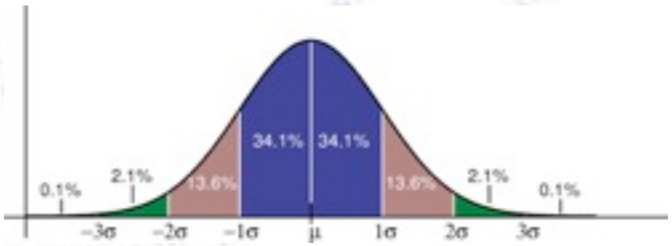
Applied Statistics

Multivariate analysis






Troels C. Petersen (NBI)

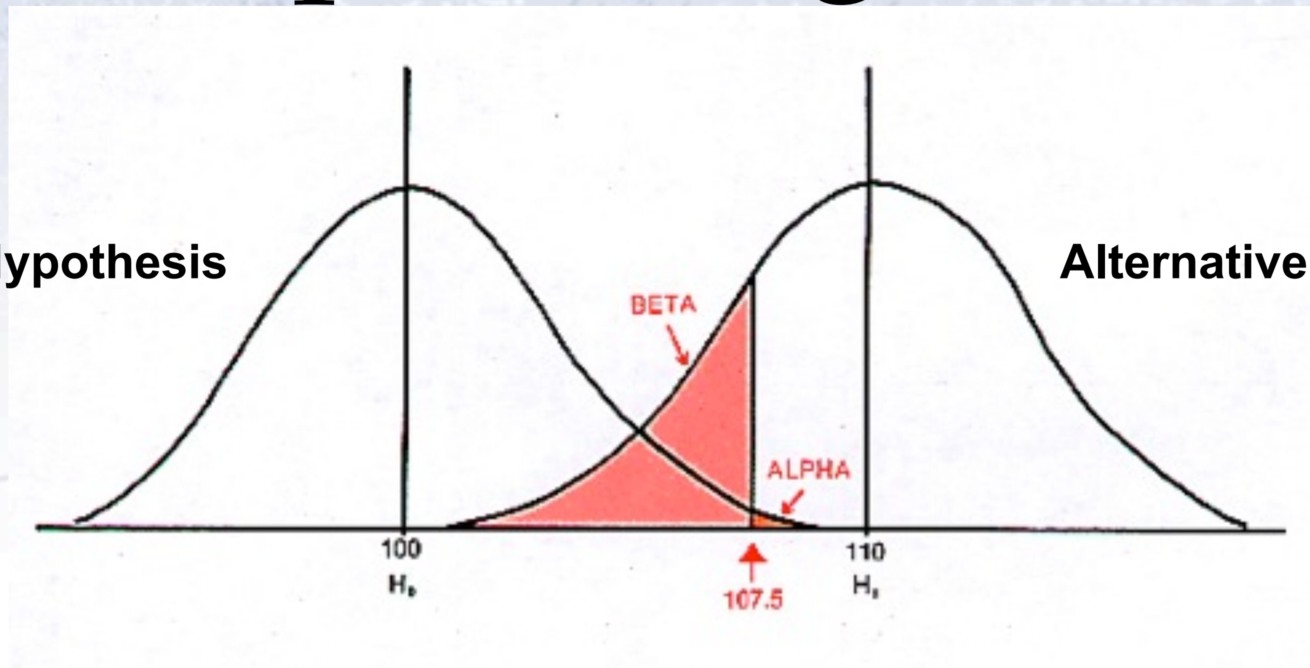


"Statistics is merely a quantization of common sense"

Separating data

Null Hypothesis

Alternative Hypothesis



REALITY

STATISTICAL DECISION:		Null is True	Null is False
	Do Not Reject Null	$1 - \alpha$ Correct	β Type II error
	Reject Null	α Type I error	$1 - \beta$ Correct

Separating data

Fisher's friend, Anderson, came home from picking Irises in the Gaspe peninsula...

