Problem Set

Summary of the problem set

General remarks

Problem 1.1

This problem was to exercise the basic PDF skills.

Almost all had it correct.

The answer can be "guessed" from simple principles.

$$f(t) = Ce^{-t/\tau} \quad t \in [t_0, \infty]$$

$$\int_{t_0}^{\infty} f(t)dt = 1 \Rightarrow$$

$$\int_{t_0}^{\infty} Ce^{-t/\tau} dt = 1 \Rightarrow$$

$$C = \frac{1}{\tau e^{-t_0/\tau}} \Rightarrow$$

$$\boxed{f(t) = \frac{1}{\tau} e^{-(t-t_0)/\tau}}$$

$$\langle t \rangle = \int_{t_0}^{\infty} t f(t) dt \Rightarrow$$

$$\langle t \rangle = \frac{1}{\tau} e^{-t_0/\tau} \int_{t_0}^{\tau} t e^{-t/\tau} dt \Rightarrow$$

$$\boxed{\langle t \rangle = \tau + t_0}$$

$$\langle t^2 \rangle = \int_{t_0}^{\infty} t^2 f(t) dt \Rightarrow$$

$$\langle t^2 \rangle = \frac{1}{\tau} e^{-t_0/\tau} \int_{t_0}^{\tau} t^2 e^{-t/\tau} dt \Rightarrow$$

$$\langle t^2 \rangle = 2\tau^2 + 2t_0\tau + t_0^2$$

$$\sigma = \sqrt{\langle t^2 \rangle - \langle t \rangle^2} \Rightarrow$$

$$\boxed{\sigma = \tau}$$

Problem 1.2

The PDF to use is a binomial (or Poisson in last problem).

Again, almost everybody solved it well.

Flipping coins follows an binomial distribution. So the chance of r successes in n trials and with a probability given by p is:

$$P(r;p,n) = p^r(1-p)^{n-r}\frac{n!}{r!(n-r)!} \quad (4)$$

The change of getting one head in one throw is 50 %, $p = 0.50$. So the probability of 14 heads or more ($20 > r > 14$) in 20 throws ($n = 20$) is given by the sum over the individual r's probabilities:

$$\sum_{r=14}^{20} P(r;p=0.5,20) = 0.0577 \quad (5)$$

So the probability of getting 14 or more heads in 20 throws is 5.77 %.
The change of getting 18 or more heads in twenty throws is given by the same distribution:

$$\sum_{r=18}^{20} P(r;p=0.5,20) = 0.000201 \quad (6)$$

So it is extremely low. Now the next I do is simply to calculate a new binomial with the new n ($n = 100$), p ($p = 0.000201$) and r ($r = 1$).

$$\sum_{r=1}^{100} P(r;p=0.000201,20) = 0.0199 \quad (7)$$

So barely two percent. Due to the low probability and high n, this could also have been calculated by the use of Poison statistics.

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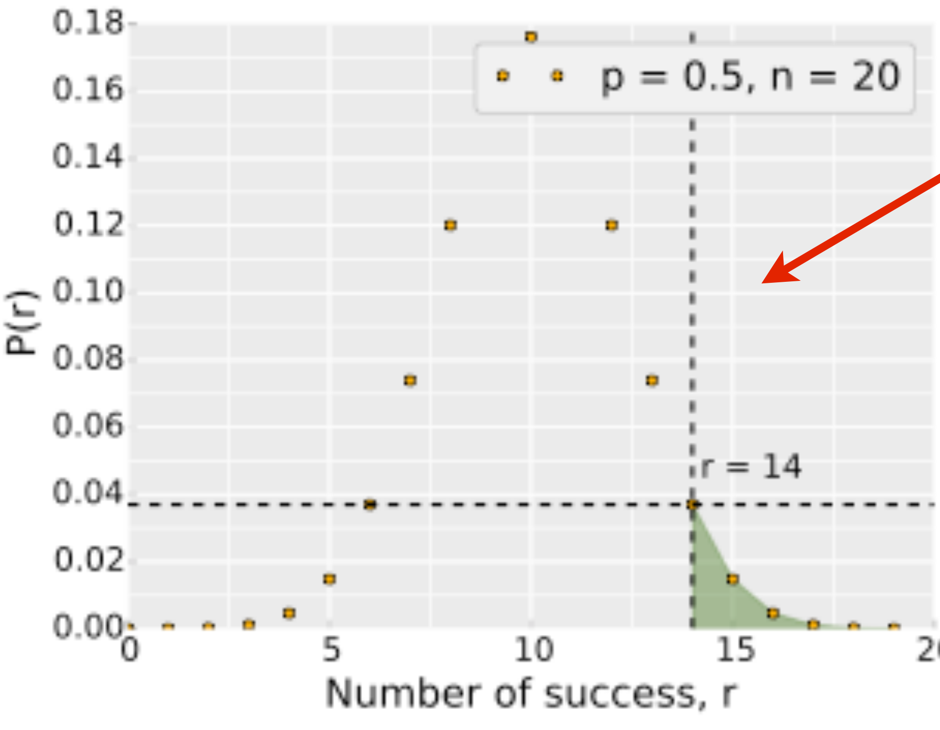

