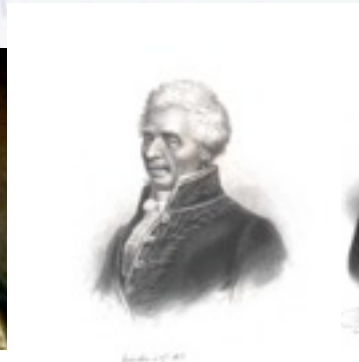
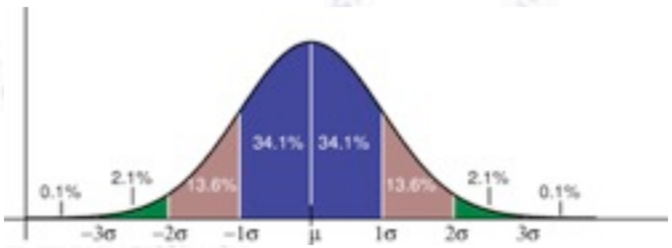
Applied Statistics

Central Limit Theorem



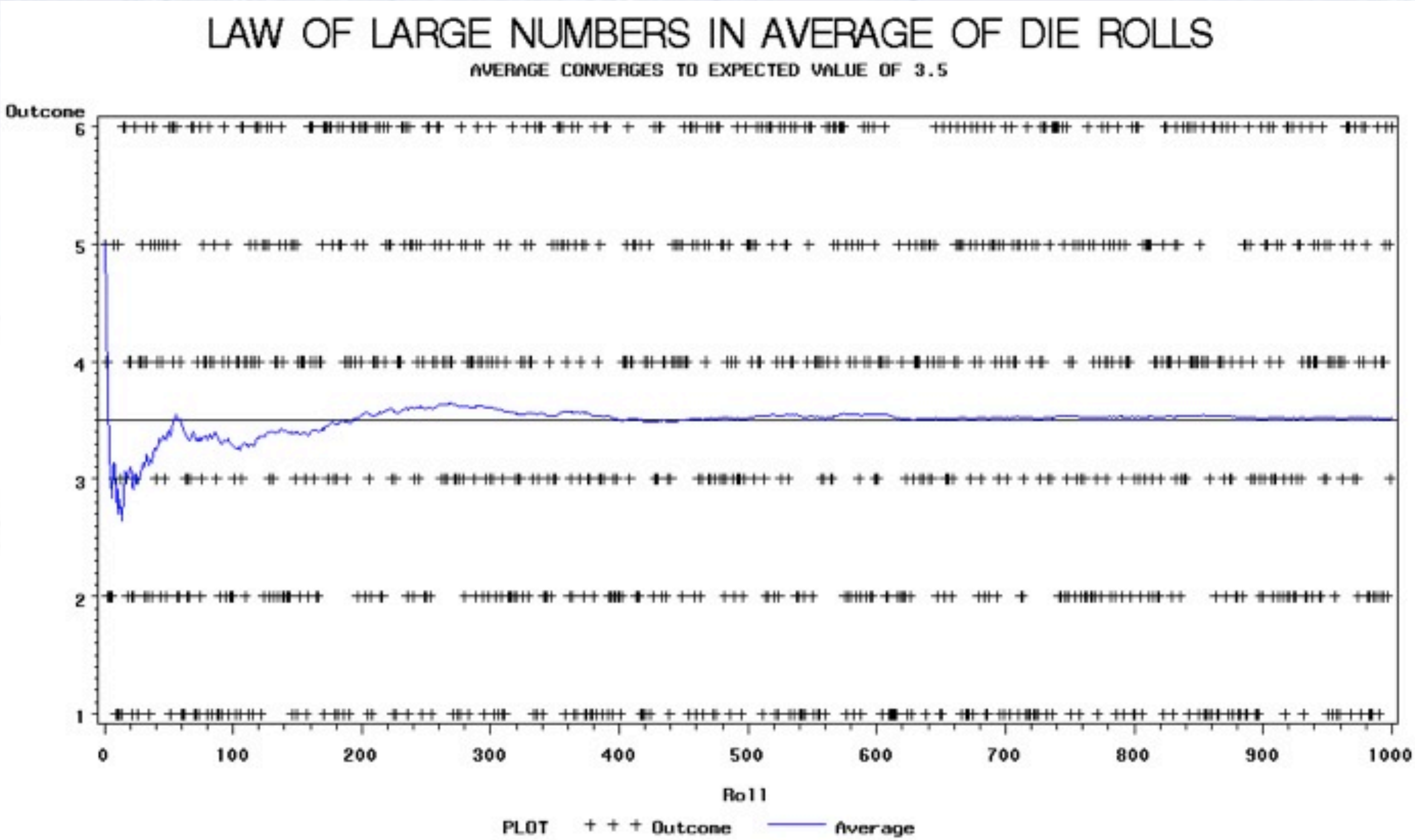



Troels C. Petersen (NBI)

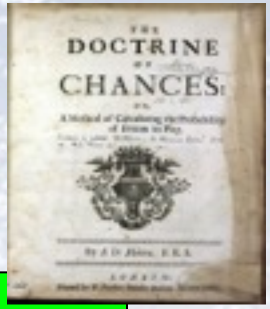


"Statistics is merely a quantization of common sense"