180 MULTIPLE MEASUREMENTS IN TAXONOMIC PROBLEMS

Table I

Iris setosa				*Iris versicolor*				*Iris virginica*			
Sepal length	Sepal width	Petal length	Petal width	Sepal length	Sepal width	Petal length	Petal width	Sepal length	Sepal width	Petal length	Petal width
5·1	3·5	1·4	0·2	7·0	3·2	4·7	1·4	6·3	3·3	6·0	2·5
4·9	3·0	1·4	0·2	6·4	3·2	4·5	1·5	5·8	2·7	5·1	1·9
4·7	3·2	1·3	0·2	6·9	3·1	4·9	1·5	7·1	3·0	5·9	2·1
4·6	3·1	1·5	0·2	5·5	2·3	4·0	1·3	6·3	2·9	5·6	1·8



5·8	4·0	1·2	0·2	5·6	2·9	3·6	1·3	5·8	2·8	5·1	2·4
5·7	4·4	1·5	0·4	6·7	3·1	4·4	1·4	6·4	3·2	5·3	2·3
5·4	3·9	1·3	0·4	5·6	3·0	4·5	1·5	6·5	3·0	5·5	1·8
5·1	3·5	1·4	0·3	5·8	2·7	4·1	1·0	7·7	3·8	6·7	2·2
5·7	3·8	1·7	0·3	6·2	2·2	4·5	1·5	7·7	2·6	6·9	2·3

Fisher Discriminant

You want to separate two types/classes of events using several measurements.

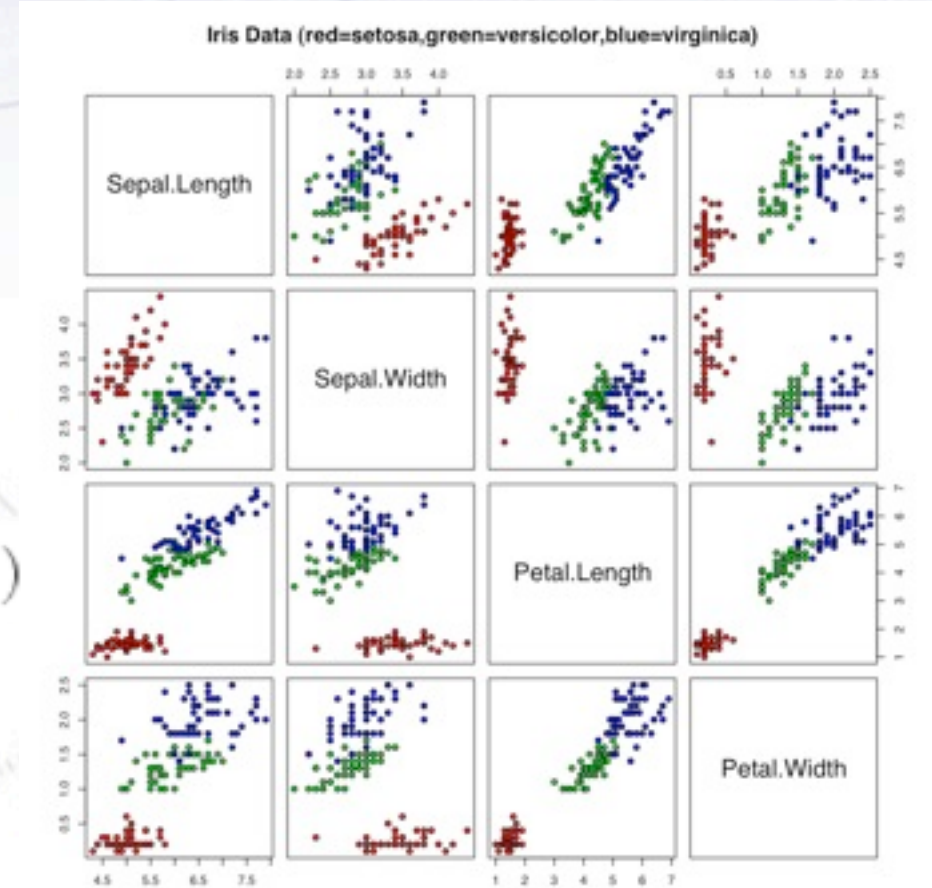
Q: How to combine the variables?
A: Use the Fisher Discriminant:

$$X = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$$

Q: How to choose the values of λ?
A: Inverting the covariance matrices:

$$\vec{w} = (\Sigma_{y=0} + \Sigma_{y=1})^{-1}(\vec{\mu}_{y=1} - \vec{\mu}_{y=0})$$

This can be calculated analytically, and incorporates the correlations into the separation capability.



Fisher Discriminant

You want to separate two types/classes of events using several measurements.

