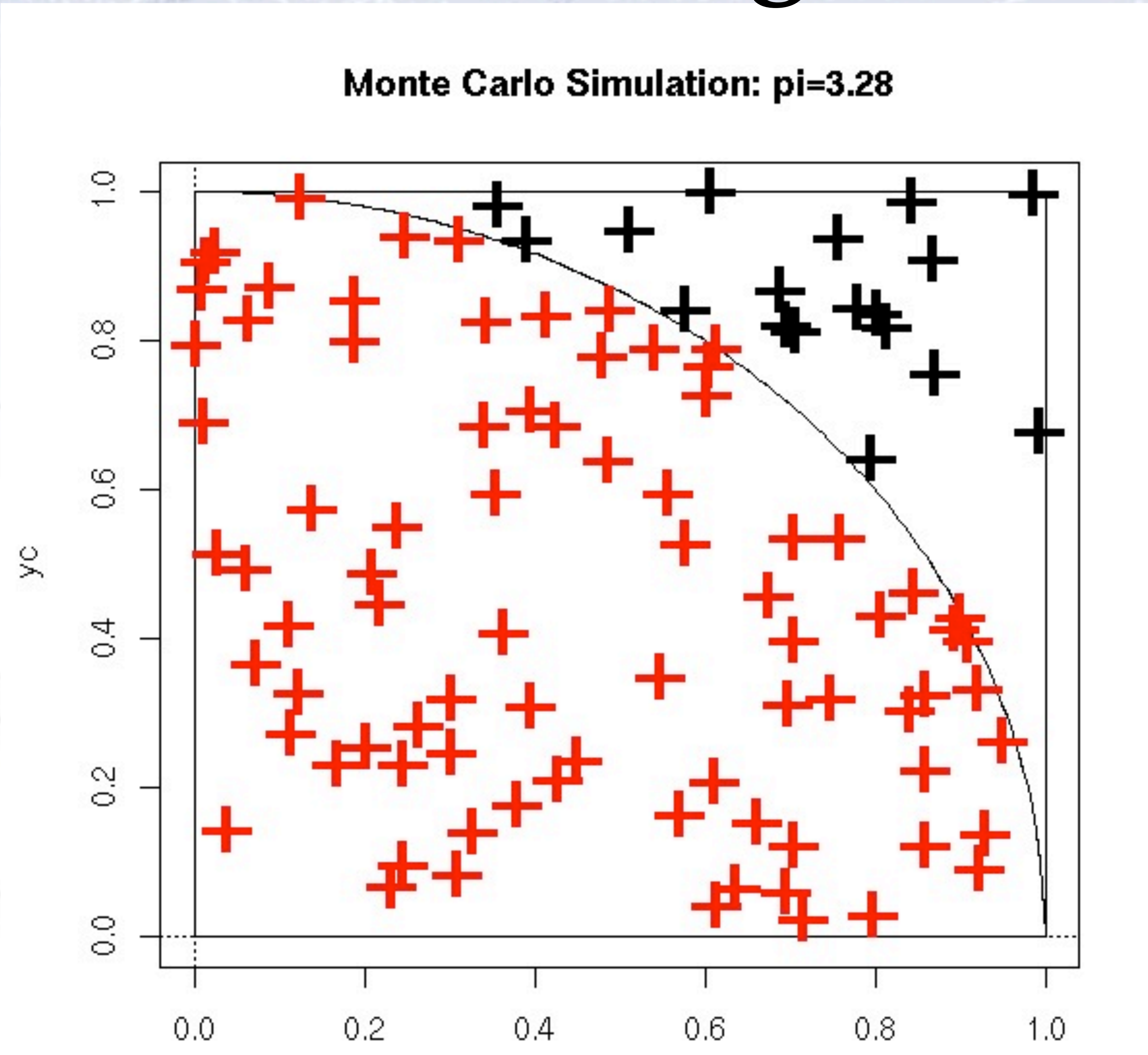
"Statistics is merely a quantization of common sense"

Random numbers

"Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin." [John Von Neumann]