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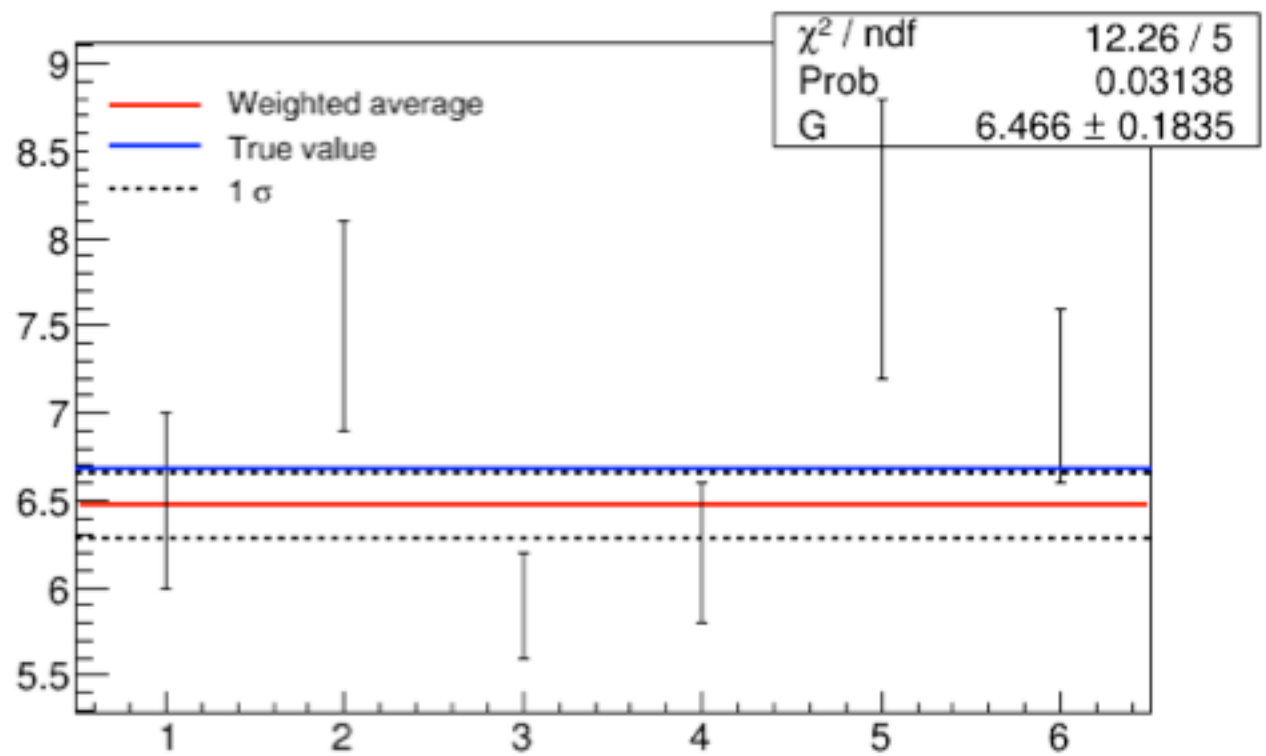
son statistics.

Problem 2.1

Combining measurements in a weighted average, with a Chi-square is a requirement!

p-value = 3.1% is suspicious, but not to be rejected ASAP.

Combination is in agreement with official G.

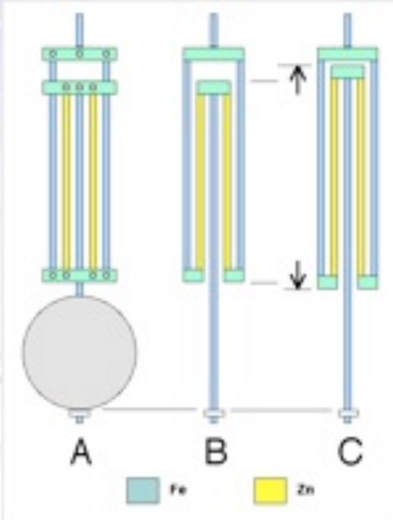


	Mean 10^{-11} m^3/kgs	Uncertainty 10^{-11} m^3/kgs	χ^2	Probability
Calculated	6.47	0.18	12.26	0.03
Fitted	6.47	0.18	12.26	0.03

The true value is $G = 6.67384 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, and therefore the measured value af G is 1.13 sigma away, which is within the 95 (which is a fair limit) % confidence level. This means that it matches the true value satisfactory.