Law of large numbers



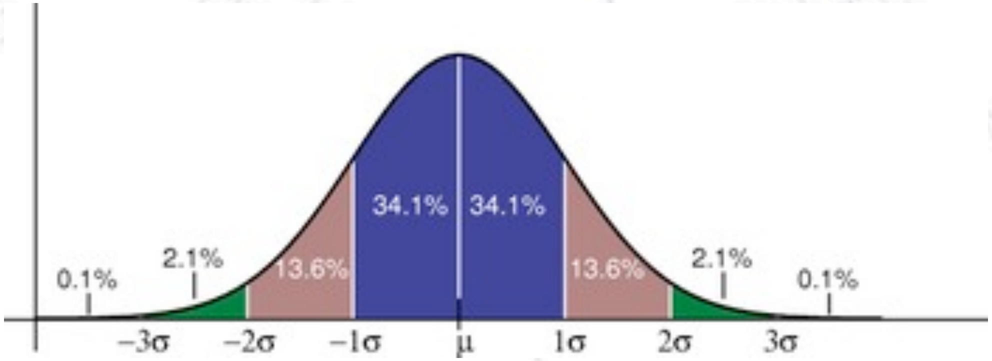
Central Limit Theorem

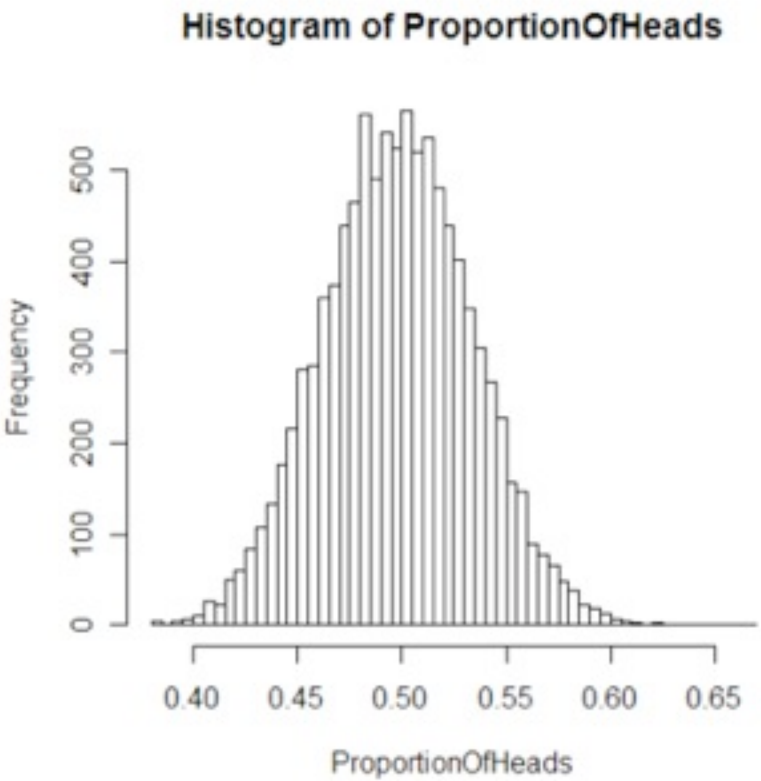


<u>Central Limit Theorem:</u>
The sum of N *independent* continuous random variables x_i with means μ_i and variances σ_i^2 becomes a Gaussian random variable with mean $\mu = \Sigma_i \mu_i$ and variance $\sigma^2 = \Sigma_i \sigma_i^2$ in the limit that N approaches infinity.

This holds under fairly general conditions, which means that the Gaussian distribution takes a central role in statistics.

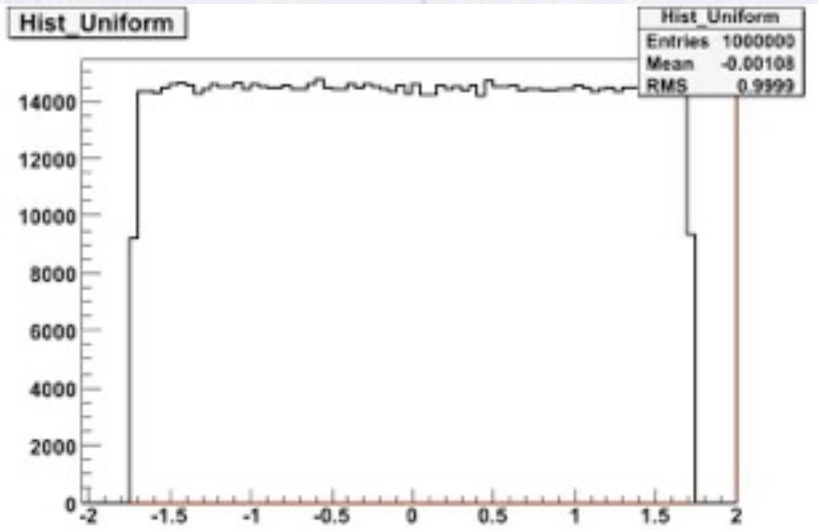
The Gaussian is "the unit" of distributions!