Q: How to combine the variables?

ments are given. We shall first consider the question: What linear function of the four measurements

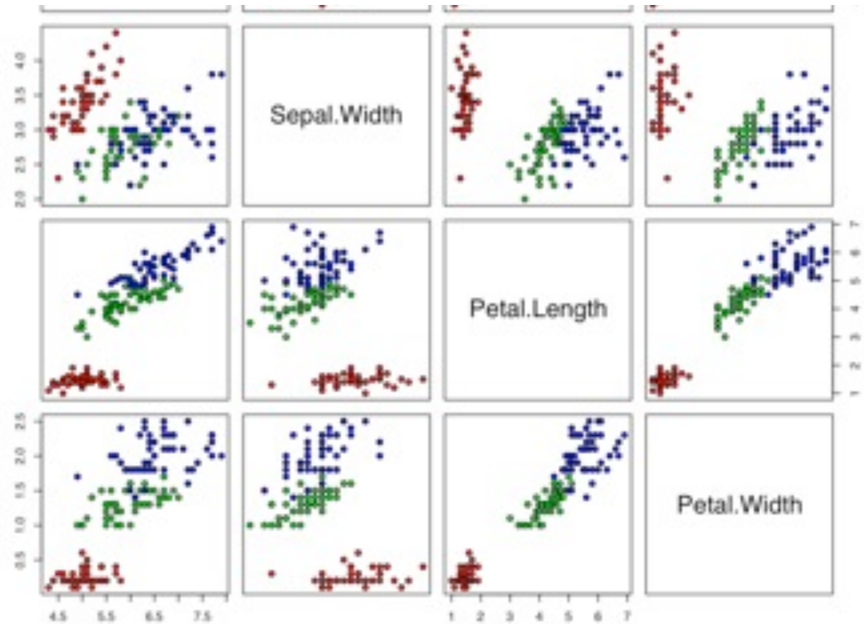
$$X = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4$$

will maximize the ratio of the difference between the specific means to the standard deviations within species? The observed means and their differences are shown in Table II.

Q: How to choose the values of λ?
A: Inverting the covariance matrices:

$$\vec{w} = (\Sigma_{y=0} + \Sigma_{y=1})^{-1}(\vec{\mu}_{y=1} - \vec{\mu}_{y=0})$$

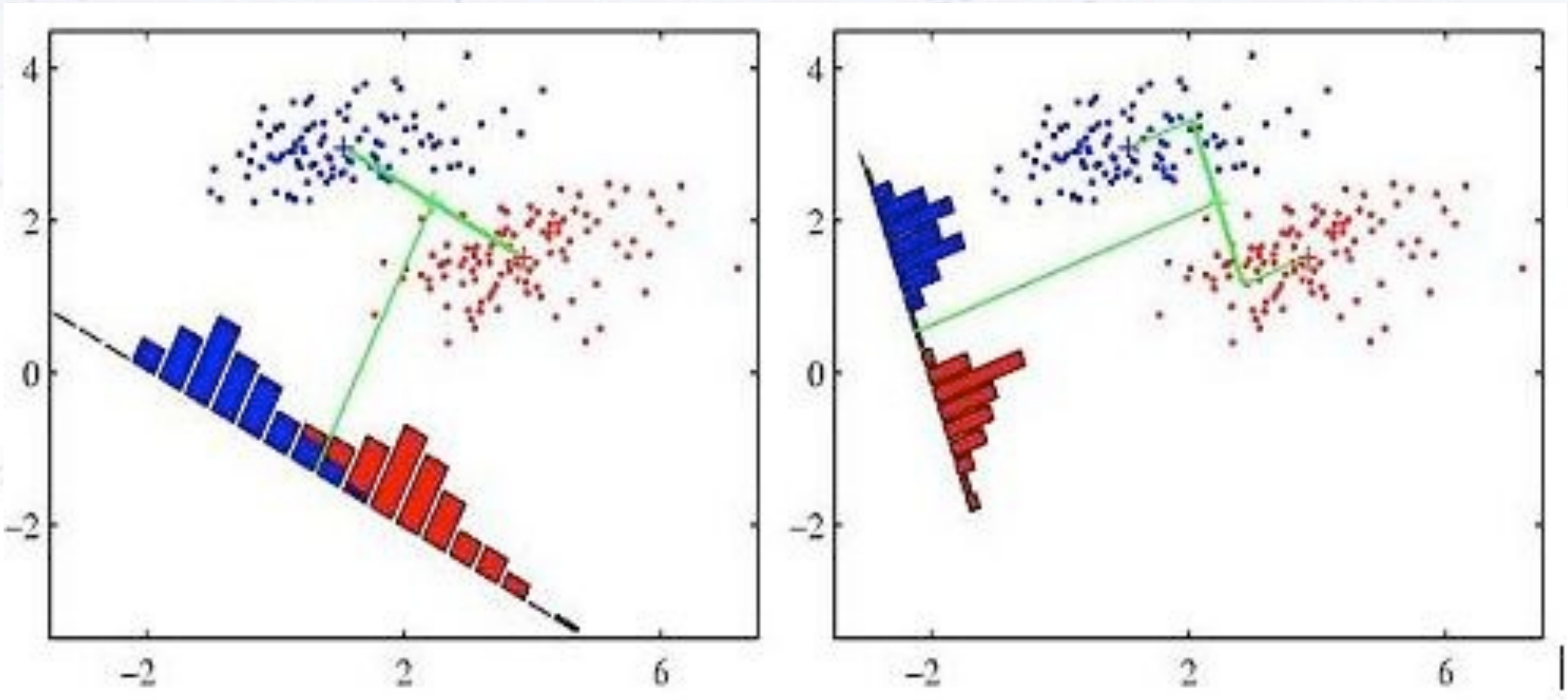
This can be calculated analytically, and incorporates the correlations into the separation capability.