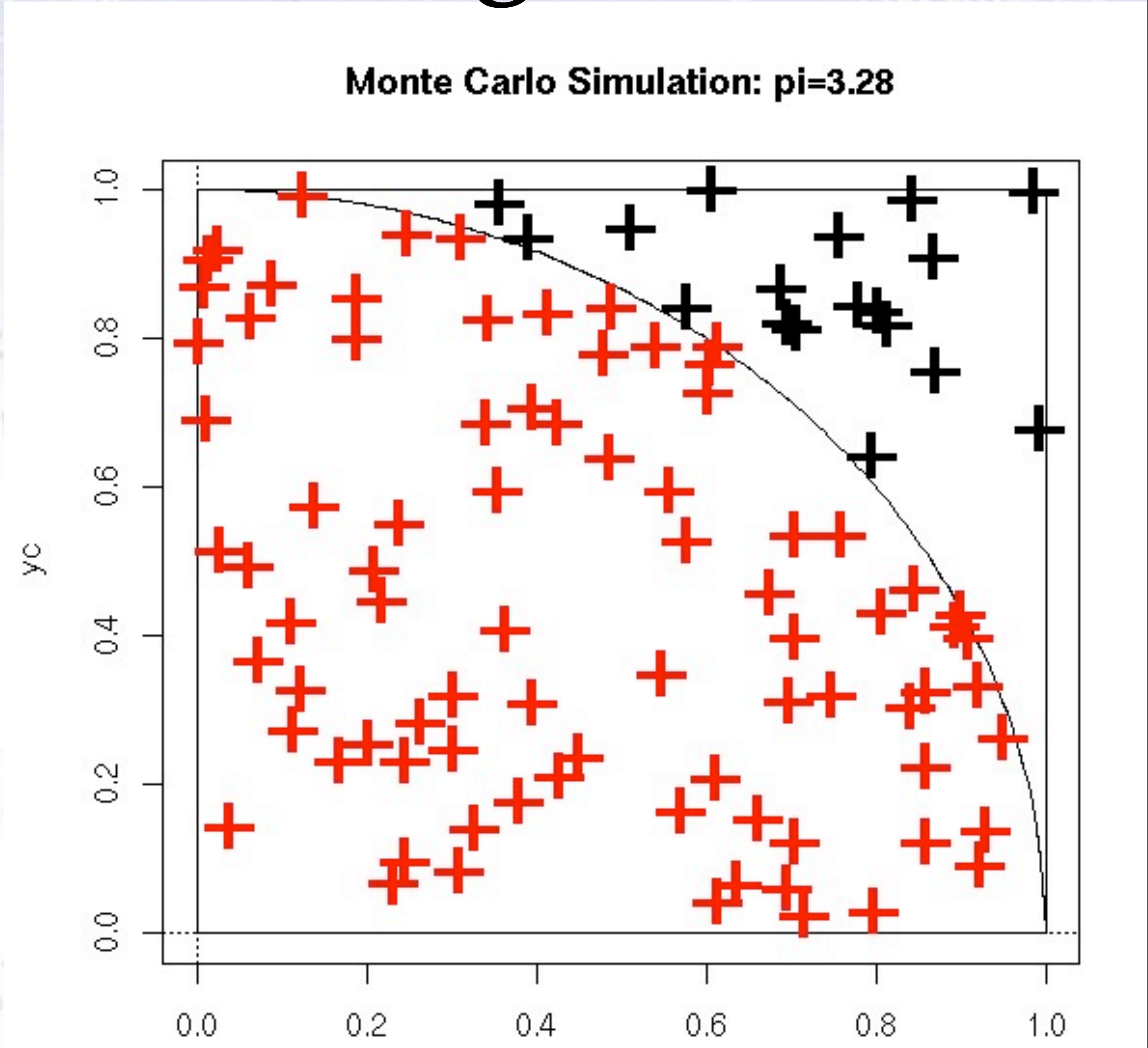
Calculating Pi



Calculating Pi

This is called the Hit-and-Miss method (Von Neumann method):

- Find min and max in both x and y.
- Generate uniform random numbers (x,y) in these ranges.
- Accept x, if y < f(x).
- Reject x, if y > f(x).



Transformation method

We have uniformly distributed random numbers r. We want random numbers x according to some distribution.

We want to try to find a function x(r), such that g(r) (uniformly distributed numbers) will be transformed into the desired distribution f(x).

It turns out, that this is only possible, if one can (in this order):

- Integrate f(x)
- Invert F(x)

As this is rarely the case, this method can rarely be used by itself.

However, in combination with the Hit-and-Miss (Von Neumann) method, it can pretty much solve all problems.

Dimensionality of problems

For simple "low dimensionality" problems, (possibly) numerical integrals are the fastest solution.

However, with increasing complexity, the Monte Carlo method win:

- Monte Carlo method: $$\frac{1}{\sqrt{N}}$$
- Numerical (e.g. trapizoidal rule): $$\frac{1}{N^{2/d}}$$

The Monte Carlo method is also easier to get uncertainties from, and usually quicker to implement.

History of the Monte Carlo

The first thoughts and attempts I made to practice [the Monte Carlo Method] were suggested by a question which occurred to me in 1946 as I was convalescing from an illness and playing solitaires. **The question was what are the chances that a Canfield solitaire laid out with 52 cards will come out successfully?** After spending a lot of time trying to estimate them by pure **combinatorial calculations**, I wondered whether a **more practical method** than "abstract thinking" might not be to lay it out say one hundred times and simply observe and count the number of successful plays. This was already possible to envisage with the beginning of the new era of fast computers, and I immediately thought of problems of neutron diffusion and other questions of mathematical physics, and more generally how to change processes described by certain differential equations into an equivalent form interpretable as a succession of random operations. Later [in 1946], I described the idea to John von Neumann, and we began to plan actual calculations.

[Stanisław Ulam]