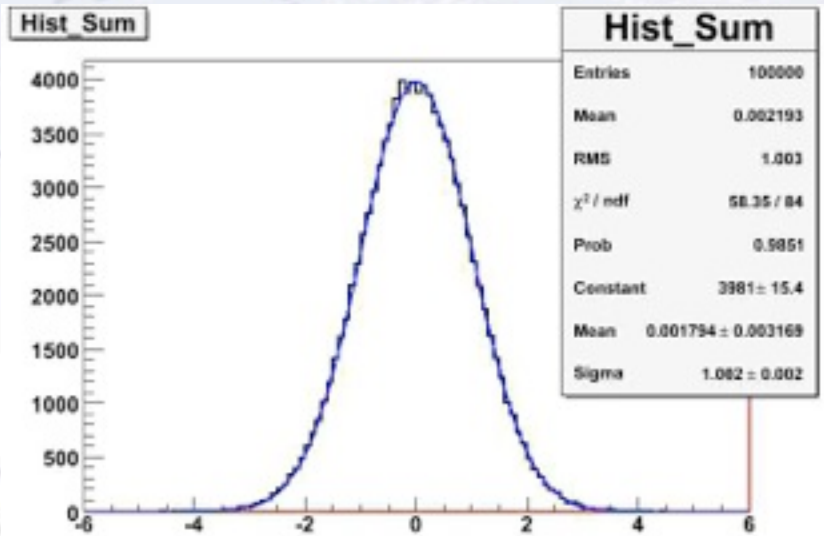




Example of Central Limit Theorem

Take the sum of 100 uniform numbers! Repeat 100000 times to see what distribution the sum has…



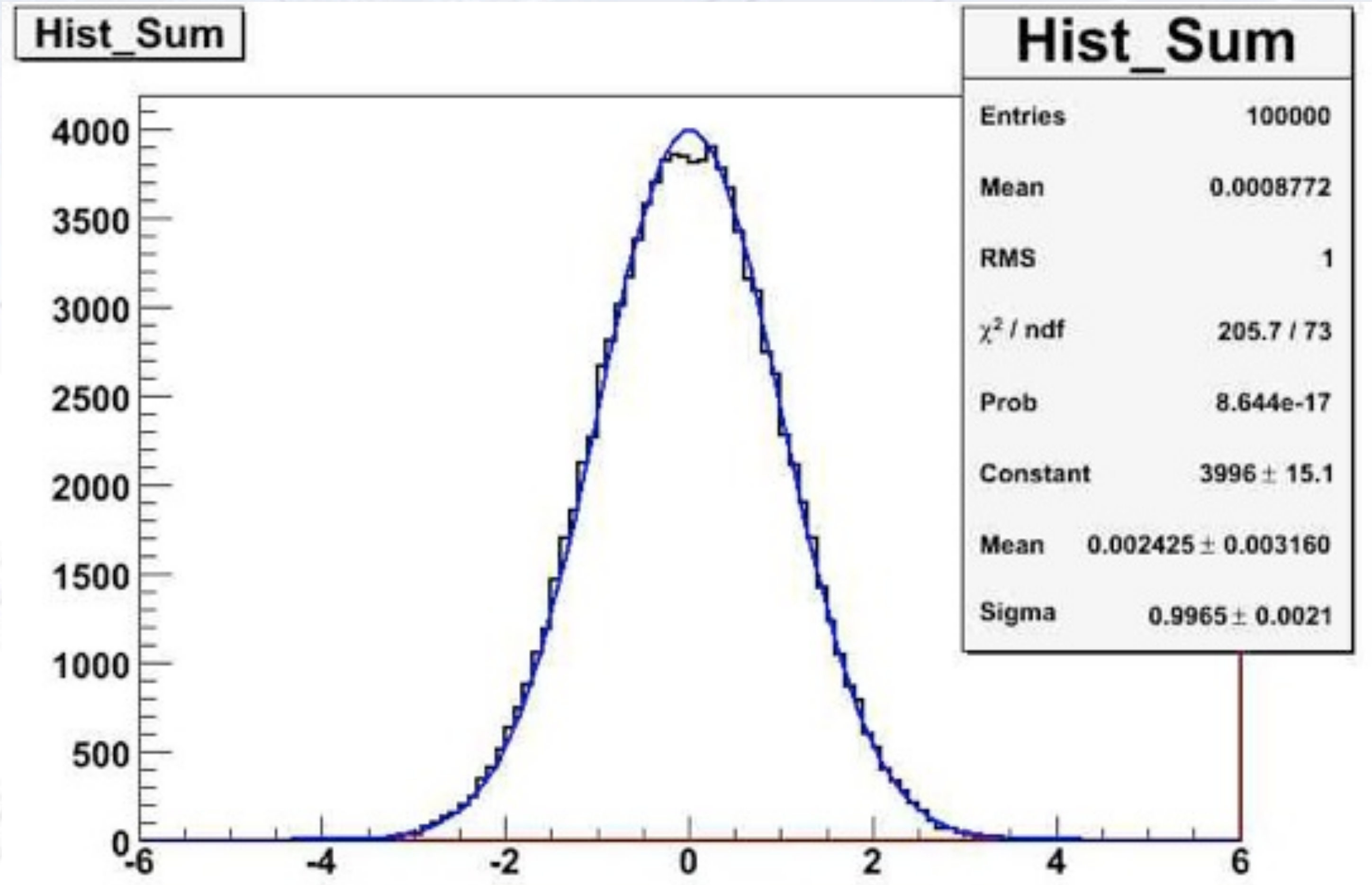


The result is a bell shaped curve, a so-called **normal** or **Gaussian** distribution.

It turns out, that this is very general!!!