Problem 2.2

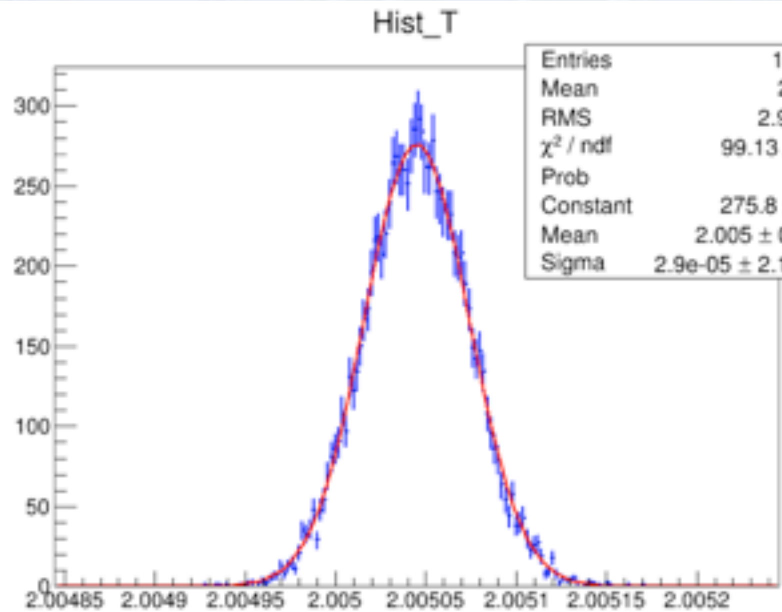
Problem might have benefitted from a figure:



Numbers chosen not to lead to too large improvement.

Most did OK...

Note: This problem could have been done/checked with MC!



$$\sigma_T = 3.14\sqrt{\frac{L}{9.82\ \mathrm{m/s^2}}}11.3\times10^{-5}\ \mathrm{m/mK}\cdot 2.5\ \mathrm{K}$$

$$\sigma_T = 2.83\times10^{-5}\sqrt{L}\ \frac{\mathrm{s}}{\sqrt{\mathrm{m}}}$$

$$\sigma_L^2 = \left(\frac{\partial L_I}{\partial T}\sigma_t\right)^2 + \left(\frac{\partial L_B}{\partial\sigma_t T}\right)^2 - 2\left(\frac{\partial L_I}{\partial T}\right)\left(\frac{\partial L_B}{\partial T}\right)\sigma_t^2$$

$$\sigma_L^2 = (1.6L\alpha\sigma_t)^2 + (0.6L\beta\sigma_t)^2 - 2\cdot1.6\cdot0.6L^2\alpha\beta\sigma_t^2$$

$$\sigma_L = L\sigma_t(1.6\alpha - 0.6\beta)$$

$$\sigma_T = \pi\sqrt{\frac{L}{g}}\left(1.6\alpha - 0.6\beta\right)\sigma_t$$

$$\sigma_T = 1.7\times10^{-5}\sqrt{L}\frac{\mathrm{s}}{\sqrt{\mathrm{m}}}$$

Problem 2.3

2.3 The index of refraction for sugar solution in water is measured to be:

$$n_s = n_{air} \frac{\sin \theta_{air}}{\sin \theta_s} = 1 \cdot \frac{\sin 25.21^\circ}{17.91^\circ} = \underline{1.385}$$

with an error calculated to be:

$$\sigma_s = \left[\left(n_{air} \frac{\cos \theta_{air}}{\sin \theta_s} \sigma_{\theta,air} \right)^2 + \left(n_{air} \frac{\cos \theta_s \sin \theta_{air}}{\sin^2 \theta_s} \sigma_{\theta,s} \right)^2 \right]^{1/2}$$
$$= \underline{0.008}$$

The percentage of the solution assuming a linear interpolation is given by $P = \frac{75(n-1.3330)}{1.4774-1.3330}$ where n is the index of refraction and P is the percentage of the solution. For the above numbers we get:

$$P = \frac{75(n_s - 1.3330)}{1.4774 - 1.3330}$$
$$= \underline{27\%}$$
$$\sigma_P = \frac{75}{1.4774 - 1.3330} \cdot 0.008$$
$$= \underline{4\%}$$

Problem 3.1

Normalization and transformation method.