Fisher Discriminant

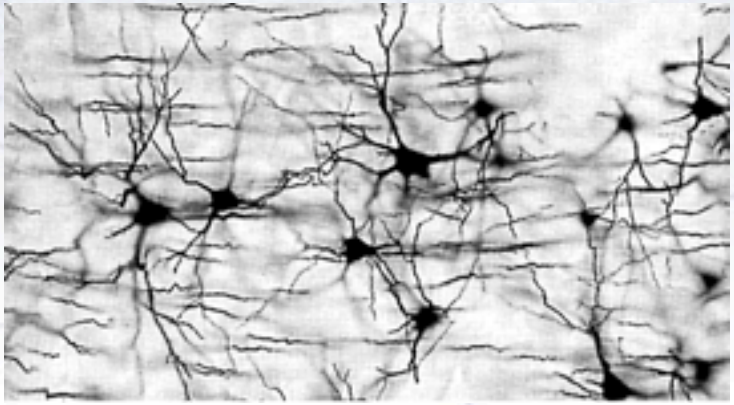
<u>Executive summary:</u>

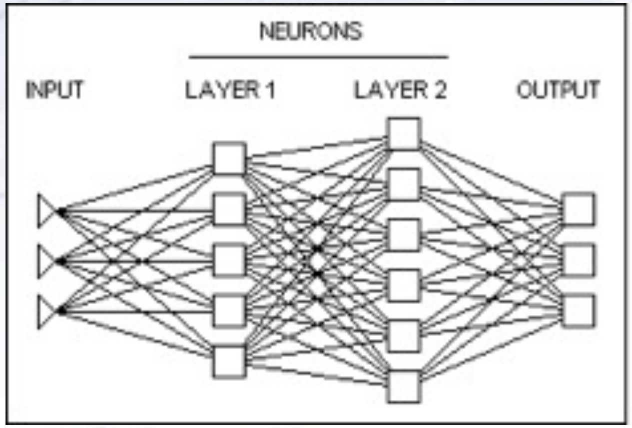
Fisher's Discriminant uses a linear combination of variables to give a single variable with the maximum possible separation (for linear combinations!).



Data Mining

Seeing patterns in data and using it!