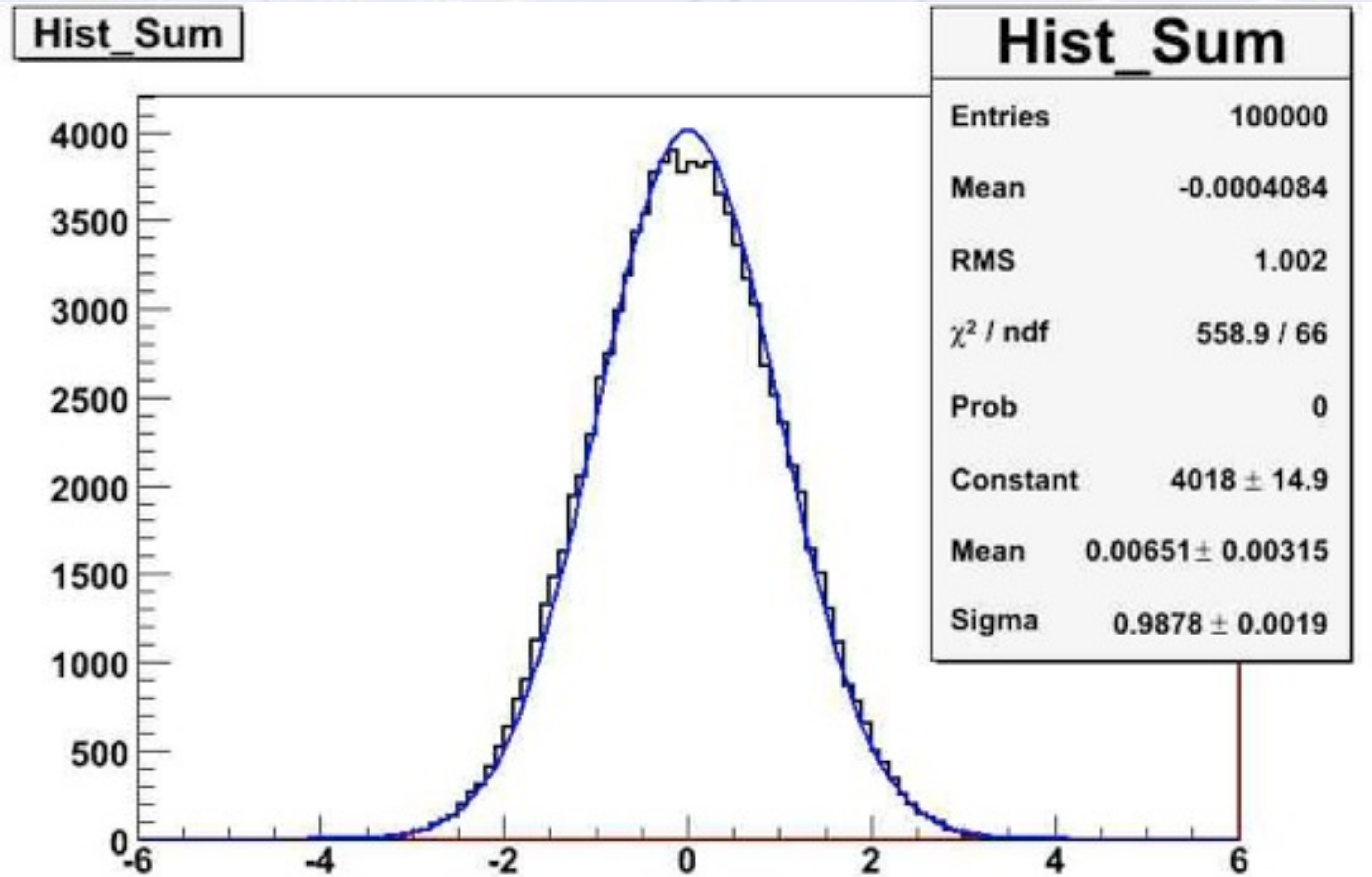
Example of Central Limit Theorem

Now take the sum of just **10** uniform numbers!