I was happy to see, that almost all of you can now produce numbers according to any distribution.

$$1 = \int_1^\infty C x^{-3} \mathrm{dx} \Updownarrow$$

$$1 = \left[-C \frac{1}{2} x^{-2} \right]_1^\infty \Updownarrow$$

$$C = 2$$

$$f(x) = 2x^{-3}$$

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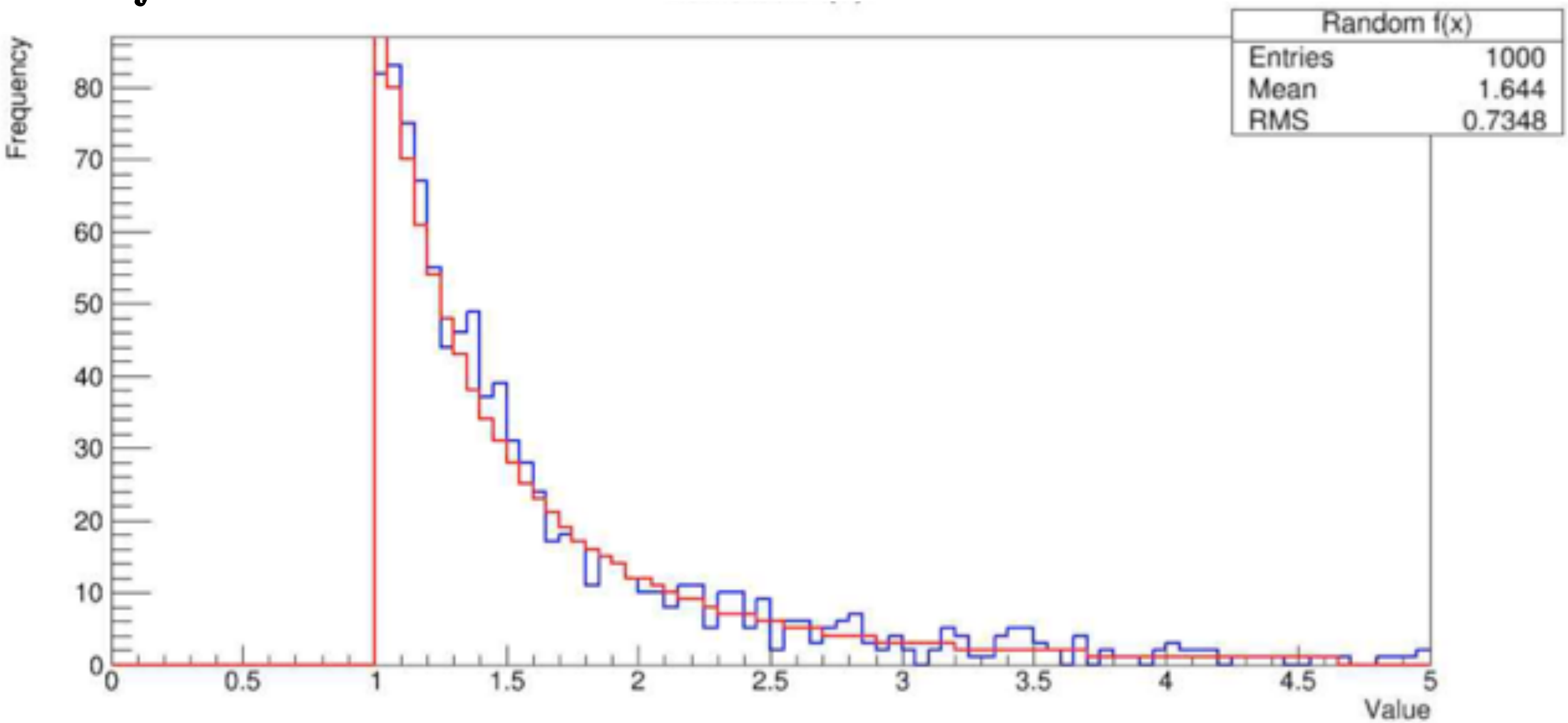
$$F(x) = \int_1^{x'} 2x^{-3} = -x'^{-2} + 1$$

$$u = F(x) = -x'^{-2} + 1$$

$$x' = \frac{1}{\sqrt{1-u}}$$



Problem 3.2

$$\langle R\rangle = 3\sum_{i=1}^{10}\frac{i}{10} + 6\sum_{j=1}^{8}\frac{j}{8} + 10\sum_{k=1}^{6}\frac{k}{6} \Rightarrow$$