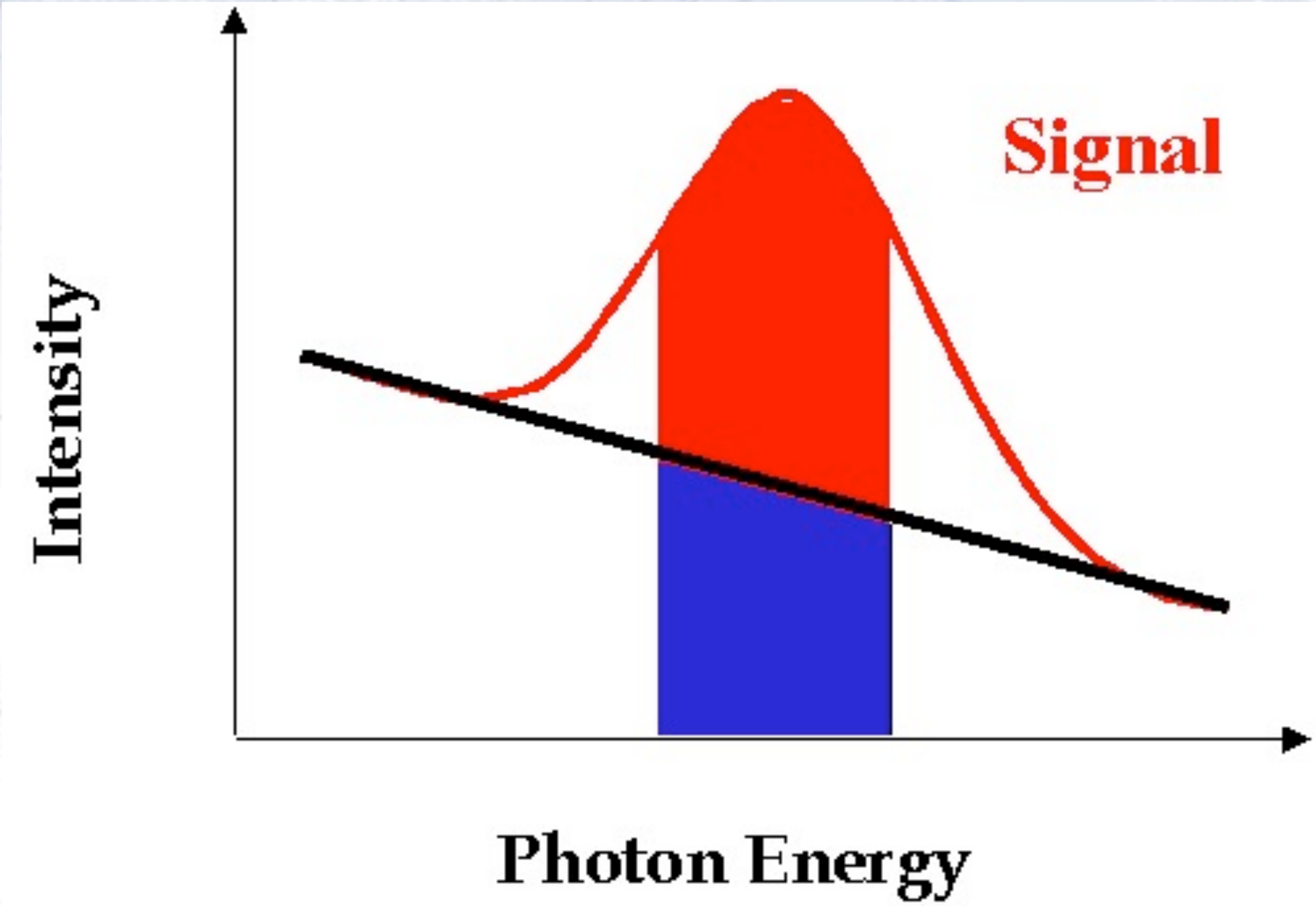




Data mining *is the process of extracting patterns from data. As more data are gathered, with the amount of data doubling every three years, data mining is becoming an increasingly important tool to transform these data into information. It is commonly used in a wide range of prolifiling practices, such as marketing, surveillance, fraud detection and* ***scientific discovery.***