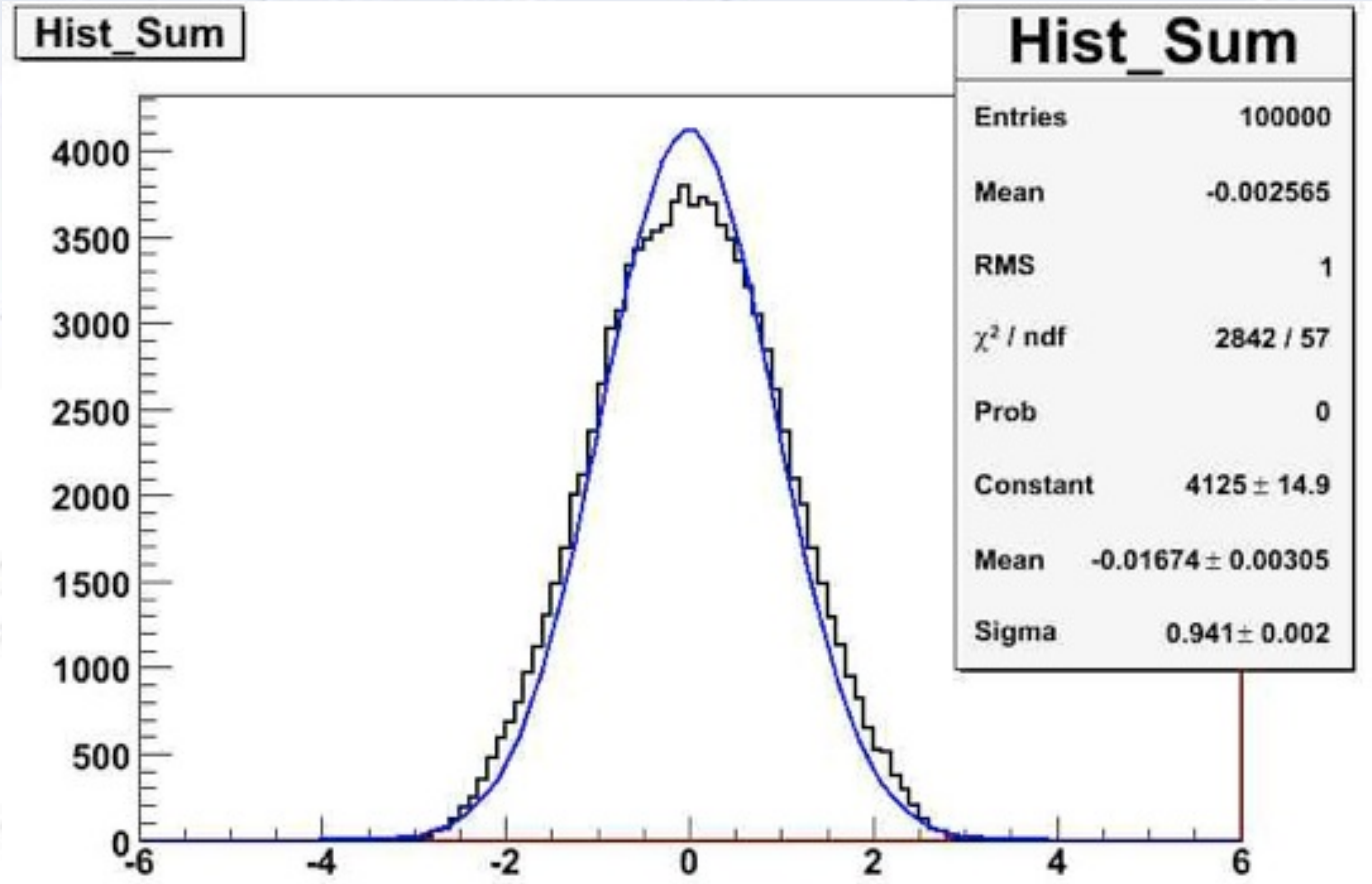
Example of Central Limit Theorem

Now take the sum of just **5** uniform numbers!



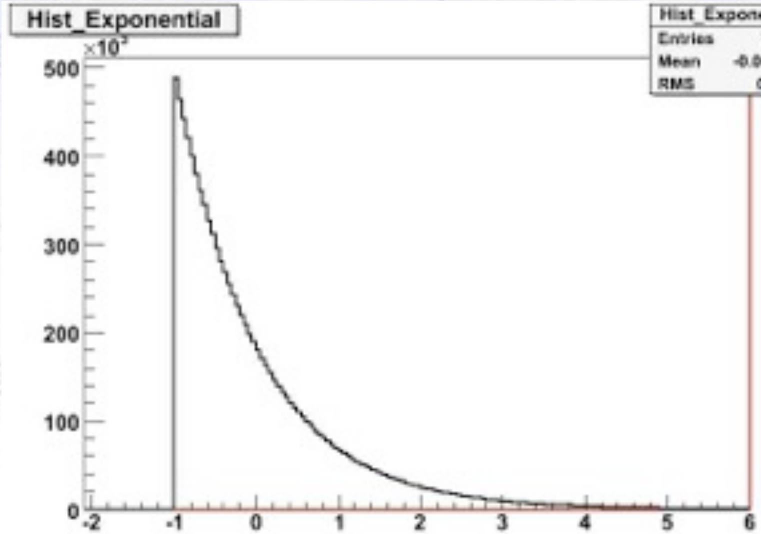
Example of Central Limit Theorem

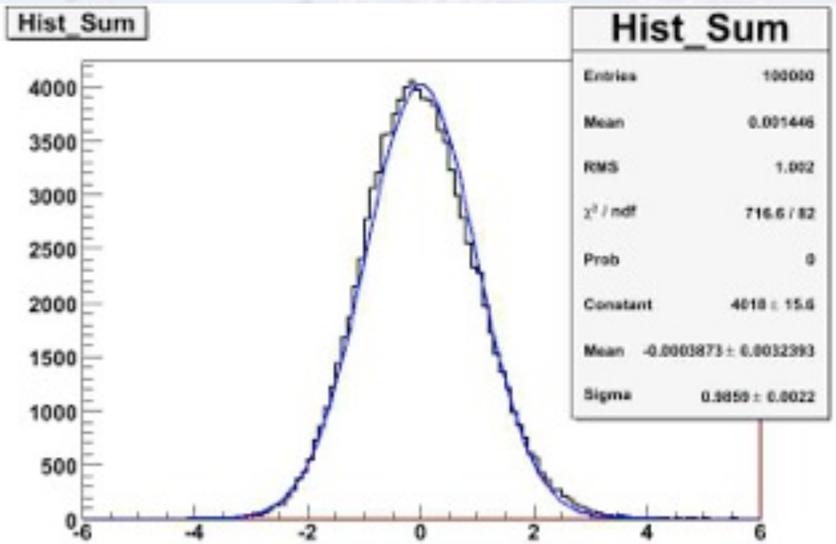
Now take the sum of just **3** uniform numbers!