$$\boxed{\langle R\rangle = \frac{157}{2} = 78.5}$$

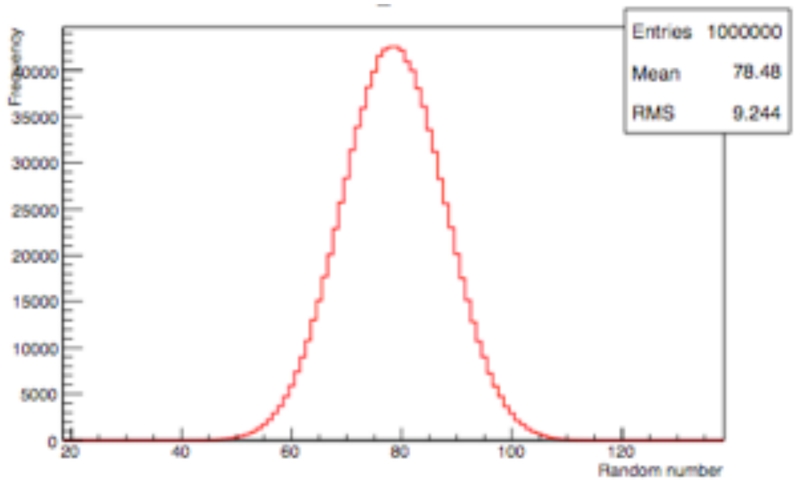
While this problem can in principle be calculated by hand (i.e. analytically), Monte Carlo is superior (or at least faster).

$$\sigma^2 = 3\left(\sum_{i=1}^{10}\frac{i^2}{10} - \left(\sum_{i=1}^{10}\frac{i}{10}\right)^2\right) + 6\left(\sum_{j=1}^{8}\frac{j^2}{8} - \left(\sum_{j=1}^{8}\frac{j}{8}\right)^2\right) + 10\left(\sum_{k=1}^{6}\frac{k^2}{6} - \left(\sum_{k=1}^{6}\frac{k}{6}\right)^2\right) \Rightarrow$$

$$\boxed{\sigma = 9.24}$$

$$P_{gaus}(x \geq 100, \mu = 78.5, \sigma = 9.242) = \int_{100}^{\infty} \frac{1}{\sqrt{2\cdot\pi * 9.242}} e^{\frac{(x-78.5)^2}{2\cdot 9.242^2}} dx$$

$$= 0.01000$$



Despite being discreet distributions, central limit theorem works well.

However, not perfectly...