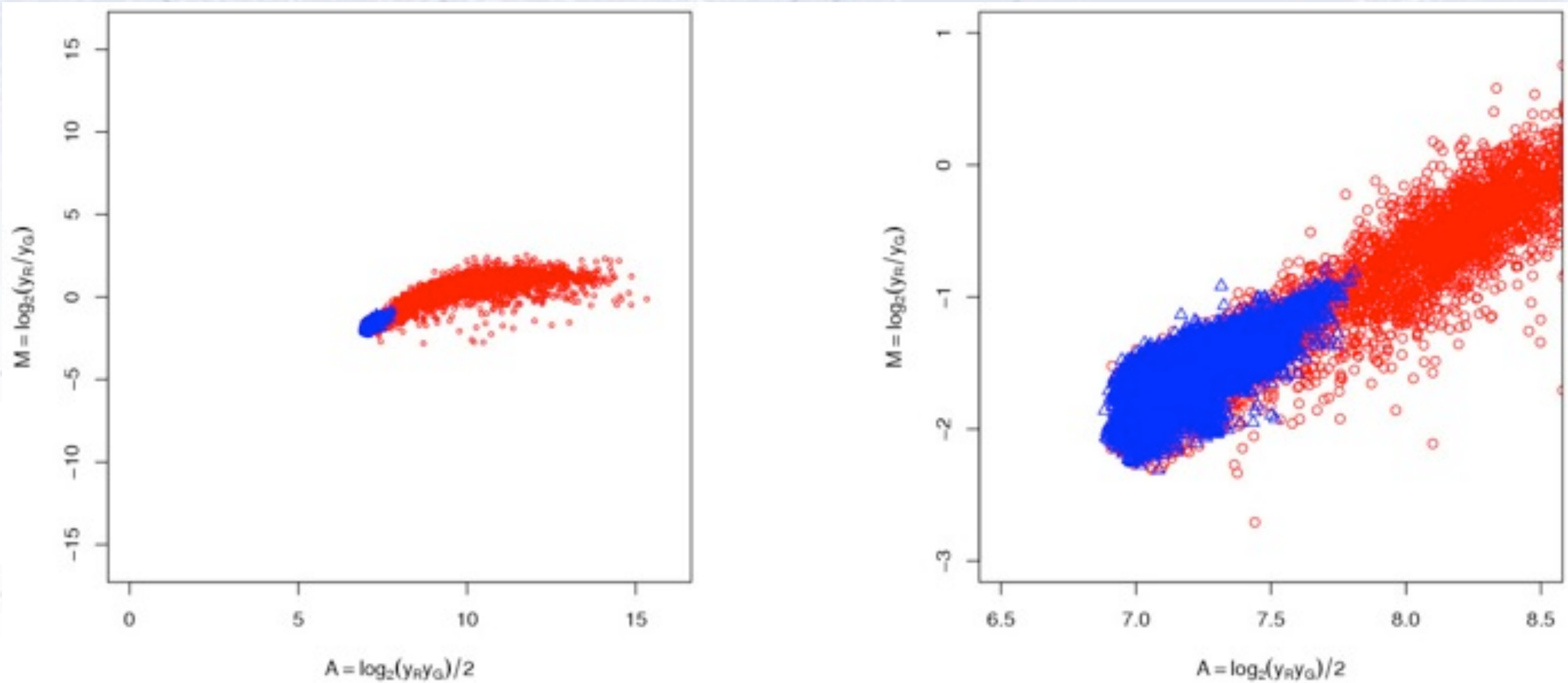
[Wikipedia, Introduction to Data Mining]