Example of Central Limit Theorem

This time we will try with a much more "**nasty**" function. Take the sum of 100 *exponential* numbers! Repeat 100000 times to see the sum's distribution…





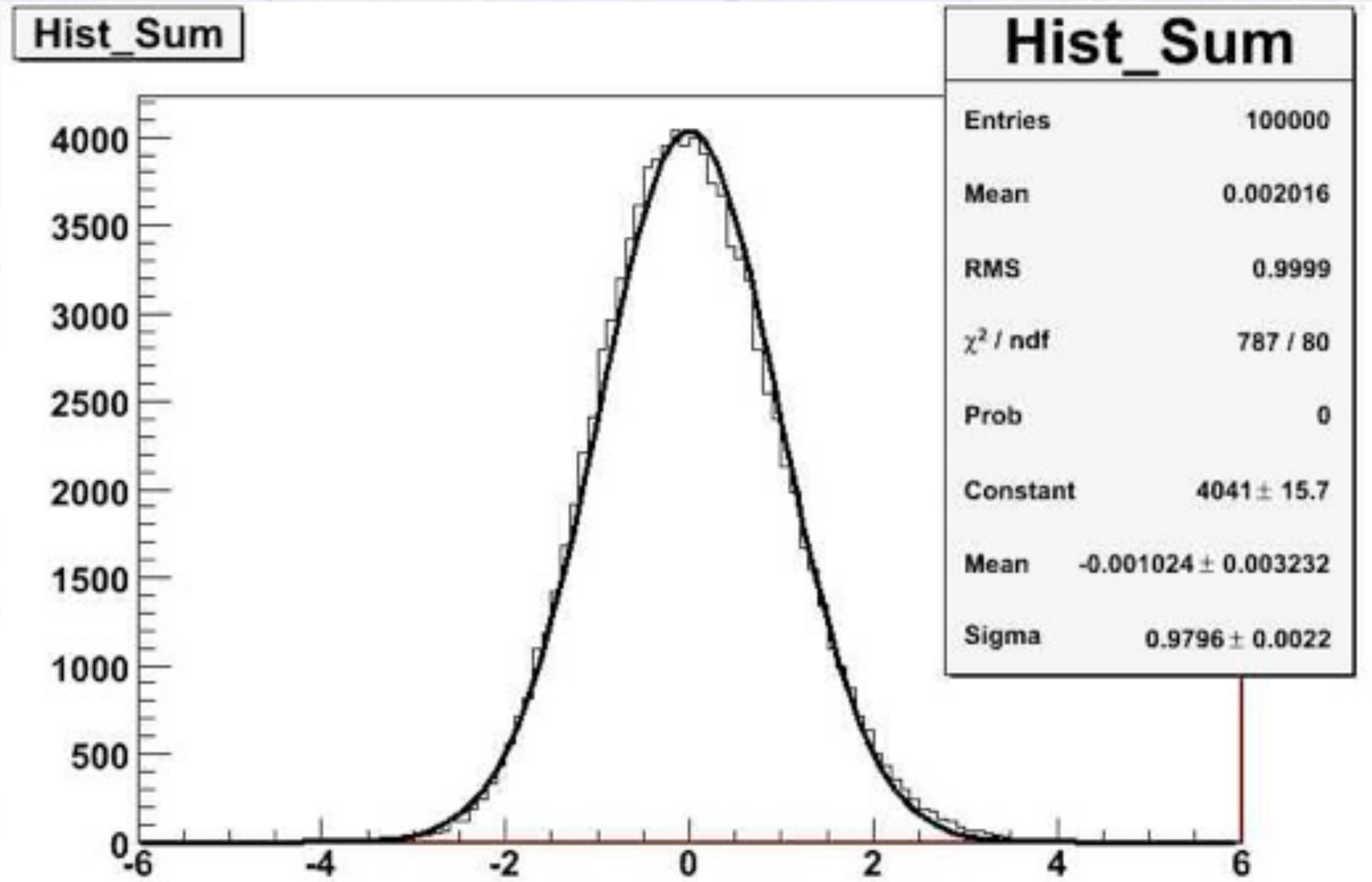
Even with such a non-Gaussian skewed distribution, the sum quickly becomes

Gaussian!!!

It turns out, that this fact saves us from much trouble: Makes statistics "easy"!