$$P_{sum \geq 100} = 1.10 \pm 0.01\%$$

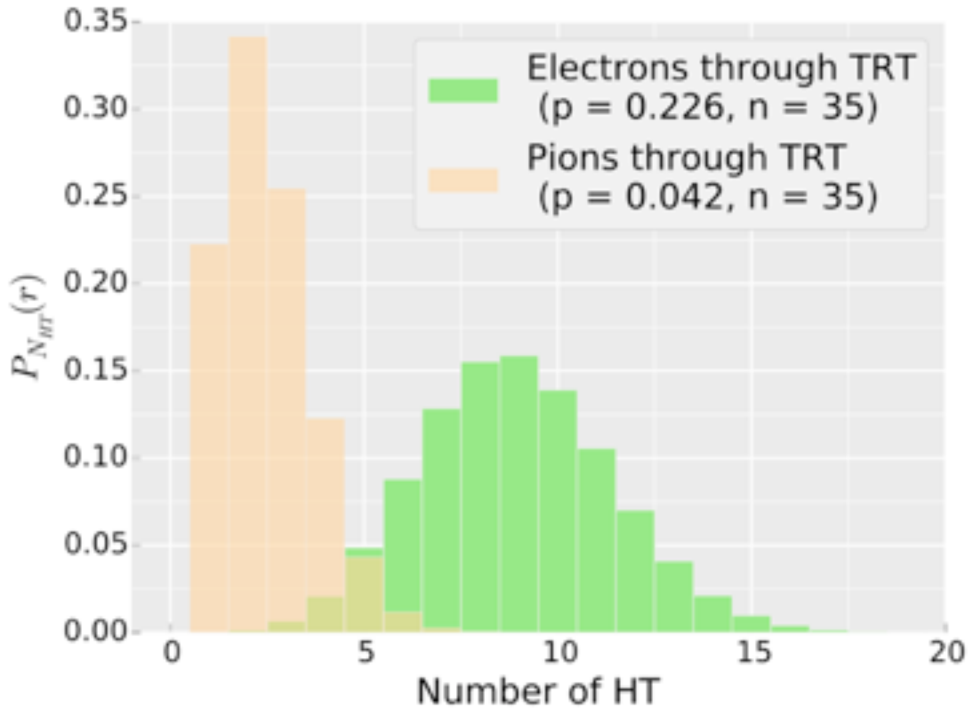
Problem 4.1

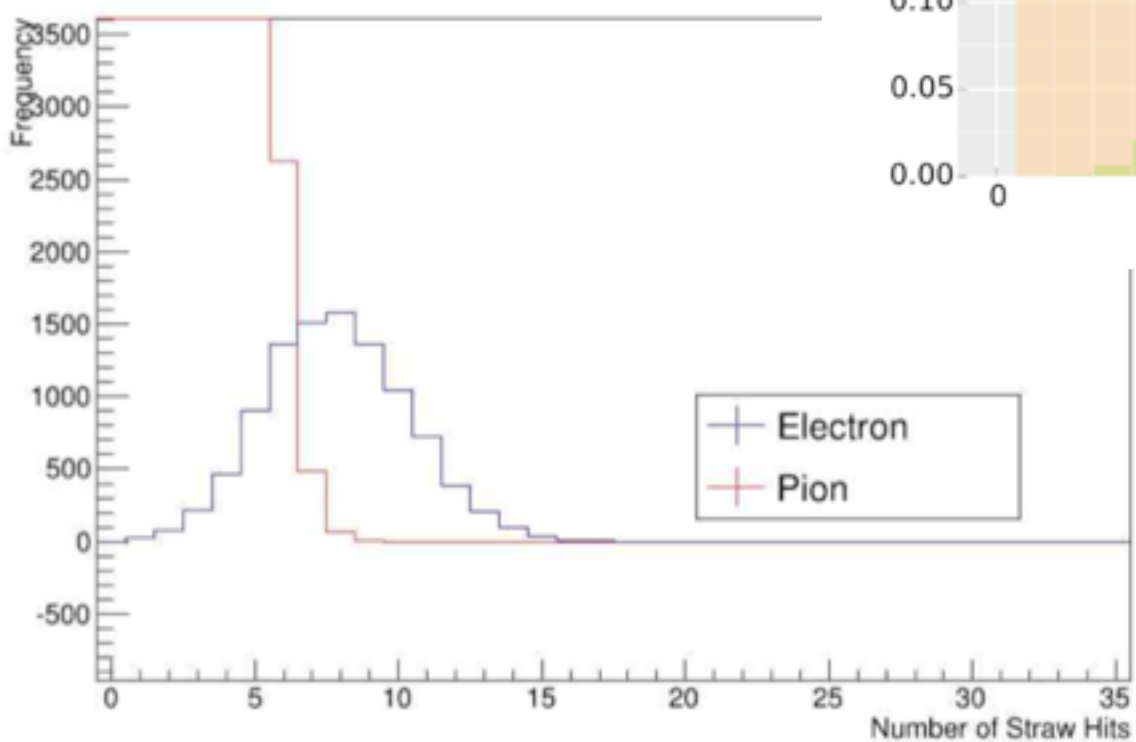
$$P_{N_{HT}>6} = \sum_{r=7}^{35} p_e^r (1-p_e)^{35-r} \frac{35!}{r!\,(35-r)!} \Rightarrow$$

$$\boxed{P_{N_{HT}>6} = 0.706}$$

$$P_{N_{HT}>6} = \sum_{r=7}^{35} p_p^r (1-p_p)^{35-r} \frac{35!}{r!\,(35-r)!} \Rightarrow$$

$$\boxed{P_{N_{HT}>6} = 0.000548}$$