Cuts



Classical case (signal peak on background)…

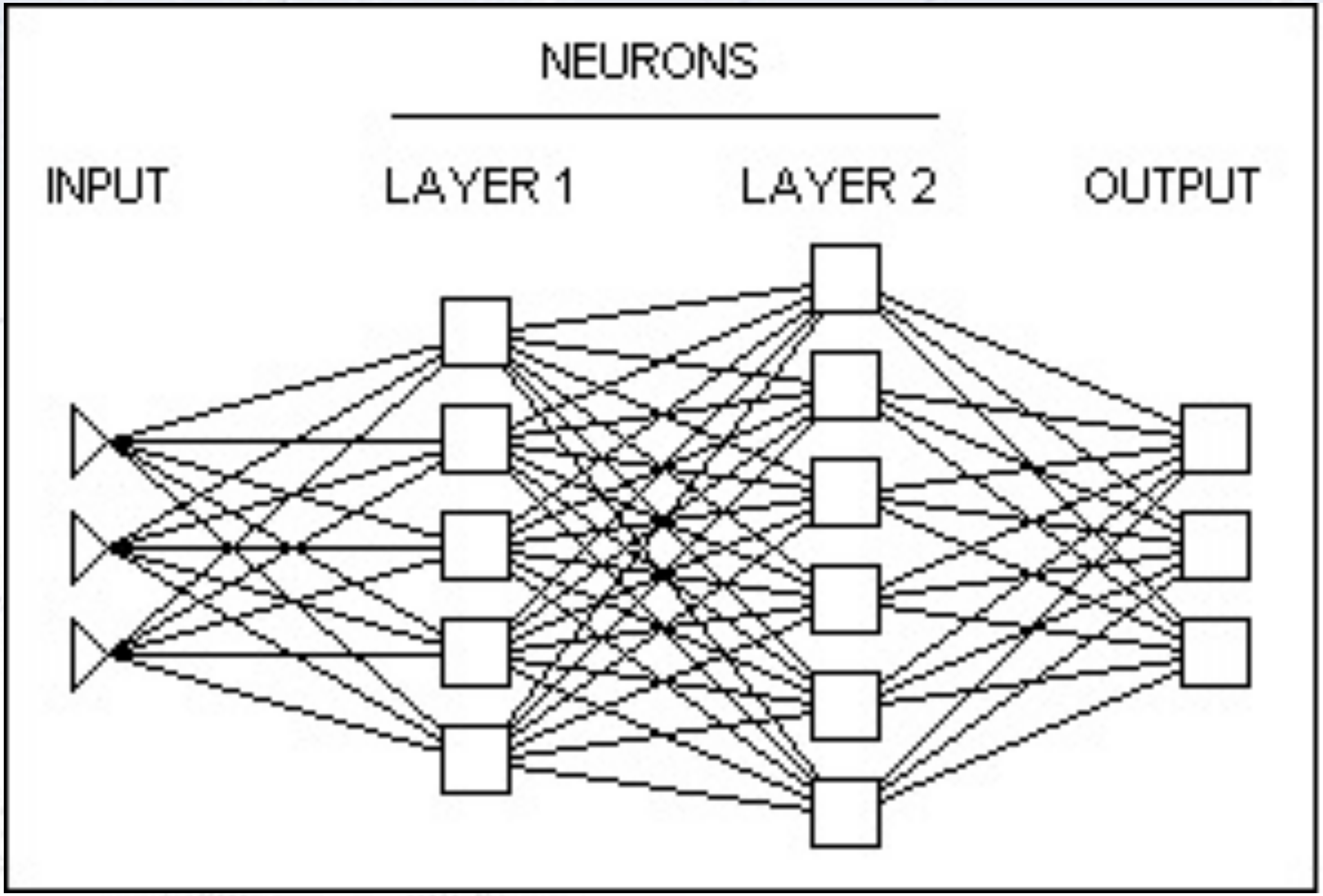
Cuts – in 2 dimensions



Not as simple as the 1 dimensional case!

Correlations now has to be taken into account.