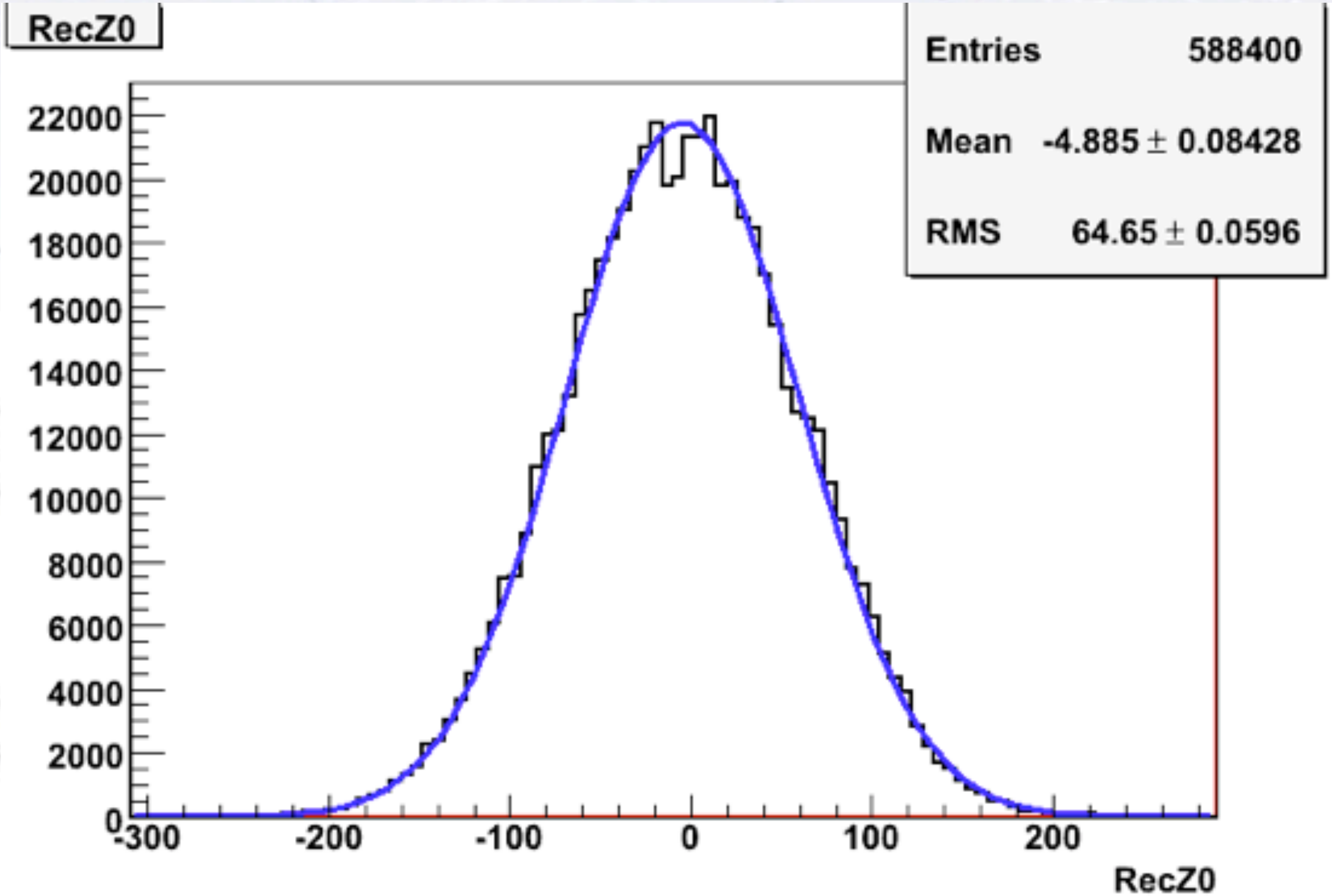
Example of Central Limit Theorem

Generally, measurements are the result of many different influences from various distributions! Here **10** uniform numbers and **10** exponential numbers:

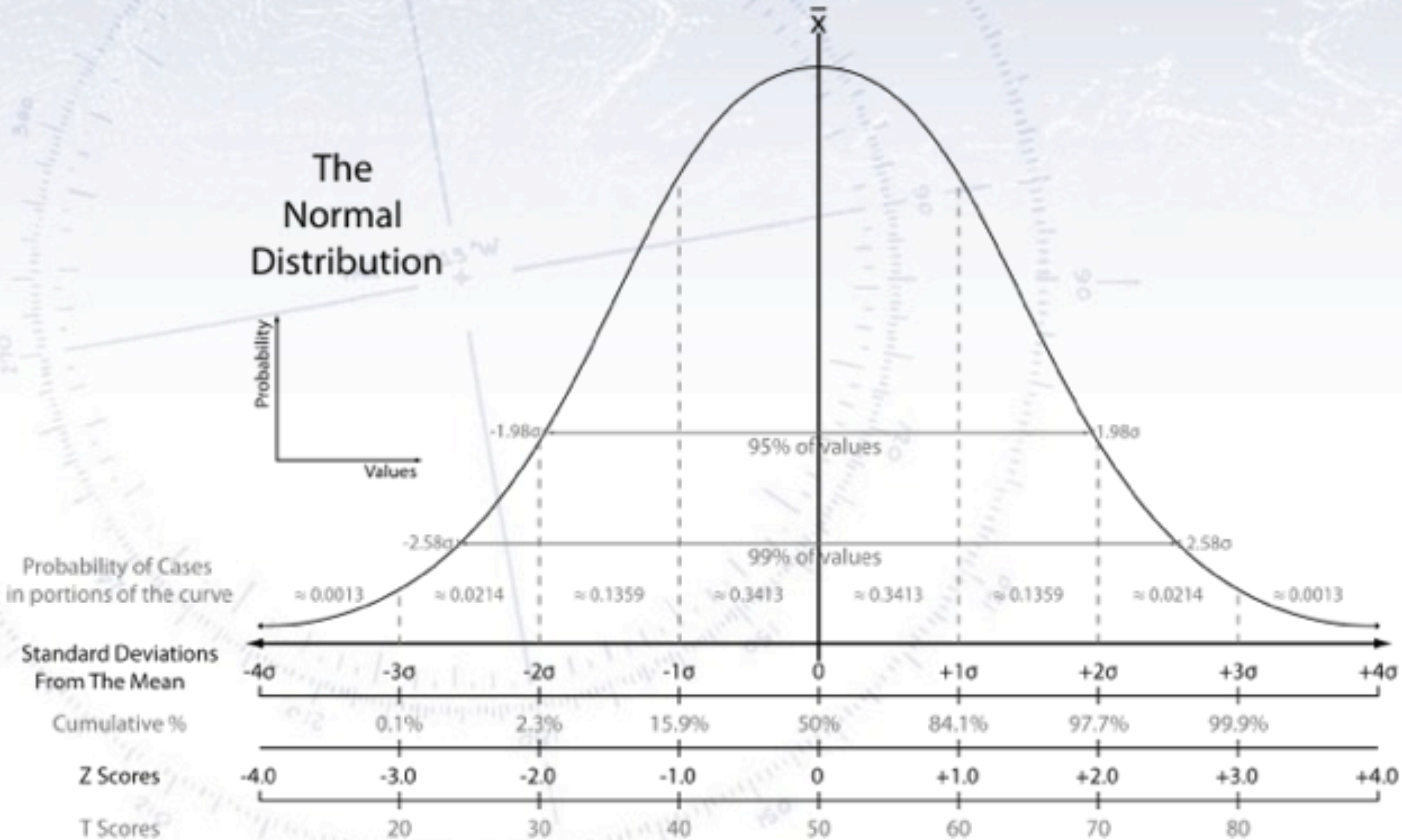


Example of Central Limit Theorem

Looking at z-coordinate of tracks at vertex from proton collisions in CERNs LHC accelerator by the ATLAS detector, this is what you get:



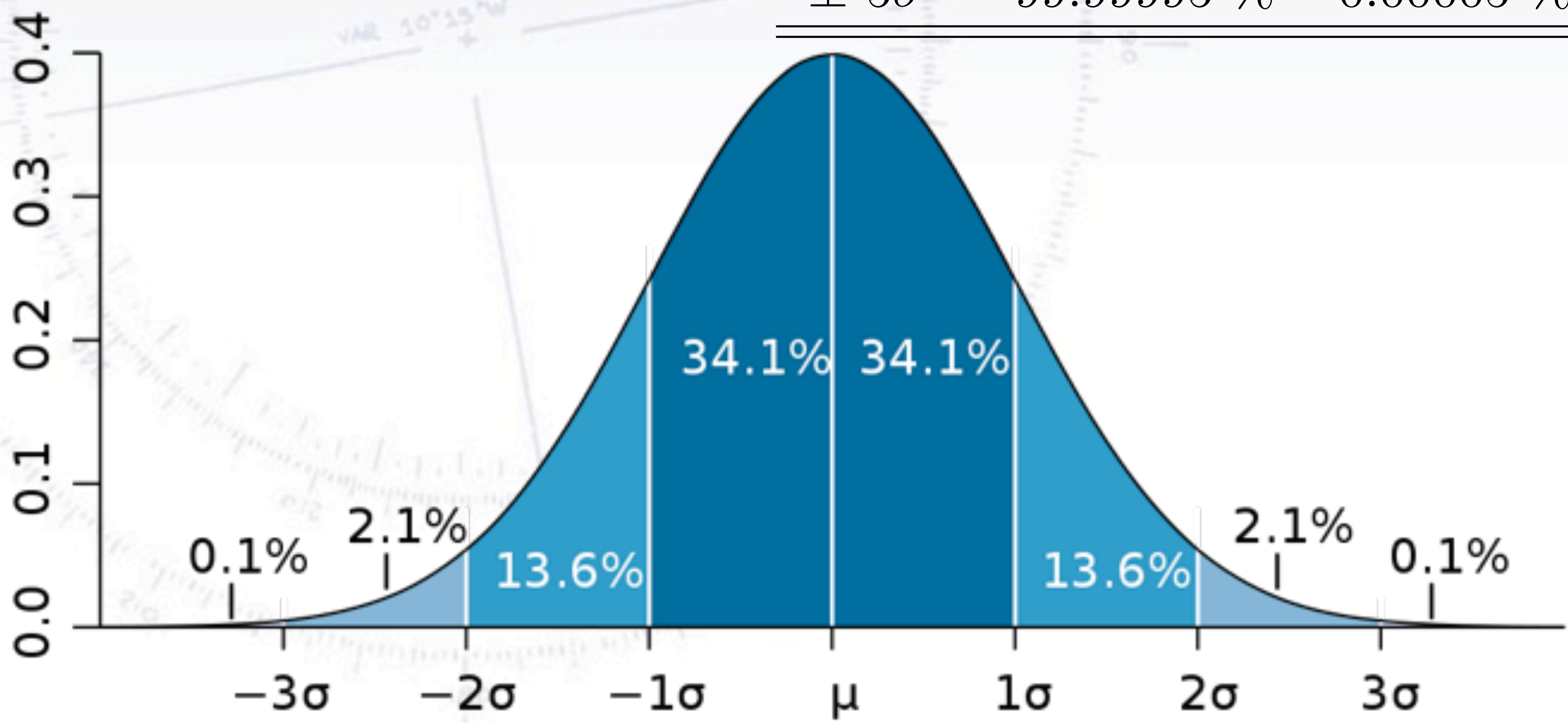
Central Limit Theorem



Central Limit Theorem

Never mind the details.
Below is what you should remember:

Range	Inside	Outside
$\pm 1\sigma$	**68** %	32 %
$\pm 2\sigma$	**95** %	5 %
$\pm 3\sigma$	**99.7** %	0.3 %
$\pm 5\sigma$	99.99995 %	0.00005 %



Central Limit Theorem

...is your good friend because it...

Ensures that uncertainties tend to be Gaussian!