The typical situation is a majority of pions to be rejected.

As it turns out, one gets 80% purity for N(HT) ≥ 7

Problem 5.1

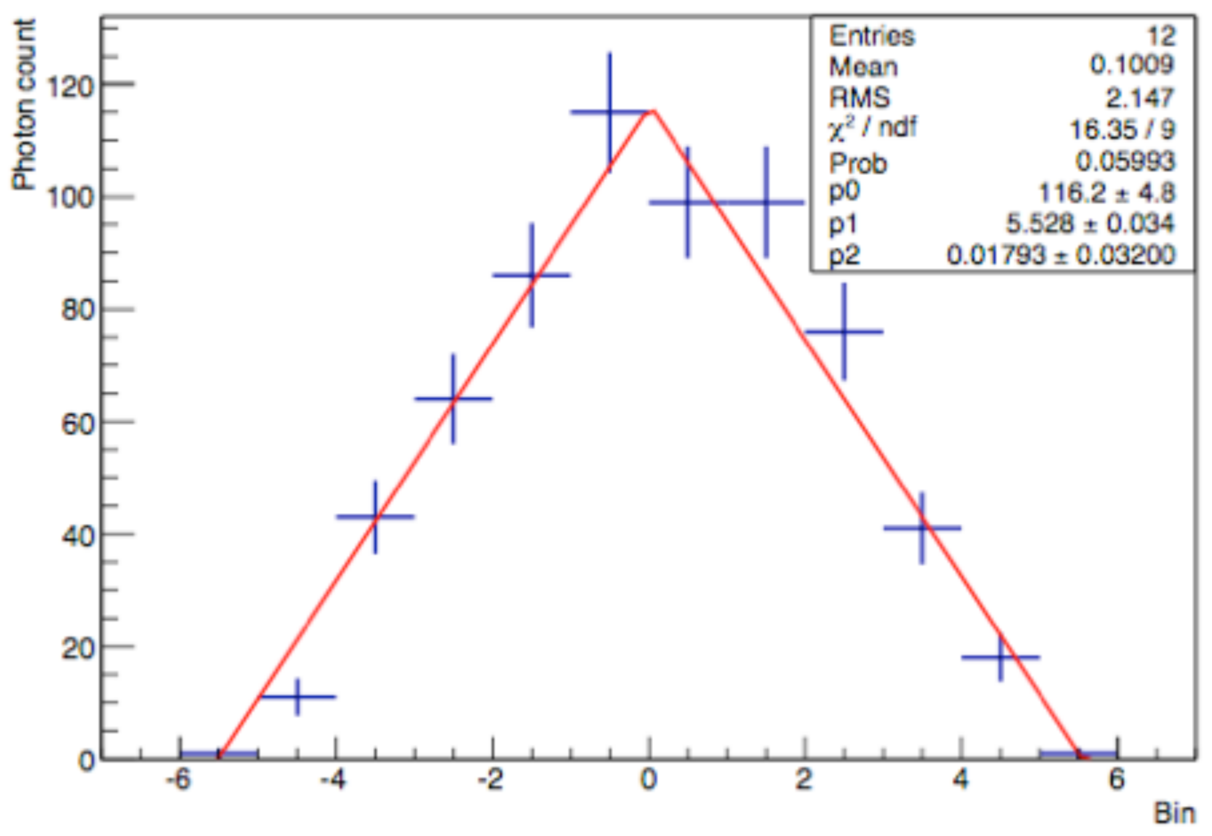
Looking at the fit for the data in Figure 5 we can see that we get χ^2 of 31.29 with 9 degrees of freedom. Yielding a χ^2-probability of 0.0003, not a good fit, I'm sure that we can do better! Lets look!