Neural Networks

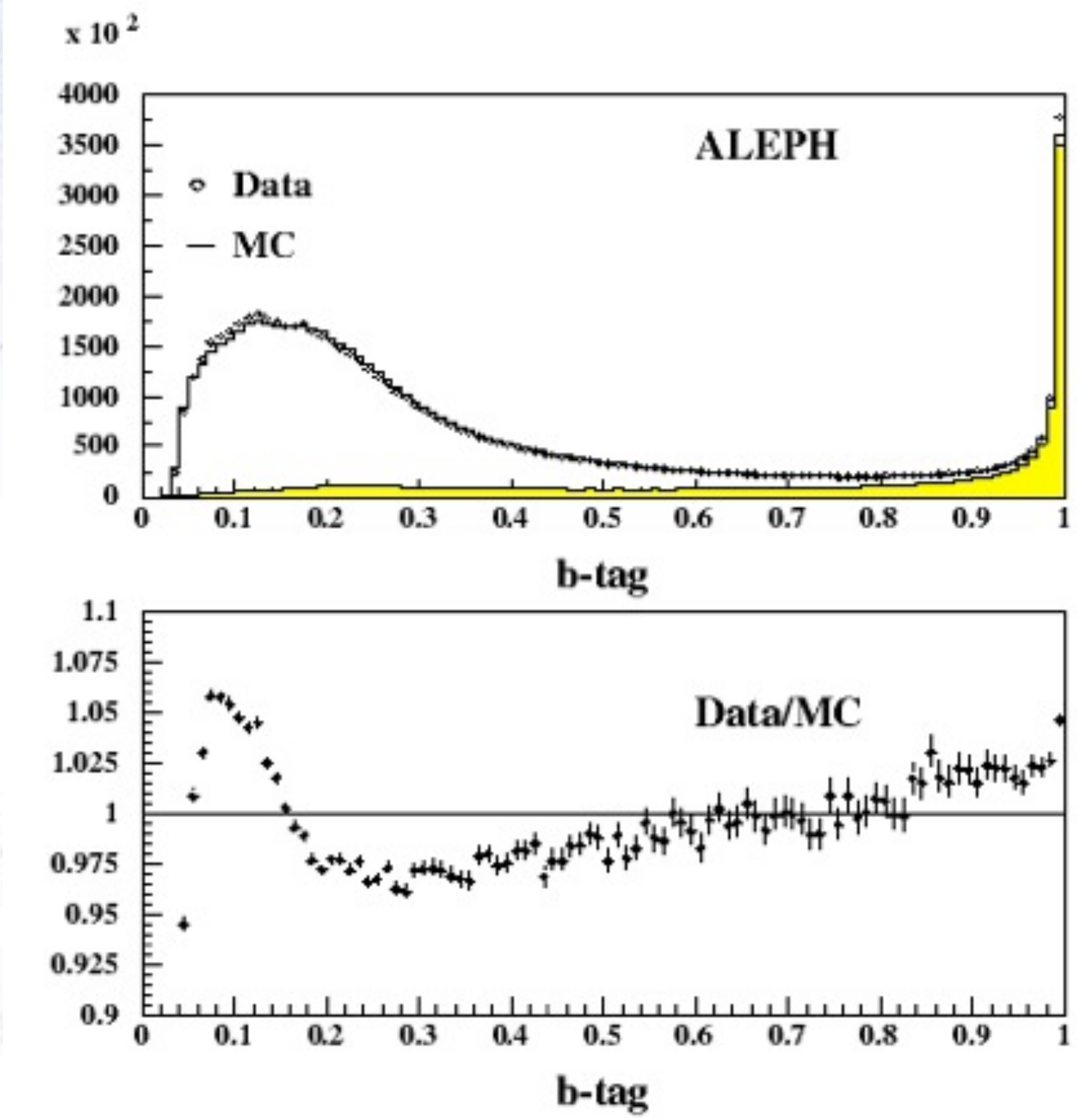


Neural Networks

An example from CERN is the ALEPH collaboration at LEP.

Used to determine if a jet is from a b-quark or not.

Very large statistics, and of very great importance.



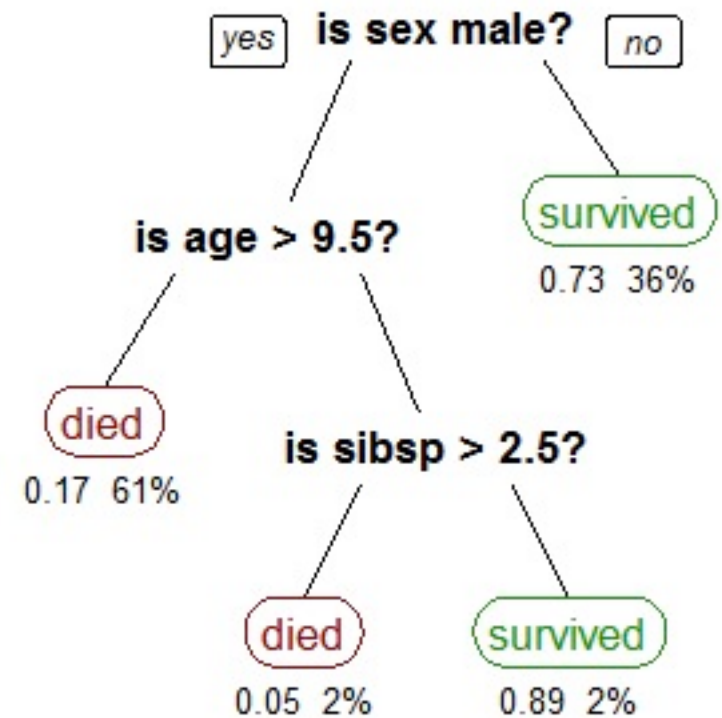
Boosted Decision Trees

Can become very complex.

Good for discrete problems.

Not always as efficient.

Boosting adds to separation.



Fisher's Exact Test

Suppose you have a (small) **contingency table**, that is an **m** by **n** table of counts:

	Men	Women	*Total*
Dieting	*a*	*b*	*a + b*
Non-dieting	*c*	*d*	*c + d*
Totals	*a + c*	*b + d*	*a + b + c + d (=n)*

If you want to test, if the rows and columns are independent, you use **Fisher's Exact Test**.

Fisher proved that the probability of obtaining the numbers a, b, c, and d when there is no dependence, are:

$$p = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}} = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!}$$