TROELS HAR UDFORDRET DIG TIL D-D-D-D-D-DUUUUUEEEEEELLL! "Using alternative hypotheses with a maximum of 3 parameters, make my day!" - Troels C. Petersen

The mean is consistent with zero: 0.10 ± 0.08

The Gaussian fit is easy, but doesn't fit: p = 0.0003

Note that any PDF can be assumed to have a mean of zero!



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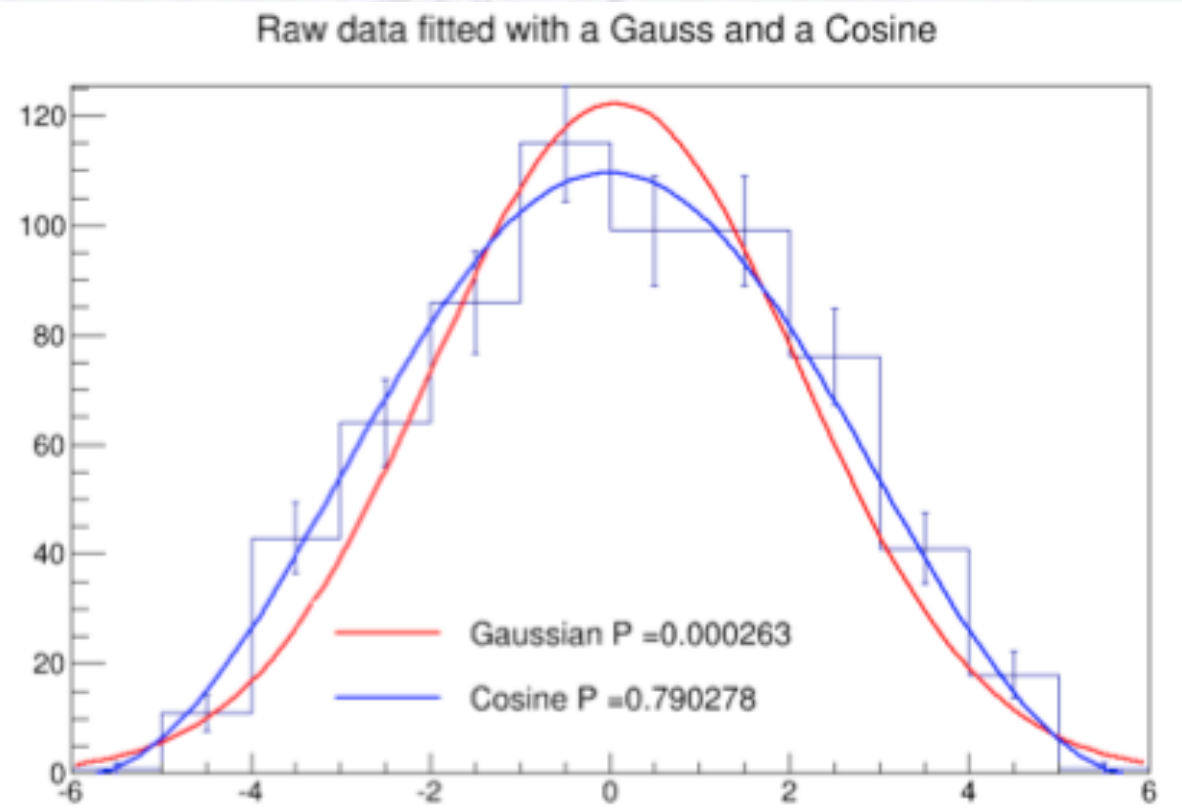
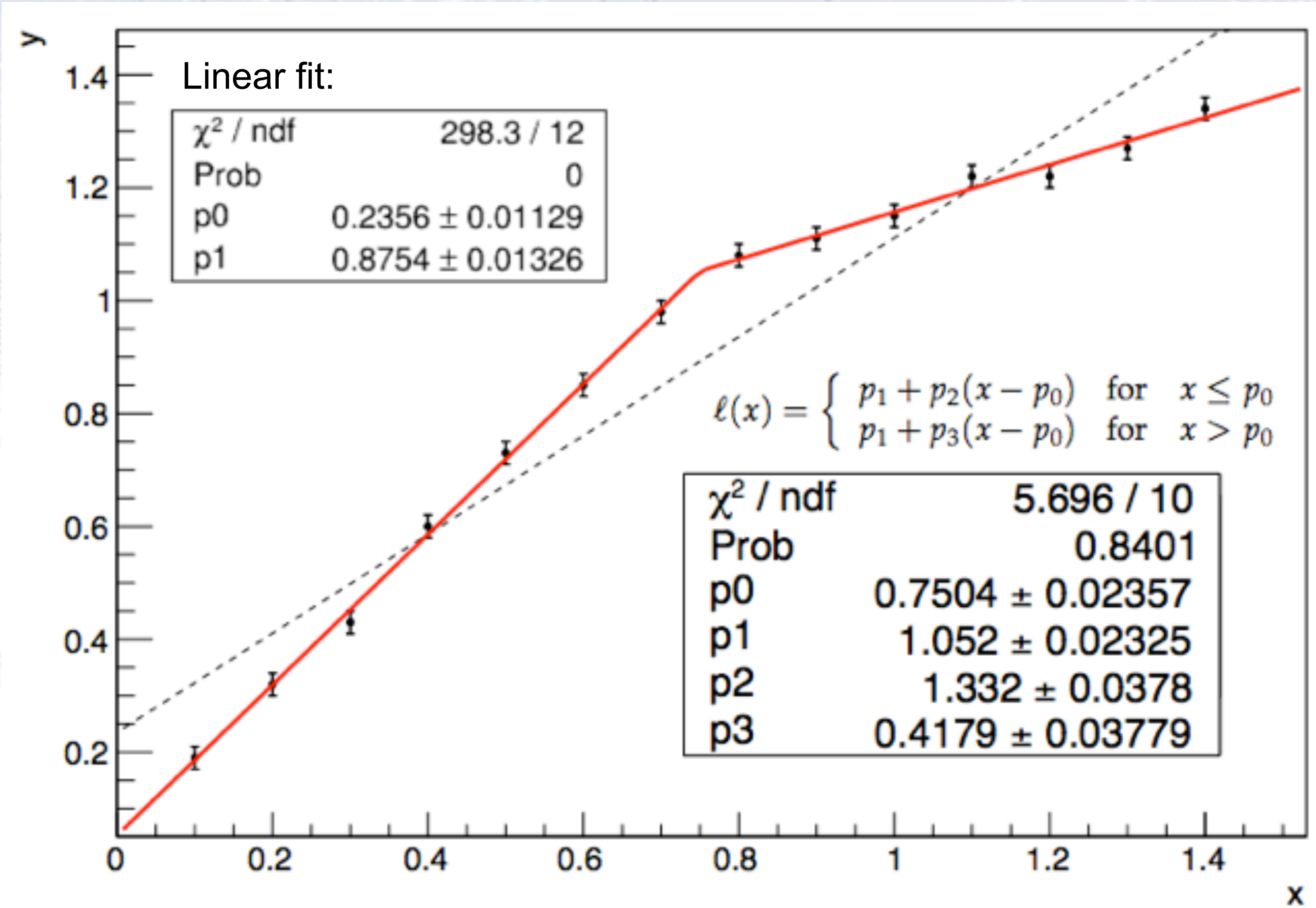


Problem 5.2



Linear fit:
χ² / ndf 298.3 / 12
Prob 0
p0 0.2356 ± 0.01129
p1 0.8754 ± 0.01326
$$\ell(x) = \begin{cases} p_1 + p_2(x - p_0) & \text{for} \quad x \le p_0 \\ p_1 + p_3(x - p_0) & \text{for} \quad x > p_0 \end{cases}$$
χ² / ndf 5.696 / 10
Prob 0.8401
p0 0.7504 ± 0.02357
p1 1.052 ± 0.02325
p2 1.332 ± 0.0378
p3 0.4179 ± 0.03779
y
x

Final Comments